Solutions to Homework #3
(Chapter 3)

Q3.1 No. Yes. The points could be widely separated. In this case, you can only determine the average velocity, which is \( \mathbf{v} = \Delta \mathbf{x} / \Delta t \).

Q3.5 At the time the second ball is launched. Yes. 1 second. No.

Q3.7 Yes. The top of the mast and the deck have the same horizontal velocity.

Q3.10 The quantities (b) acceleration and (c) horizontal component of velocity remain constant for a projectile in the absence of air resistance.

Q3.13 The projectile on the Moon has greater range and reaches greater altitude. The Apollo astronauts did the experiment with golf balls.

Q3.15 (a) Drive straight ahead. (b) Hold the steering wheel still to drive straight ahead or to follow any other path of constant curvature.

\[
\begin{align*}
3.3 & \quad (a) \quad r & = \begin{pmatrix} 18.0 \text{ m} \end{pmatrix} + \begin{pmatrix} 4.00 \text{ m} - 4.90t^2 \end{pmatrix} \begin{pmatrix} 0 \text{ m} \end{pmatrix} + \begin{pmatrix} 0 \text{ m} \end{pmatrix} \begin{pmatrix} 1 \text{ m} \end{pmatrix} \\
 & \quad (b) \quad v & = \begin{pmatrix} 18.0 \text{ m/s} \end{pmatrix} + \begin{pmatrix} 4.00 \text{ m/s} - 9.80 \text{ m/s}^2 \end{pmatrix} \begin{pmatrix} 0 \text{ m/s} \end{pmatrix} + \begin{pmatrix} 0 \text{ m/s} \end{pmatrix} \begin{pmatrix} 0 \text{ m/s} \end{pmatrix} \\
 & \quad (c) \quad a & = \begin{pmatrix} -9.80 \text{ m/s}^2 \end{pmatrix} \begin{pmatrix} 0 \text{ m/s} \end{pmatrix} \\
 & \quad (d) \quad r(3.00 \text{ s}) & = \begin{pmatrix} 54.8 \text{ m} \end{pmatrix} - \begin{pmatrix} 32.1 \text{ m} \end{pmatrix} \\
 & \quad (e) \quad v(3.00 \text{ s}) & = \begin{pmatrix} 18.0 \text{ m/s} \end{pmatrix} - \begin{pmatrix} 25.4 \text{ m/s} \end{pmatrix} \\
 & \quad (f) \quad a(3.00 \text{ s}) & = \begin{pmatrix} -9.80 \text{ m/s}^2 \end{pmatrix} \begin{pmatrix} 0 \text{ m/s} \end{pmatrix}
\end{align*}
\]
3.7 (a) For the x-component of the motion we have

\[ x_f = x_i + v_{ix}t + \frac{1}{2}a_xt^2 \]

\[ 0.01 \text{ m} = 0 + \left(1.80 \times 10^7 \text{ m/s}\right)t + \frac{1}{2}\left(8 \times 10^{14} \text{ m/s}^2\right)t^2 \]

\[ 4 \times 10^{14} \text{ m/s}^2 \right)t^2 + \left(1.80 \times 10^7 \text{ m/s}\right)t - 10^{-2} \text{ m} = 0 \]

\[ t = \frac{1.80 \times 10^7 \pm \sqrt{\left(1.80 \times 10^7 \text{ m/s}\right)^2 - 4 \left(4 \times 10^{14} \text{ m/s}^2\right) \left(-10^{-2} \text{ m}\right)}}{2 \left(4 \times 10^{14} \text{ m/s}^2\right)} \]

\[ t = \frac{-1.80 \times 10^7 \pm 1.84 \times 10^7 \text{ m/s}}{8 \times 10^{14} \text{ m/s}^2} \]

We choose the + sign to represent the physical situation \( t = \frac{4.39 \times 10^7 \text{ m/s}}{8 \times 10^{14} \text{ m/s}^2} = 5.49 \times 10^{-10} \text{ s} \)

Here

\[ y_f = y_i + v_{iy}t + \frac{1}{2}a_yt^2 = 0 + 0 + \frac{1}{2} \left(1.6 \times 10^{15} \text{ m/s}^2\right)\left(5.49 \times 10^{-10} \text{ s}\right)^2 = 2.41 \times 10^{-4} \text{ m} \]

So, \( r_f = (10.0 \text{ i} + 0.241 \text{ j}) \text{ mm} \)

(b) \( v_f = v_i + at = 1.80 \times 10^7 \text{ m/s} \text{ i} + \left(8 \times 10^{14} \text{ m/s}^2 \text{ i} + 1.6 \times 10^{15} \text{ m/s}^2 \right)\left(5.49 \times 10^{-10} \text{ s}\right) \)

\[ = \left(1.80 \times 10^7 \text{ m/s}\right) \text{ i} + \left(4.39 \times 10^7 \text{ m/s}\right) \text{ j} + \left(8.78 \times 10^7 \text{ m/s}\right) \text{ j} \]

\[ = \left(1.84 \times 10^7 \text{ m/s}\right) \text{ i} + \left(8.78 \times 10^7 \text{ m/s}\right) \text{ j} \]

(c) \( |v_f| = \sqrt{\left(1.84 \times 10^7 \text{ m/s}\right)^2 + \left(8.78 \times 10^7 \text{ m/s}\right)^2} = 1.85 \times 10^7 \text{ m/s} \]

(d) \( y = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{8.78 \times 10^7}{1.84 \times 10^7}\right) = 73.7^\circ \)

3.18 We interpret the problem to mean that the displacement from fish to bug is 2.00 m at \( 30^\circ = (2.00 \text{ m})\cos30^\circ \text{ i} + (2.00 \text{ m})\sin30^\circ \text{ j} = (1.73 \text{ m})\text{ i} + (1.00 \text{ m}) \text{ j} \). If the water should drop 0.03 m during its flight, then the fish must aim it at a point 0.03 m above the bug. The initial velocity of the water then is directed through the point with displacement \((1.73 \text{ m})\text{ i} + (1.00 \text{ m}) \text{ j}\) at \( 30^\circ \).

For the time of flight of a water drop we have

\[ x_f = x_i + v_{ix}t + \frac{1}{2}a_xt^2 \]

\[ 1.73 \text{ m} = 0 + \left(2 \times 1.00 \text{ m/s}\right) t + \frac{1}{2} \left(9.8 \text{ m/s}^2\right)t^2 \]

so \( t = \frac{1.73 \text{ m}}{2 \times 1.00 \text{ m/s}} \)

The vertical motion is described by

\[ y_f = y_i + v_{iy}t + \frac{1}{2}a_yt^2 \]

The "drop on its path" is

\[ -3.00 \text{ cm} = \frac{1}{2} \left(9.8 \text{ m/s}^2\right) \left(\frac{1.73 \text{ m}}{2 \times 1.00 \text{ m/s}}\right)^2 \]

Thus, \( v_i = \frac{1.73 \text{ m}}{\cos30^\circ} \sqrt{\frac{9.8 \text{ m/s}^2}{2 \times 0.03 \text{ m}}} = 2.015 \text{ m/s} \left(12.8 \text{ s}^{-1}\right) = 25.8 \text{ m/s} \)

3.28

\( \psi = \frac{v^2}{r} \)

\( v = \sqrt{a_f} = \sqrt{3 \left(9.8 \text{ m/s}^2\right)(9.45 \text{ m})} = 16.7 \text{ m/s} \)

Each revolution carries the astronaut over a distance of \( 2\pi r = 2\pi(9.45 \text{ m}) = 59.4 \text{ m} \). Then the rotation rate is \( \frac{1 \text{ rev}}{59.4 \text{ m}} = 0.281 \text{ rev/s} \)
3.32  (a) See figure to the right.

(b) The components of the 20.2 and the 22.5 m/s² along the rope together constitute the centripetal acceleration:

\[
\alpha = \left[22.5 \text{ m/s}^2\right] \cos(90.0° - 36.9°) + \left[20.2 \text{ m/s}^2\right] \cos 36.9° = 29.7 \text{ m/s}^2
\]

(c) \(\alpha = \frac{v^2}{r}\) so \(v = \sqrt{\alpha r} = \sqrt{29.7 \text{ m/s}^2 (1.50 \text{ m})} = 6.67 \text{ m/s} \) tangent to circle

\[r = \frac{6.67 \text{ m/s}}{36.9°} \text{ above the horizontal}\]

3.39 The satellite is in free fall. Its acceleration is due to gravity and is by effect a centripetal acceleration.

\[a_c = g\]

so \(\frac{v^2}{r} = g\)

Solving for the velocity:

\[v = \sqrt{\frac{8g}{5}} = \sqrt{\frac{6400 + 600}{10^3 \text{ m}}} \sqrt{8.21 \text{ m/s}^2} = 7.58 \times 10^3 \text{ m/s}\]

\[T = \frac{2\pi}{v}\]

and

\[T = \frac{2\pi}{\frac{2\pi(7000 \times 10^3 \text{ m})}{7.58 \times 10^5 \text{ m/s}}} = 5.80 \times 10^5 \text{ s}\]

\[T = 5.80 \times 10^5 \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 98.7 \text{ min}\]

3.42 After the string breaks the ball is a projectile, and reaches the ground at time \(t\):

\[-1.20 \text{ m} = 0 + \frac{1}{2}[-9.80 \text{ m/s}^2]t^2 \quad \text{so} \quad t = 0.495 \text{ s}\]

Its constant horizontal speed is:

\[v_x = \frac{x}{t} = \frac{2.00 \text{ m}}{0.495 \text{ s}} = 4.04 \text{ m/s}\]

so before the string breaks

\[a_x = \frac{v_x^2}{r} = \frac{(4.04 \text{ m/s})^2}{0.300 \text{ m}} = 54.4 \text{ m/s}^2\]

3.47 \(x_f = v_x t = 4.04 \cos 40.0° \)

Thus, when \(x_f = 10.0 \text{ m}\),

\[T = \frac{10.0 \text{ m}}{v_x \cos 40.0°}\]

At this time, \(y_f\) should be

\[3.05 \text{ m} = 2.00 \text{ m} = 1.05 \text{ m}\]

Thus,

\[1.05 \text{ m} = \left[\frac{v_x \sin 40.0°}{v_x \cos 40.0°}\right] 10.0 \text{ m} \left|\frac{1}{2}[-9.80 \text{ m/s}^2]\right| 10.0 \text{ m} \left[\frac{v_x \sin 40.0°}{v_x \cos 40.0°}\right] \]

From this,

\[v_x = 10.7 \text{ m/s}\]
Equation 3.15:
\[ h = \frac{v_0^2 \sin^2 \theta_0}{2g} \]

Equation 3.16:
\[ R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{2v_0^2 \sin \theta_0 \cos \theta_0}{g} \]

If \( h = R/6 \), Equation 3.15 yields
\[ v_0 \sin \theta_0 = \sqrt{\frac{gR}{3}} \quad (1) \]

Substituting Equation (1) above into Equation 3.16 gives
\[ R = \frac{2(\sqrt{gR/3}) v_0 \cos \theta_0}{g} \]

which reduces to
\[ v_0 \cos \theta_0 = \frac{1}{2} \sqrt{gR} \quad (2) \]

(a) From \( v_{yf} = v_{yi} + gt \), the time to reach the peak of the path (where \( v_{yf} = 0 \)) is found to be

\[ t_{peak} = \frac{v_0 \sin \theta_0}{g} \]

Using Equation (1), this gives

\[ t_{peak} = \sqrt{\frac{R}{3g}} \]

The total time of the ball’s flight is then

\[ t_{flight} = 2t_{peak} = 2\sqrt{\frac{R}{3g}} \]

(b) At the path’s peak, the ball moves horizontally with speed

\[ v_{x} = v_{x0} = v_0 \cos \theta_0 \]

Using Equation (1), this becomes

\[ v_{x0} = \frac{1}{2} \sqrt{3gR} \]

(c) The initial vertical component of velocity is

\[ v_{yi} = v_{y0} = v_0 \sin \theta_0 \]

From Equation (1),

\[ v_{yi} = \sqrt{\frac{gR}{3}} \]

(d) Squaring Eq. (1) and (2) and adding the results,

\[ v_0^2 \left( \sin^2 \theta_0 + \cos^2 \theta_0 \right) = \frac{8R}{3} + \frac{3gR}{4} = \frac{13gR}{12} \]

Thus, the initial speed is

\[ v_0 = \sqrt{\frac{13gR}{12}} \]

(e) Dividing Equation (1) by (2) yields

\[ \tan \theta_0 = \frac{v_0 \sin \theta_0}{v_0 \cos \theta_0} = \frac{\left( \sqrt{\frac{gR}{3}} \right)}{\left( \frac{1}{2} \sqrt{3gR} \right)} = \frac{2}{3} \]

Therefore,

\[ \theta_0 = \tan^{-1} \left( \frac{2}{3} \right) = 33.7^\circ \]

(f) For a given initial speed, the projection angle yielding maximum peak height is \( \theta_0 = 90.0^\circ \). With the speed found in (d), Equation 3.15 then yields

\[ h_{max} = \frac{(13gR/12) \sin^2 90.0^\circ}{2g} = \frac{13}{22} R \]

(g) For a given initial speed, the projection angle yielding maximum range is \( \theta_0 = 45.0^\circ \). With the speed found in (d), Equation 3.16 then gives

\[ R_{max} = \frac{(13gR/12) \sin 90.0^\circ}{g} = \frac{13}{12} R \]
3.50 (a) \[ \Delta y = -\frac{1}{2}gt^2; \quad \Delta x = v_x t \]

Combine the equations eliminating \( t \):

\[ \Delta y = -\frac{1}{2N} \left( \frac{\Delta x}{v_x} \right)^2 \]

From this,

\[ (\Delta x)^2 = \frac{-2\Delta y}{g} \]

thus

\[ \Delta x = v_x \sqrt{\frac{-2\Delta y}{g}} = 275 \sqrt{\frac{2(-3000)}{9.80}} = 6.80 \times 10^3 = 6.80 \text{ km} \]

(b) The plane has the same velocity as the bomb in the \( x \) direction.

Therefore, the plane will be 3000 m directly above the bomb when it hits the ground.

(c) When \( \phi \) is measured from the vertical, \( \tan \phi = \frac{\Delta x}{\Delta y} \)

therefore,

\[ \phi = \tan^{-1} \left( \frac{\Delta x}{\Delta y} \right) = \tan^{-1} \left( \frac{6800}{3000} \right) = 66.2^\circ \]