Useful Equations

\[ A_x = A \cos(\theta) \quad A_y = A \sin(\theta) \quad \theta = \tan^{-1} \left( \frac{A_x}{A_y} \right) \quad A = \sqrt{A_x^2 + A_y^2} \]

\[ \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \quad \vec{A} \cdot \vec{B} = |A||B| \cos(\theta) \quad |\vec{A} \times \vec{B}| = |A||B| \sin(\theta) \]

\[ \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \]

\[ \vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \]

\[ v = \frac{d}{\Delta t} \quad \vec{v}_x = \frac{\Delta x}{\Delta t} \quad v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad x_f = x_i + v_x t \quad \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \]

\[ a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad v_{xf} = v_{xi} + a_x t \quad x_f = x_i + \frac{1}{2} (v_{xf} + v_{xi}) t \]

\[ x_f = x_i + v_x t + \frac{1}{2} a_x t^2 \quad v_f^2 = v_i^2 + 2a(x_f - x_i) \]

\[ \vec{r} = x \hat{i} + y \hat{j} \quad v_y = v_i \cos(\theta) = \text{constant} \quad v_y = v_i \sin(\theta) - gt \quad x_f = x_i + v_i \cos(\theta) t \]

\[ y_f = y_i + v_i \cos(\theta) t - \frac{1}{2} gt^2 \quad a_c = \frac{v^2}{r} \quad a_i = \frac{d|\vec{v}|}{dt} \quad y_f = (\tan(\theta))x_f - \left( \frac{g}{2v^2 \cos^2(\theta)} \right)x_f^2 \]

\[ T = \frac{2 \pi r}{v} \quad h = \frac{v_i^2 \sin^2(\theta)}{2g} \quad R = \frac{v_f^2 \sin(2\theta)}{g} \quad a_r = -a_c = \frac{-v^2}{r} \quad \vec{v}_{r0} = \vec{v}_{p0} - \vec{v}_{o0} \]

\[ \sum \vec{F} = 0 \quad \vec{F}_{12} = -\vec{F}_{21} \quad \vec{F}_g = m \vec{g} \quad \sum \vec{F} = m \vec{a} \quad \frac{m_1}{m_2} = \frac{a_2}{a_1} \]

Useful Constants

\[ g = 9.80 \text{ m/s}^2 = 32.0 \text{ ft/s}^2 \quad c = 2.99 \times 10^8 \text{ m/s} \]
1. (5 pts) In the computation \( \frac{(47.787-42.1)}{(5999.6-874.326)} \times 0.00375 \) what is the correct answer, giving the proper number of significant figures (at each stage of the calculation, apply significant figures)?

   a.) 0.000004125
   b.) 0.000004161
   c.) 0.00000413
   d.) 0.0000041
   e.) 0.0000042

2. (5 pts) After a long night of studying for your midterms, you leave a stack of three books on your desk, Physics on the bottom, Chemistry in the middle, and Biology on top. Order these books by the magnitude of their vertical normal force, smallest magnitude first:

   a.) Chemistry, Biology, Physics
   b.) Biology, Chemistry, Physics
   c.) Physics, Chemistry, Biology
   d.) All have the same vertical normal force.
   e.) Not enough information to tell.

3. (5 pts) Consider a particle moving along the positive x-axis (where positive x is right, and positive y is up). If the particle's acceleration in y is positive and its acceleration in x is zero, then

   a.) the particle's speed will increase.
   b.) the particle's velocity will “turn” downward
   c.) the particle's velocity will “turn” upward
   d.) the particle's speed will decrease.
   e.) None of the above.
4. (5 pts) A truck loaded with gravel is accelerating along a highway at 0.55 m/s². Assume the engine of the truck supplies a constant force. If the truck develops a leak and gravel starts to leave the truck, what happens to the acceleration?

a.) The acceleration remains constant.

b.) The acceleration increases.

c.) The acceleration decreases.

d.) Not enough information to tell.

e.) None of the above.

5. (5 pts) You can swim in still water at a speed of 1.50 m/s. You want to cross a 15.0 m wide river as quickly as possible, going from the west bank to the east bank. The current of the river flows south at 1.50 m/s. In what direction should you swim?

a.) 45 degrees north of east

b.) directly east

c.) 45 degrees south of east

d.) for the first half swim 45 degrees north of east, then second half 45 degrees south of east.

e.) Not enough information to tell

6. (5 pts) A box with mass m is sitting on the floor. You push the box with a force of 2mg. The direction of your push makes an angle of 30 degrees above the horizontal. What is the force of the floor pushing on the box? (assume positive is upward)

a.) -mg

b.) mg

c.) 0

d.) -2mg

e.) 2mg

7. $\vec{A}$ and $\vec{B}$ are two vectors that are not parallel or perpendicular to each other and the angle between them is $\theta$. What is $(\vec{A} \times \vec{B}) \times (\vec{B} \times \vec{A})$?

a.) $|\vec{A}|^2 |\vec{B}|^2 \sin^2(\theta)$

b.) $\vec{A} + \vec{B}$

c.) $(\vec{A} \cdot \vec{B})^2$

d.) 0

e.) not enough information to tell
8. (5 pts) A particle moves from point A to point B. If the speed of the particle is equal to the magnitude of the average velocity, then

a.) The instantaneous acceleration must be zero at every point from A to B.

b.) **The instantaneous acceleration must be parallel or anti-parallel to the instantaneous velocity at every point from A to B.**

c.) The instantaneous acceleration must be perpendicular to the instantaneous velocity at every point from A to B.

d.) None of the above are true.

9. (5 pts) You shoot an arrow up into the air at some initial speed v. If you want the arrow to stay in the air as long as possible, at what angle relative to the horizontal should you aim the arrow?

a.) 0 degrees
b.) 30 degrees
c.) 45 degrees
d.) 60 degrees
e.) 90 degrees

10. Your infinitely intelligent physics instructor gives you a graph of position versus time for an object moving in one dimension. You correctly tell him that to find whether the acceleration is positive, negative or zero at a given instant, you

a.) can not say anything about the acceleration from a position versus time plot.
b.) you draw a line tangent to the curve and the value of the slope is the acceleration
c.) you draw a line from the origin to the position at that instant and the slope of that line is the acceleration.

d.) **you look at how tangent lines to the curve before and after the given instant are changing and that change is the acceleration.**
e.) none of the above.
11. (5 pts) If \((\vec{A} - \vec{B}) \cdot \vec{C} = 0\), and \(\vec{A} \neq \vec{B}\), then which of the following statements is true?

a.) The component of \(\vec{A}\) in the direction of \(\vec{C}\) is equal to the component of \(\vec{B}\) in the direction of \(\vec{C}\).

b.) The magnitudes of \(\vec{A}\) and \(\vec{B}\) are equal.

c.) \(\vec{A}\) is perpendicular to \(\vec{B}\).

d.) all of the above are true

e.) None of the above are true.

12. (5 pts) A ball is attached to one end of a string. The other end of the string is attached to a spool (like a spool of thread). If the ball is going around the spool with a constant speed so that the string is winding onto the spool, then the acceleration of the ball is

a.) increasing

b.) decreasing

c.) stays the same

d.) not enough information to tell

e.) none of the above.
13. (20 pts) Consider the situation in the figure below, where the inclined plane is frictionless, the rope is massless and non-stretching, and the pulley is frictionless and massless. Assume \( m_1 \) is 12.5 kg, and theta is 45.0°. a.) What should \( m_2 \) be so that \( m_1 \) does not move?  
b.) What is the tension in the rope with this value of \( m_2 \)?  
c.) Would your result for a.) change if the inclined plane were located on the moon? Explain why or why not.  
d.) Finally, assume the rope breaks. What is the acceleration of \( m_1 \) and \( m_2 \)? (circle your final answers and show all work)

**Diagram:**

- \( m_1 \) on an inclined plane with an angle of 45°.
- \( m_2 \) hanging from a rope passing over a pulley.
- Free body diagrams showing forces in the horizontal (x) and vertical (y) directions.

**Work:**

a.) Since rope does not stretch, \( a_{1x} = a_{2y} \)

Free body diagrams & use \( F = ma \):

\[
T \quad m_2 \quad m_1 \\
\downarrow \quad \downarrow \quad \downarrow \\
x: 0 = 0 \quad x: m_1 g \sin \theta - T = m_1 a_x \\
y: m_1 g \cos \theta = m_1 a_y, \ y = 0 \\
\]

Solve for \( T \) from \( m_2 \) and put into x equation for \( m_1 \), then solve for \( m_2 \):

\[
T = m_2 g + m_2 a \\
m_1 g \sin \theta - (m_2 g + m_2 a) = m_1 a \\
(m_1 + m_2) a = g (m_1 \sin \theta - m_2) \\
\]

\[
a = \frac{g (m_1 \sin \theta - m_2)}{m_1 + m_2} \quad \text{So set: } m_2 = m_1 \sin \theta = 12.5 \sin (45°) = 8.84 \text{ kg}
\]

b.) From a.) above: \( T = m_2 g + m_2 a = m_2 g = (9.80)(8.84) = 86.6 \text{ N} \)

c.) No, because \( g \) does not enter into the equation for \( m_2 \).

d.) When the rope breaks, \( m_2 \) would be in free fall, so: \( a = g = 9.80 \text{ m/s}^2 \)

for \( m_1 \): \( m_1 g \sin \theta = m_1 a \quad \text{so} \quad a = g \sin \theta = 9.80 \sin (45°) = 6.93 \text{ m/s}^2 \)
14. (20 pts) At a pistol shooting range a policeman fires a gun, aiming horizontally, directly at the center of a block of wood 18.2 m away (see diagram below). The block is nailed to a table so it can't move. The bullet strikes the wood block and comes to rest inside the block, 12.5 cm from the front. If the initial speed of the bullet when it left the gun was 500. m/s and the bullet's mass is 75.0 g, then compute a.) the stopping acceleration of the bullet, b.) the average force of the wooden block on the bullet, c.) the time the bullet was in the air before hitting the block, and d.) how far below the center of the front of the wood block did the bullet strike.  (circle your final answers and show all work)

\[ a = \frac{v_f^2 - v_i^2}{2 \Delta x} = \frac{0 - 500^2}{2 \times 0.125} = -100000 = -1.00 \times 10^6 \text{ m/s}^2 \]

b.) Force which causes above \( a \)? Use \( F = ma \):
\[ F = ma = (0.075 \text{ kg})(-1.00 \times 10^6) = -75000 \text{ N} \]

c.) concept: projectile motion, in x direction, no acceleration during flight use:
\[ x_f = x_i + v_{ix} t \quad \text{or} \quad \Delta x = v_{ix} t \quad \text{so} \quad t = \frac{\Delta x}{v_{ix}} = \frac{18.2}{500} = 0.0364 \text{ s} \]

d.) concept: projectile motion, in y direction: use:
\[ y_f = y_i + v_{iy} t - \frac{1}{2} g t^2 \quad \text{or} \quad \Delta y = v_{iy} t - \frac{1}{2} g t^2 = -\frac{1}{2} g t^2 = -\frac{1}{2} (9.80)(0.0364)^2 = 0.00649 \text{ m} \]