Physics 3A: Energy
- So far we have explored motion using velocity, acceleration, forces
  - Forces cause changes in motion => accelerations which are changes in velocities.
- We can apply another approach to motion which in many cases is much simpler than forces, accelerations, etc.
  - This uses the concept of energy
  - It won't tell us all the details, but is still very useful
- To use energy, we must separate the things we want to analyze from those outside that we don't want to analyze.
  - Define the System as being the things we want to study.
  - A single object or particle
  - A collection of objects or particles
  - A region of space
  - It can vary in size and shape
  - Define the environment to be everything else outside the system

Physics 3A: Energy
- Work
  - We use the term work frequently in everyday life
  - In physics we define work differently and more precisely
  - Consider: You just ran out of gas and have to push your car to the gas station.
    - Case 1:
      - You push your car with a force of 50 N for 100 m.
      - Physics says you just did a significant amount of work "on the car"
    - Case 2:
      - You push and push on your car with a 50 N force but the car never moves.
      - Physics says you did zero work "on the car", even though you might have exerted a lot of effort

Physics 3A: Energy
- Examples of systems and environments:
  - Choice 1:
    - system is ball
    - environment is everything else
    - the environment influences the system by the tension in the rope & by gravity
  - Choice 2:
    - system is the ball, block, rope and pulley
    - environment is everything else
    - so tension in rope is now an internal force, not external
    - earth's gravity influences the system by external forces on both the block and ball

Physics 3A: Energy
- In physics we define work as (for constant forces):
  "The work done by an agent exerting a constant force on a system is the product of the component Fcos(θ) of the force along the direction of the displacement of the point of application of the force and the magnitude Δr of the displacement"
- in equation form: \[ W = F \Delta r \cos(\theta) \] (6.1)
- some properties of work:
  - Dimensions? \( \text{ML}^2\text{T}^{-2} \)
  - Units? \( \text{N m} = \text{Joule (J)} \)
  - Vector or scalar? scalar
  - if component of F in direction of displacement is zero, then work is zero.
**Physics 3A: Energy**

- **Question:** Order the following situations in order of decreasing work done on block, displacement is always to right:

(a) \( F \rightarrow \)  
(b) \( F \leftarrow \)  
(c) \( F \rightarrow \)  
(d) \( F \leftarrow \)  

\( \text{(c), (a), (d), (b)} \)

- **Work as a Scalar Product (dot product)**

  - Recall the scalar product of two vectors:  
  \( \vec{A} \cdot \vec{B} = AB \cos(\theta) \)

  - From the previous definition of work, we can express it as the **scalar product** of the two vectors \( \mathbf{F} \) and \( \mathbf{\Delta r} \) as:
  \[ W = \mathbf{F} \cdot \mathbf{\Delta r} = F \Delta r \cos(\theta) \] (6.4)

- **Is work done by (and if so, is it positive or negative):**
  - You on a heavy box as you lift it \( \text{Yes, positive} \)
  - Gravity on the box as you lift it \( \text{Yes, negative} \)
  - Gravity on the box as you carry it \( \text{No} \)
  - You on the box as you carry it \( \text{No} \)
  - You on the box as you lower it to the ground \( \text{Yes, negative} \)
  - Gravity on the box as you lower it to the ground \( \text{Yes, positive} \)
  - Frictional force between a car’s tires and the road on a car as it undergoes uniform circular motion \( \text{No} \)

- **Some properties of scalar product** \( \vec{A} \cdot \vec{B} = AB \cos(\theta) \)

  - **commutative law:** \( \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \) (6.5)
  - **distributive law:** \( \vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} \) (6.6)
  - **unit vectors:**
    \[ \hat{i} \cdot \hat{i} = 1, \hat{j} \cdot \hat{j} = 1, \hat{k} \cdot \hat{k} = 1 \] (6.7)
    \[ \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = \hat{k} \cdot \hat{k} = 0 \] (6.8)
  - **Component form:**
    \[ \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \] (6.9)
  - **Special case:**
    \[ \vec{A} \cdot \vec{A} = A_x^2 + A_y^2 + A_z^2 = A^2 \]
**Physics 3A: Energy**

- Equation (6.1) is only valid if the force is constant.
- **What are some situations where the force is not constant?**
  - Gravity: \( F_g = \frac{G m_1 m_2}{r^2} \)
  - Electrostatic force: \( F_e = \frac{k q_1 q_2}{r^2} \)
  - Springs: \( F_s = -kx \)
- In these situations, we need to extend our mathematics.
- Consider \( F(x) \) on a block moving in \( x \)-direction:

![Diagram](image1)

- **We break motion into tiny pieces where force is almost constant:**

![Diagram](image2)

- Sum up all pieces: \( W = \sum F_x \Delta x \)
- Take the limit as \( \Delta x \) goes to zero: \( W = \lim_{\Delta x \to 0} \sum F_x \Delta x = \int F_x \, dx \)
- So:

\[
W = \int_{x_i}^{x_f} F_x \, dx \quad (6.11) \]

**Physics 3A: Energy**

- We can generalize further to case where force and displacement are not parallel:

![Diagram](image3)

- can break into \( x \) & \( y \) if convenient: \( W = W_x + W_y \)

\[
W_x = \int_{x_i}^{x_f} F_x \, dx \quad \text{and} \quad W_y = \int_{y_i}^{y_f} F_y \, dy
\]

- **Note:** If \( W \) is to be the total work done, then \( F \) in above is the resultant (net) force.
- Can compute work done by individual forces as well

**Physics 3A: Energy**

- **Example:**
  - Consider case of a block attached to a spring, horizontally
  - define \( x = 0 \) when spring not compressed or stretched (equilibrium)
  - pull block to right, stretches spring and spring pulls to left
  - pull block to left, compresses spring and spring pushes to right
  - spring force always forces block back to equilibrium position
  - thus spring force is called a restoring force
  - measurements show that if displacement (\( x \)) is small, force is:

\[
F = -kx \quad (6.13)
\]

- Thus, \( k \) is a spring constant or force constant.
Let a = dv/dt, so:

\[ W = \int F \, dx \]

\[ W = \int_{x_i}^{x_f} (-kx) \, dx \]

\[ W = \left[-\frac{1}{2} k x^2 \right]_{x_i}^{x_f} = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2 \]

\[ W_i = \frac{1}{2} k x_{\text{max}}^2 \quad (6.14) \]

For an arbitrary move from \( x_i \) to \( x_f \), we get:

\[ W_i = \int_{x_i}^{x_f} (-kx) \, dx = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2 \]

What work do we do to stretch a spring?
- assume no acceleration
- then \( F_{\text{app}} = -F \)
- so

\[ W_f = \int_{x_i}^{x_f} F_{\text{app}} \, dx \]

\[ W_f = \int_{x_i}^{x_f} kx \, dx = \frac{1}{2} k x_{\text{max}}^2 \]

\[ W_f = \frac{1}{2} k x_{\text{max}}^2 - \frac{1}{2} k x_i^2 \quad (6.15) \]

We define \( \frac{1}{2} m v^2 \) as the kinetic energy of the block:

\[ \frac{1}{2} m v^2 = \frac{1}{2} k x_{\text{max}}^2 \quad (6.18) \]
Physics 3A: Energy

- So now we see that the net work done on an object goes into changing the object's kinetic energy (K):

\[ W_{net} = K_f - K_i = \Delta K \]  \hspace{1cm} (6.19) \hspace{1cm} K_i = \frac{1}{2} m v_i^2 \hspace{1cm} K_f = \frac{1}{2} m v_f^2

- This is called the work-kinetic energy theorem:

"When work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system."

- Note: If work done is positive, kinetic energy increases (and so does speed). If work done is negative, kinetic energy decreases (and so does speed)

- Also note that this only involves scalars, not vectors!

Physics 3A: Energy

- Nonisolated Systems
  - Any system which is "influenced" by its environment
  - Examples:
    - Book pushed on a table
    - Pencil picked up by you after you drop it
  - Here the "influence" causes a change in the system
  - In our current context, the change in the system is its "energy"
  - Energy is either transferred into or out of the system
  - This transfer of energy is the work done on the system
  - We can expand our concept of the system's energy from just kinetic energy to include both kinetic and internal energy
  - Internal energy - related to system's temperature (it is motion of system's "internal parts")

Physics 3A: Energy

- But wait! Is the work-kinetic energy theorem always valid?

- Consider book sliding across a table with friction.

- Frictional force does work on book, changing its kinetic energy

- But doesn't friction also do work on the table?

- Does the table's kinetic energy change?

- So system's energy now is:

\[ E_{system} = K + E_{ext} \]

- For nonisolated systems, energy can be transferred into or out of the system by the environment

- Types of energy transfers are:
  - Work
  - Pushing your out-of-gas car
  - Mechanical waves - propagation of a disturbance in a media such as air or water
  - Sound waves from your radio transferring energy into your ear
  - Heat - transfer of internal energy
  - Placing your hand on the campfire to see if it is hot
  - Matter transfer - matter crosses the system boundary
  - "fuel" into your car, or "exhaust" out of a rocket
  - Electrical transfer - electrical current flowing from your kitchen wall outlet into your finger
  - Electromagnetic - radio waves received from local station
Physics 3A: Energy

- Once we include internal energy and other forms of energy transfer, we can revive the "work-kinetic energy theorem" into the concept of "Conservation of Energy":
- Energy cannot be created or destroyed, it can only change forms or be transferred into or out of a system, i.e.:

\[
\Delta K + \Delta E_{int} = \sum H \tag{6.20}
\]

- where \(\Sigma H\) is the sum of all the ways energy can be transferred into or out of the system:

\[
\Sigma H = W + Q + H_{MW} + H_{MT} + H_{ET} + H_{ER}
\]

\(W\): work  
\(Q\): heat  
\(H_{MW}\): mass transfer  
\(H_{MT}\): mechanical wave  
\(H_{ET}\): electrical transfer  
\(H_{ER}\): electromagnetic radiation

Physics 3A: Energy

- We said that frictional forces can change the internal energy of a system and the "old" work-kinetic energy theorem doesn't hold.
- Let's see how to relate a frictional force to this change in energy.
- Consider the book sliding across a table:

Apply Newton's 2nd law:

\[
f_k = ma \quad \text{multiply by } \Delta x: \quad -f_k \Delta x = (ma) \Delta x
\]

- From chapter 2:

\[
a_x = \frac{v_i - v_f}{t} \quad \Delta x = \frac{1}{2} (v_i + v_f) t
\]

- If other forces also act on the book, then:

\[
\Delta K = -f_k \Delta x + \sum W_{other \ forces} \tag{6.23}
\]

Now consider the book and the table as the system:
- no external forces do any work on the system and so \(\Sigma H = 0\) in (6.20) and so:

\[
\Delta K + \Delta E_{int} = 0
\]

\[
-f_k \Delta x + \Delta E_{int} = 0
\]

\[
\Delta E_{int} = f_k \Delta x \tag{6.24}
\]

- The kinetic frictional force transforms kinetic energy into internal energy.
We have said work is the transfer of energy into or out of a system.
A logical question to ask is "how fast or slow is the energy transferred?"
This is the concept of Power:

- **Power** is the time rate of energy transfer.
- The average power is defined as the work done divided by the time interval ($\Delta t$) over which the work was done:
  \[ P = \frac{W}{\Delta t} \]

Just as in other rate quantities, we can define the instantaneous power by taking the limit:

\[ P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} \quad (6.26) \]

We know from before, a small piece of work ($dW$) done over a small displacement ($dr$) by a force $F$ is:

\[ dW = \mathbf{F} \cdot d\mathbf{r} \]

divide by $dt$

\[ \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \dot{\mathbf{r}} \]

\[ P = \mathbf{F} \cdot \dot{\mathbf{r}} \quad (6.27) \]

In general form, power is rate of transfer of any energy ($E$):

\[ P = \frac{dE}{dt} \quad (6.28) \]

Units of power are J/s, or watts (W): 1 W = 1 J/s = 1 kg m²/s³

1 hp = 550 ft lb/s ≈ 746 W