Solutions to Homework #5
(Chapter 5)

Q5.1. No. The book takes a shorter time in its upward motion. When it is sliding up the ramp its acceleration is larger in magnitude, \( g(\sin \theta + \mu_k \cos \theta) \), than when the book is sliding down with acceleration \( g(\sin \theta - \mu_k \cos \theta) \).

Q5.4. The car exerts the same force as twenty people. (This is twice as much as ten people on each end.)

Q5.7. The ball would not behave as it would when dropped on the Earth. As the astronaut holds the ball, she and the ball are moving with the same angular velocity. The ball, however, being closer to the center of rotation, is moving with a slower tangential velocity. Once the ball is released, it acts according to Newton's first law, and simply drifts with constant velocity in the original direction of its velocity when released — it is no longer "attached" to the rotating space station. Since the ball follows a straight line and the astronaut follows a circular path, it will appear to the astronaut that the ball will "fall to the floor." But other dramatic effects will occur. Imagine that the ball is held so high that it is just slightly away from the center of rotation. Then, as the ball is released, it will move very slowly along a straight line. Thus, the astronaut may make several full rotations around the circular path before the ball strikes the floor. This will result in three obvious variations with the Earth drop. First, the time to fall will be much larger than that on the Earth, even though the feet of the astronaut are pressed into the floor with a force that suggests the same force of gravity as on Earth. Second, the ball may actually appear to bob up and down if several rotations are made while it "falls." As the ball moves in a straight line while the astronaut rotates, sometimes she is on the side of the circle on which the ball is moving toward her and other times she is on the other side, where the ball is moving away from her. The third effect is that the ball will not drop straight down to her feet. In the extreme case we have been imagining, it may actually strike the surface while she is on the opposite side, so it looks like it ended up "falling up." In the less extreme case, in which only a portion of a rotation is made before the ball strikes the surface, the ball will appear to move backward relative to the astronaut as it falls.

Q5.11. The speed changes. The tangential force component causes tangential acceleration.

Q5.16. If astronauts were indeed weightless, meaning that there were no gravitational force on them, they would move in a straight-line path tangent to the orbit rather than following the orbit around the Earth. In the space shuttle just above the atmosphere, gravity is only slightly weaker than at the Earth's surface. There is no way to get "beyond" a long-range force described by an inverse square law. Gravity does its job most clearly on an orbiting spacecraft, because the craft feels no other forces and is in free fall.

\[ \text{Case 1, impeding upward motion} \]
Setting \( \Sigma F_y = 0 \):

\[ P \cos 50^\circ - n = 0 \]

\[ f_{t, \text{max}} = \mu_s n \]

\[ f_{t, \text{max}} = \mu_s P \cos 50^\circ = 0.250(0.643)P = 0.161P \]

Setting \( \Sigma F_y = 0 \):

\[ P \sin 50^\circ - 0.161P - 3.00(9.80) = 0 \]

\[ P_{\text{max}} = 48.6 \text{ N} \]

\[ (\text{Case 2, impeding downward motion}) \]

As in Case 1, \( f_{t, \text{max}} = 0.161P \)

Setting \( \Sigma F_y = 0 \):

\[ P \sin 50^\circ + 0.161P - 3.00(9.80) = 0 \]

\[ P_{\text{min}} = 31.7 \text{ N} \]
(a) $a = \frac{v^2}{r} = \frac{(1.57 \text{ m/s})^2}{3.00 \text{ m}} = 0.822 \text{ m/s}^2$ toward the center

(b) For no sliding motion, $f_s = ma = 45.0 \text{ kg}(0.822 \text{ m/s}^2) = 37.0 \text{ N}$ toward the center

(c) $f_c = \mu mg$

\[ \mu = \frac{37.0 \text{ N}}{45.0 \text{ kg}(9.80 \text{ m/s}^2)} = 0.0839 \]

5.21

(a) $a_c = \frac{v^2}{r}$

\[ r = \frac{v^2}{a_c} = \frac{(13.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 8.62 \text{ m} \]

(b) Let $n$ be the force exerted by the rail.

Newton's law gives

\[ Mg + n = \frac{Mv^2}{r} \]

\[ n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = Mg, \text{ downward} \]

(c) $a_c = \frac{v^2}{r}$

\[ a_c = \frac{(13.0 \text{ m/s})^2}{20.0 \text{ m}} = 8.45 \text{ m/s}^2 \]

If the force exerted by the rail is $n_1$,

then

\[ n_1 + Mg = \frac{Mv^2}{r} = Ma_c \]

\[ n_1 = M(a_c - g) \]

which is $< 0$, since $a_c = 8.45 \text{ m/s}^2$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars. To be safe we must require $n_1$ to be positive. Then $a_c > g$.

We need

\[ \frac{v^2}{r} > g \quad \text{or} \quad v > \sqrt{rg} = \sqrt{(20.0 \text{ m})(9.80 \text{ m/s}^2)} \quad v > 14.0 \text{ m/s} \]
(a) At terminal velocity, \[ R = \nu_T b = mg \]
\[ \therefore \frac{b}{\nu_T} = \frac{mg}{2.00 \times 10^{-2} \text{ m/s}} = 1.47 \text{ N} \cdot \text{s/m} \]

(b) From Equation 5.6, the velocity of the bead is
\[ \nu = \nu_T \left(1 - e^{-kt/m}\right) \]
\[ \nu = 0.632 \nu_T \text{ when } e^{-kt/m} = 0.368 \]
or at time
\[ t = \left(\frac{m}{b}\right) \ln(0.368) = 2.04 \times 10^{-3} \text{ s} \]

(c) At terminal velocity, \[ R = \nu_T b = mg = 2.94 \times 10^{-2} \text{ N} \]

5.32
\[ F = k \frac{m_1 m_2}{(r \mu)^2} = \left(8.99 \times 10^9\right) \left(\frac{(+40)}{(-40)}\right) \left(\frac{2000}{200}\right)^2 = -3.60 \times 10^6 \text{ N (attractive) } = 3.60 \times 10^6 \text{ N downward} \]

5.40
With motion impending, \[ n + T \sin \theta - mg = 0 \]
\[ f = \mu_s (mg - T \sin \theta) \]
and
\[ T \cos \theta - \mu_s mg + \mu_s T \sin \theta = 0 \]
so
\[ T = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta} \]
To minimize \( T \), we maximize \( \cos \theta + \mu_s \sin \theta \)
\[ \frac{d}{d\theta} (\cos \theta + \mu_s \sin \theta) = 0 = -\sin \theta + \mu_s \cos \theta \]

(a) \[ \theta = \tan^{-1} \mu_s = \tan^{-1} 0.350 = 19.3^\circ \]

(b) \[ T = \frac{0.350(1.30 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 19.3^\circ + 0.350 \sin 19.3^\circ} = 4.21 \text{ N} \]
**5.41** For the system to start to move when released, the force tending to move \( m_2 \) down the incline, \( m_2 g \sin \theta \), must exceed the maximum friction force which can retard the motion:

\[
f_{\text{max}} = f_{1,\text{max}} + f_{2,\text{max}} = \mu_{1,\text{max}} n_1 + \mu_{2,\text{max}} n_2
\]

\[
f_{\text{max}} = \mu_1 m_1 g \pm \mu_2 m_2 g \cos \theta
\]

From Table 5.1, \( \mu_{1,1} = 0.610 \) (aluminum on steel)

and

\( \mu_{2,2} = 0.530 \) (copper on steel).

With \( m_1 = 2.00 \text{ kg}, m_2 = 6.00 \text{ kg}, \theta = 30.0^\circ \),

the maximum friction force is found to be \( f_{\text{max}} = 38.9 \text{ N} \).

This exceeds the force tending to cause the system to move,

\[
m_2 g \sin \theta = 6.00 \text{ kg} \times (9.80 \text{ m/s}^2) \sin 30^\circ = 29.4 \text{ N}
\]

Hence, the system will not start to move when released.

The friction forces increase in magnitude until the total friction force retarding the motion, \( f = f_1 + f_2 \), equals the force tending to set the system in motion. That is, until \( f = m_2 g \sin \theta = 29.4 \text{ N} \).

**5.44** (a)

\[
\begin{align*}
5.00 \text{ kg} \quad & \quad F = 45.0 \text{ N} \\
10.0 \text{ kg} \quad & \quad T = 49.0 \text{ N}
\end{align*}
\]

\( f_1 \) and \( n_1 \) appear in both diagrams as action-reaction pairs.

(b) 5.00 kg: \( \Sigma F_x = ma \):

\[
n_1 = m_1 g = 5.00 \times (9.80) = 49.0 \text{ N}
\]

\[
f_1 - T = 0
\]

\[
T = f_1 = \mu \, m g = 0.200 \times (5.00 \times 9.80) = 98.0 \text{ N}
\]

10.0 kg: \( \Sigma F_y = ma \):

\[
45.0 - f_1 - f_2 = 10.0a
\]

\[
\Sigma F_x = 0: \quad n_2 - n_1 - 98.0 = 0
\]

\[
f_2 = \mu n_2 = \mu (n_1 + 98.0) = 0.20 (49.0 + 98.0) = 29.4 \text{ N}
\]

\[
45.0 - 9.80 - 29.4 = 10.0a
\]

\[
a = 0.580 \text{ m/s}^2
\]
5.47 (a) Since the object of mass $m_2$ is in equilibrium, \[ \Sigma F_y = T - m_2g = 0, \]
or \[ T = m_2g \]

(b) The tension in the string provides the required centripetal acceleration of the puck. Thus, \[ F_c = T = m_2g \]

c) From \[ F_c = \frac{m_1v^2}{R} \]
we have \[ v = \sqrt{\frac{m_2gR}{m_1}} \]

5.50 On the level road, the centripetal acceleration must be provided by a force of friction between car and road. However, if the road is banked at an angle $\theta$, the normal force, $n$, has a horizontal component $n \sin \theta$ pointing toward the center of the circular path followed by the car. We assume that only the component $n \sin \theta$ causes the centripetal acceleration. Therefore, the banking angle we calculate will be one for which no frictional force is required. In other words, a car moving at the correct speed (13.4 m/s) can negotiate the curve even on an icy surface. Newton’s second law written for the radial direction gives

\[ n \sin \theta = \frac{m_2v^2}{r} \quad (1) \]

The car is in equilibrium in the vertical direction.

Thus, from $\Sigma F_y = 0$, we have \[ n \cos \theta = mg \quad (2) \]

Dividing (1) by (2) gives \[ \tan \theta = \frac{v^2}{rg} \]

\[ \theta = \tan^{-1} \left[ \frac{(13.4 \text{ m/s})^2}{50 \text{ m/rad} \times 80 \text{ m/s}^2} \right] = 20.1^\circ \]

If a car rounds the curve at a speed lower than 13.4 m/s, the driver will have to rely on friction to keep from sliding down the incline. A driver who attempts to negotiate the curve at a speed higher than 13.4 m/s will have to depend on friction to keep from sliding up the ramp.