Physics 3A: 1-D Motion

- **Kinematics** – study of the motion of objects, without concern about what causes that motion.
- **Dynamics** – study of relation between motion and its causes.
- Start with one dimensional (straight-line) motion

**Average Velocity**
- Consider the motion of the car in the following:

<table>
<thead>
<tr>
<th>Position (s)</th>
<th>x (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
</tr>
<tr>
<td>D</td>
<td>30</td>
</tr>
<tr>
<td>E</td>
<td>40</td>
</tr>
<tr>
<td>F</td>
<td>50</td>
</tr>
</tbody>
</table>

Positions of the Car at Various Times

- We define the average speed (scalar) as:
  \[ v = \frac{d}{\Delta t} \]  \hspace{1cm} (2.1)

  where \( d \) is the distance traveled and \( \Delta t \) is the time interval during the movement.

- We define the average velocity (vector) as "displacement vector divided by the change in time":
  \[ \vec{v}_a = \frac{\Delta \vec{x}}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \]  \hspace{1cm} (2.2)

  note that \( d \) depends on path, but \( \Delta \vec{x} \) and thus \( \vec{v}_a \) does not!

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Physics 3A: 1-D Motion

- What are the dimensions of \( v \)? \( [v] = \frac{\text{length}}{\text{time}} = \frac{L}{T} \)

- What are the units of \( v \) in SI system? \( \frac{\text{m}}{\text{s}} \)

- How do you specify the direction of \( v \) in 1-D motion?
  - with plus or minus sign.
    - if \( (x_f > x_i) \) then \( v \) is positive,
    - if \( (x_i < x_f) \) then \( v \) is negative.
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- What if we want to know an object's velocity at an exact time? 
  *Compute object's "instantaneous velocity"*

- **Instantaneous velocity** ($v$) – the velocity of an object at a specific instant of time.

- Consider:

- Take the limit:

$$ v_i = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} $$

- In calculus:

$$ v_i = \frac{dx}{dt} $$ (2.3)

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- What if the velocity is not constant? => **Acceleration!**

- We define average acceleration as the "time-rate-of-change" of velocity, averaged over some time interval:

$$ \overline{a}_v = \frac{v_{t_f} - v_{t_i}}{t_f - t_i} \Delta t $$ (2.5)

- What are the dimensions of acceleration?
  
  $[a] = \text{length} / \text{time}^2$

- Measures how rapidly the velocity is changing, i.e. $a = 2 \text{ m/s}^2$ means velocity is changing 2 m/s in each second

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Physics 3A: 1-D Motion

- Summary for constant velocity, 1-D:
  - Displacement (vector):
    $$ \overrightarrow{\Delta x} = \vec{x}_f - \vec{x}_i $$
  - Average velocity (vector):
    $$ \overrightarrow{v_{\text{avg}}} = \frac{\Delta \vec{x}}{\Delta t} $$
  - Instantaneous velocity (vector):
    $$ \overrightarrow{v} = \frac{d\vec{x}}{dt} $$
  - Speed (scalar):
    $$ v = \frac{d}{dt} $$
  - Model for constant velocity:
    $$ v_{\text{avg}} = \frac{x_f - x_i}{t_f - t_i} \Delta t $$
    
    Setting $t_f = 0$

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Physics 3A: 1-D Motion

- Just like instantaneous velocity, we can define instantaneous acceleration which is the **acceleration at one instant in time**:

$$ \overrightarrow{a} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{d\overrightarrow{v}}{dt} $$ (2.6)

- Properties of acceleration:
  - Since its a derivative of velocity wrt time, graphically its the slope of a velocity versus time plot at any point:
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- Properties of acceleration (cont.):
  - Can also obtain acceleration from displacement vector:
    \[ \vec{\alpha} = \frac{d \vec{v}}{dt} = \frac{d}{dt} \left[ \frac{d \vec{x}}{dt} \right] = \frac{d^2 \vec{x}}{dt^2} \]

- Match the graphs:
  - What is meant by "de-acceleration?"

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- Another equation comes from v versus t plot, and that area under curve is displacement:
  \[ \Delta x = \text{area square + area triangle} \]
  \[ \Delta x = v_s \Delta t + \frac{1}{2} (v_w - v_s) \Delta t \]
  \[ \Delta x = (v_s + \frac{1}{2} v_w - v_s) \Delta t = \frac{1}{2} (v_w + v_s) \Delta t \]
  
  but \( \Delta x = v_s \Delta t \) so \( v_s \Delta t = \frac{1}{2} (v_w + v_s) \Delta t \)

- What is meant by "de-acceleration?"
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- Now put in value of acceleration: \( v_f = v_i + at \)
  \[ x_f = x_i + \frac{1}{2} \left( v_i + v_f \right) t \]
  \[ x_f = x_i + \frac{1}{2} \left( v_i + v_f \right) t \]

(2.11)

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- Now we can pull out value of time: \( v_f = v_i + at \)
  \[ x_f = x_i + \frac{1}{2} \left( v_i + v_f \right) t \]
  \[ t = \frac{v_f - v_i}{a} \]
  \[ x_f = x_i + \frac{1}{2} \left( v_i + v_f \right) \frac{v_f - v_i}{a} = x_i + \frac{v_f^2 - v_i^2}{2a} \]
  \[ v_f^2 = v_i^2 + 2a(x_f - x_i) \] (2.12)

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- Summary:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Contains x v v_i v_f a t</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_f = v_i + at )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>( v_f = \frac{1}{2} \left( v_i + v_f \right) )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>( x_f = x_i + \frac{1}{2} \left( v_i + v_f \right) t )</td>
<td>( \checkmark ) ( \checkmark ) ( \checkmark ) ( \times )</td>
</tr>
<tr>
<td>( x_f = x_i + v_i t + \frac{1}{2} a t^2 )</td>
<td>( \checkmark ) ( \checkmark ) ( \checkmark )</td>
</tr>
<tr>
<td>( v_f^2 = v_i^2 + 2a(x_f - x_i) )</td>
<td>( \checkmark ) ( \checkmark ) ( \checkmark ) ( \times )</td>
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Physics 3A: 1-D Motion

- Your Mission? => Problem solving:
  - 1. Draw a picture (Model)
  - 2. Extract from "words" useful information
  - 3. Decide on concept (i.e. constant acceleration?)
  - 4. Select equation(s) which use info you have and also info you want.
  - 5. Solve for unknown.
  - 6. Adjust for significant figures and include units!

- Example of constant acceleration => Free-fall Motion
  - We all know that objects released near earth fall freely toward earth's center.
  - Due to gravity (see later lectures) free-falling objects undergo a constant acceleration (vector), pointed toward earth's center with magnitude:
    \[ \left| \mathbf{a} \right| = g = -9.80 \text{ m/s}^2 = -32 \text{ ft/s}^2 \]
Converting our constant acceleration equations for free-fall motion gives:

\[ v_f = v_i - gt \]

\[ y_f = y_i + v_i t - \frac{1}{2} gt^2 \]

\[ v_f^2 = v_i^2 - 2g(y_f - y_i) \]

\[ v_f = \frac{v_i + v_f}{2} \]

\[ y_f = y_i + \frac{1}{2}(v_i + v_f)t \]

With \( g = 9.80 \text{ m/s}^2 \) or \( 32 \text{ ft/s}^2 \)