High Energy Astrophysics

Problem Set 2

Jan 28 2004

1. Compute the maximal value of radius for a planet which occurs when the pressure due to gravity becomes so large as to ionize atoms. Assume the relation between density and average atomic mass and also that the planet is made up mostly of Hydrogen. Compare it with radius of Jupiter.

2. If the pressure due to the weight of a mountain on a planet exceeds the yield pressure of the material then the mountain will "sink". Assume that the planet has an average atomic mass of A=20. To calculate the pressure due to weight you have to know local g at the surface of the planet and the formula for energy density for yield, which is

\[
\epsilon_{\text{yield}} = 0.03 \epsilon_{\text{binding}}^H / A^{0.8}
\]

in kg/m\(^3\). Using the values of M and R of planets and Newton's gravitational constant, show that the maximum height of mountains are:

(a) Earth : 64 km
(b) Mars : 150 km
(c) Moon : 300 km

Why are there no mountains as high as these numbers predict? Also note that if H \sim R than the object will not have spherical shape.

3. Sun is a sphere of fully ionized atoms. Let X, Y and Z be the concentrations of the atoms of H, He and heavier elements by mass and assume the average atomic mass of the heavier elements is A. If the density of this plasma is \( \rho \) Kg/m\(^3\), calculate value of n, the number of particles per unit volume in the equation of state which describes this non-degenerate classical gas, \( P = n k T \) in terms of \( X, Y, Z, \rho \) and \( m_H \) the mass of the proton. Here T is to be taken as the mean temperature of the sun.
To this must be added the pressure due to radiation which is in equilibrium. Write down the expression for total pressure.

If the mean density of the sun is 1600 Kgm/m³ and the effective temperature is 10⁷ °K calculate the magnitudes of the particle pressure and radiative pressure. Assume X=0.73, Y=0.25, Z=0.02 and A=20.

4. Sketch the Planck energy density distribution law for the case of our sun whose surface temperature is about 6000° K.

(a) At what frequency is the energy density maximum?

(b) How much energy is radiated per square meter of the sun?

(c) Where would the energy distribution be a maximum if the temperature of an object was only 100° K?