PARTICLE CREATION BY BLACK HOLES

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- Three Types of Black Holes (BH)
- Unruh Effect
- Particle Creation by BH
- Black Hole Thermodynamics
- Back-Reaction Effect
- Conclusion and Outlook
- Artificial BH

General Relativity Workshop
UC IRVINE, June 13, 2003

THREE TYPES OF BLACK HOLES

- supernova explosion, X-ray Binaries
  - collapse of massive stars can result in BH with mass up to $M_{BH} \approx 10 M_\odot$
  - if member of binary systems $\longrightarrow$ detected by influence of companion star

- supermassive black holes
  - deep gravitational potential well at center of galaxy $\longrightarrow$ gravitational collapse
  - supermassive BH at center of galaxies up to mass $M_{BH} \approx M_\odot^9$

- primordial black holes
  - early fluctuations might have grown and collapsed under gravitation
  - production of small primordial black holes with mass of order $M_{BH} \approx M_\odot^{-19}$ would explode today quantummechanically due to Hawking radiation

THE UNRUH EFFECT

- Unruh Effect $\longrightarrow$ Hawking Effect
- Quantum Field Theory (QFT) in flat and curved spacetime
- Vacuum state $|0>_I$ (Rindler vacuum) at rest
- Particle state $|0>_M$ in QFT of flat spacetime seen by observer who undergo to acceleration a
- Start with pure state $|0>_I \longrightarrow$ mixed state $|0>_M$
- Solving density matrix of the new particle $\longrightarrow$ thermal density matrix $\rho = exp(-H_I/T)$.
- Particle $|0>_M$ precisely a thermal state with temperature

$$T = \frac{a}{2\pi} \tag{1}$$

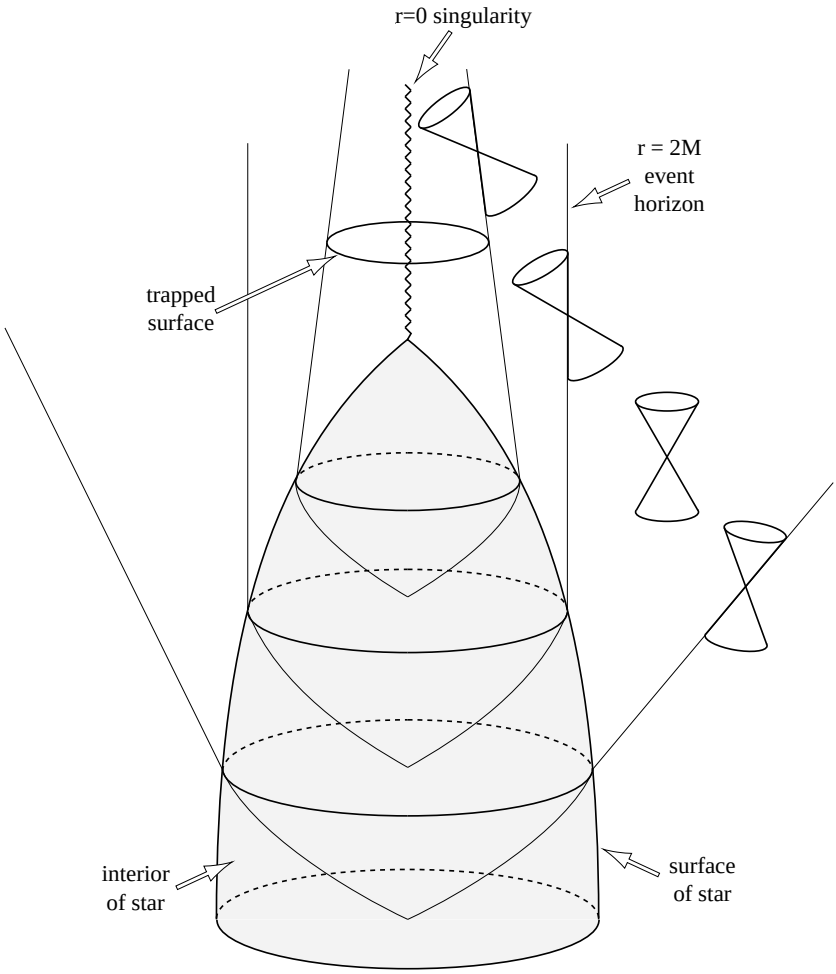
- This is called the Unruh effect
- Interpretation: An accelerating observer feels himself immersed in a thermal bath of particles at temperature $T = a/2\pi$

$$\frac{T}{1K} = \frac{a}{10^{21}\mathrm{cmsec}^{-2}} \tag{2}$$

- Extremely small effect for accelerations achievable by macroscopic bodies

BLACK HOLE FORMATION

- Collapsing star $\longrightarrow$ BH: so strong gravitational field that light cones will be bent inward



- Outgoing light rays are converging rather that diverging $\longrightarrow$ closed trapped surface

- Trapped surface and singularity cannot be visible from far away

- Region of spacetime where is impossible to escape to infinity
- This Region is called Black Hole (BH)
- Its boundary is called Event Horizon (EH)
- EH is a Null surface $\longrightarrow$ light fails to get away to infinity
- Area A of EH can never decrease $\longrightarrow$ BH is in a stationary state
- Body collapses$\longrightarrow$ BH $\longrightarrow$ large amount of information is lost (no hair theorem)

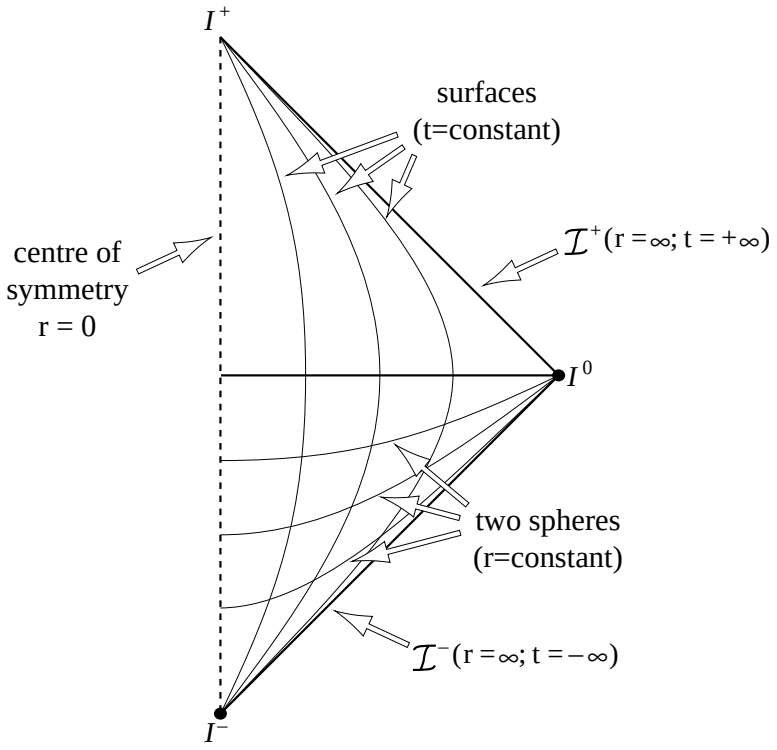
BH DISAPPEAR LOSS OF INFO

- No problem for classical theory, BUT not for QFT
- Body sends out limited amount of photons until cross the EH
- Too few to carry out all information
- in Classical Theory: no way for an outside observer can measure the state of the collapsing body (no big deal!)
- BUT QFT important $\longrightarrow$ only in QFT BH radiate and lose mass
- Eventually BH disappear completely, taking with them all information (ZAG!)
- Loss of information introduces $\longrightarrow$ new level of uncertainty above the usual uncertainty of the quantum theory
- BUT: very difficult to prove it experimentally

SPHERICAL COLLAPSE

- Spherical collapse $\longrightarrow$ spacetime doesn't depend on angles θ and ϕ, all geometry in $r - t$ plane

- 2-dim plane is conformal to flat space: Null lines are at 45^o



- Two diagonal sides: Past and Future Null Infinities

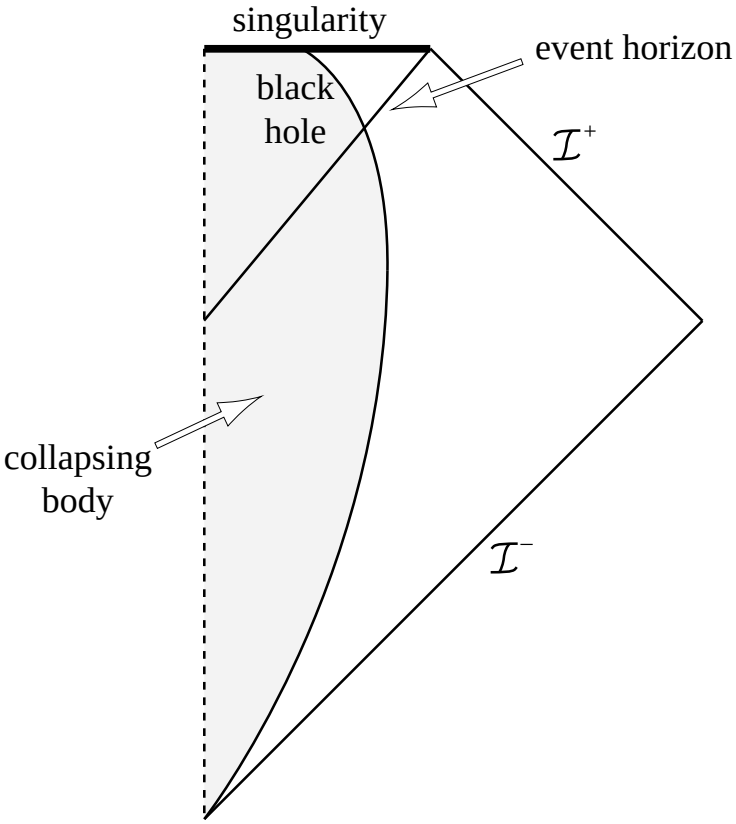
- Each point of Triangle corresponds to a two sphere of radius r

- $r = 0$ on the left, $r = \infty$ on th right

- In Minkowski space every point is in the past of Null Infinity $\mathscr{I}^+ \longrightarrow$ no BH and no EH

GRAVITATIONAL COLLAPSE

- BUT diagram change if body collapsing



- Cut Off $\longrightarrow$ horizontal boundary: singularity (Hawking-Penrose Th)

- Points under singularity not in the $\Im^+ \longrightarrow$ this is a BH

PARTICLE CREATION BY BH

- Quantum Field effect of gravitational collapse$\longrightarrow$ BH
- Spacetime geometry: Schwarzschild Black Hole
- QFT construction "in" H_{in} and "out" H_{out} representation
- Cauchy Surface intersecting the EH defines two group of solutions: inside BH (early-time); outside BH (late-time).
- Calculate particle creation: S-matrix needed U : $F(H_{in}) \longrightarrow F(H_{out})$
- Relation In-Out representation $\longrightarrow$ Relation In-Out at the Event Horizon of BH.
- Calculate $U|0_{in} > \longrightarrow$ density matrix ρ in the Fock space $F(H_{out} \longrightarrow$ spontaneous creation of particle

PARTICLE CREATION BY BH

- Scalar field ϕ on this background
- Spacetime time independent, ϕ solution of WE, only pos. freq. on $\Im^-$ would give pos. freq on $\Im^+$ too.
- In this case no mixing of frequency $longrightarrow$ no particle creation!
- NOW: metric time-dependent during collapse
- then ϕ solution of the WE. pos. freq. on $\Im^-$ and partly neg. freq. when it gets to on $\Im^+$
- MIXING: take wave $e^{-i\omega u}$ on $\Im^+$ and propagate it back in neg. and pos. frequency to $\Im^-$
- ϕ mixed state of positive and negative frequencies
- the mixing depends only on the surface gravity κ which measure the strength of the gravitational field on the Horizon of the BH
- MIXING of positive and negative frequencies $\longrightarrow$ PARTICLE CREATION

$$\Psi_{in} = \Psi_{1_{in}} + e^(-\pi\omega/\kappa)\Psi_{2_{in}} \quad (3)$$

$$\Psi_{out} = \Psi_{1_{out}} + e^(-\pi\omega/\kappa)\Psi_{2_{out}} \quad (4)$$

Ψ_{in} dump into BH; Ψ_{out} escapes to infinity

UNRUH EFFECT-HAWKING T

- Unruh effect in curved spacetime $\longrightarrow$ Hawking Temperature

$$T = \frac{\kappa}{2\pi} \tag{5}$$

- κ: surface gravity of BH: limiting value of the force at EH

- Unruh effect in curved Schwarzschild spacetime gives the value: $\kappa = 14M$ where M is the mass of the Schwarzschild spacetime

- Non-singular vacuum state of the Schwarzschild spacetime: Hartle-Hawking vacuum $U|0_{in} >$ thermal state at temperature $T = \kappa/2\pi$

- Schwarzschild BH formed by gravitational collapse radiates precisely as a thermal blackbody (BB) at temperature:

$$T = \frac{1}{8\pi M} = \frac{\hbar c^3}{8\pi k G M} = 6\text{x}10^{-8} \left(\frac{\mathrm{M}_\odot}{\mathrm{M}}\right) \mathrm{K} \tag{6}$$

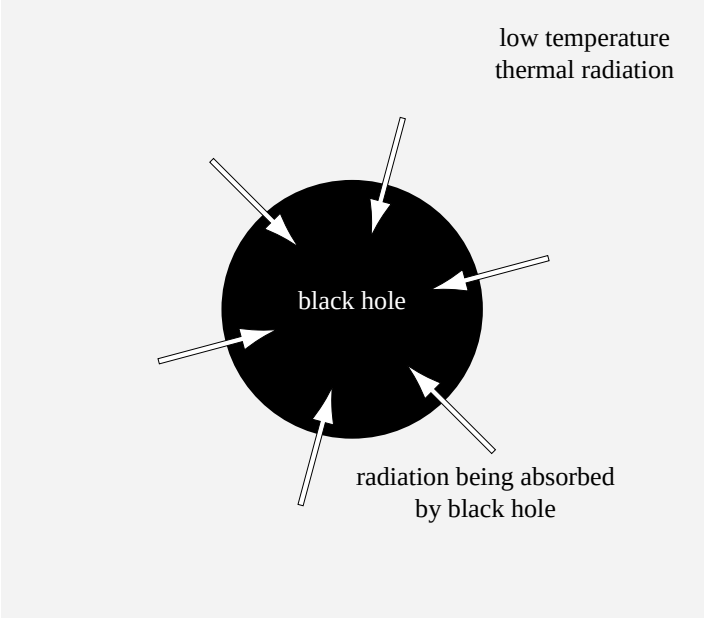
- Corresponds to particle creation by BH at late time after formation of BH, which appear to emerge from BH at this temperature.

- this holds only at late time, i.e for all time s.t. $t - t_0 >> t_D$ where $t_0 =$ BH formation and $t_D =$ dynamical timescale

$$t_D \approx 10^{-5} \frac{M}{M_\odot} sec \tag{7}$$

- BH approaches its perfect BB state very rapidly!

- So big surprise that a very messy calculation of GR leads to particle emission which is exactly thermal!



BLACK HOLE THERMODYNAMICS

- Hawking Effect $\longrightarrow$ connection BH mechanics and Law of thermodynamics

- First Law of Thermodynamics

$$T\Delta S = \Delta E + P\Delta V \tag{8}$$

- First Law of BH Mechanics

$$\kappa\Delta A = 8\pi(\Delta M - \Omega\Delta J) \tag{9}$$

- ΔA: change of the area of EH; ΔM:change in mass of a BH; ΔJ: change of its angular momentum.

- Mass of the BH plays the same role as the total energy of thermodynamical system

- Mass-Energy $\longrightarrow$ same physical quantity $\longrightarrow$ 1. Law of BH mechanics might have some physical content

- Physical temperature T of a BH is the absolute zero

- BUT: Hawking Effect shows $\kappa/2\pi$ is the true physical temperature of BH

- $M \longleftrightarrow E$ and $\kappa/2\pi \longleftrightarrow T$
- so $A/4 \longrightarrow$ must be the physical entropy of a BH!

ENTROPY OF A BLACK HOLE

- Physical entropy of a BH:

$$S_{BH} = \frac{kc^3}{4G\hbar}A \tag{10}$$

- Enormous entropy, for BH of $M_\odot$, $S_{BH}/k \approx 10^{77}$, while for the sun $S_\odot/k \approx 10^{58}$

- Is this valid? Answer: only with a complete quantum theory of gravity

- However Euclidean approach to Quantum Gravity and Canonical Ensemble $\longrightarrow$partition function which precisely gives the same results!

- Temperature of a BH $\kappa \propto 1/M$ (energy-mass)$\longrightarrow$ Schwarzschild BH has a negative heat capacity

- BUT self-graviting stars have also negative heat capacity $\longrightarrow$ in fact remove energy from star, it contracts and heats up!

GENERALIZED SECOND LAW

- Second Law of Thermodynamics

$$\Delta S \geq 0 \tag{11}$$

- Second Law of BH Mechanics

$$\Delta A \geq 0 \tag{12}$$

- BUT when BH forms a problem arises

- take matter and dump into BH $\longrightarrow$ it will disappear into singularity within BH

- Total Entropy of matter S_{matter} in the universe $\longrightarrow$ decreases$\longrightarrow$ one would expect A to increase

- BUT A actually decreases (back-reaction effect) $\longrightarrow$ one would expect S_{matter} increasing: paradox!

- Something needed which makes S_{matter} and A_{BH} decreasing individually and some general Entropy which never decreases:

$$S^{'} = S_{matter} + \frac{1}{4} A_{BH} \tag{13}$$

- Generalized second law, Bekestein (1974).
- This is the ordinary second law applied to a system containing BH
- BH have intrinsic entropy and lose information from our region of the universe
- Would be possible that information about the quantum state of a system could be lost?
- BUT: how to get out information from a BH?
- BH entropy: lose of information $\longrightarrow$ BH radiation.

BACK-REACTION EFFECT

- Energy properties of the quantum field are described by semi-classically Einstein Equation

$$G_{ab} = 8\pi < T_{ab} > \tag{14}$$

- Energy must be conserved.

- However the particle created escaping to infinity must be connected with a flux of the form

$$F = \frac{\alpha}{M^2} \tag{15}$$

- Since thermal emission $\longrightarrow$ Stefan-Boltzmann flux law

$$F_{S-B} = \sigma A T^4 \; T = \frac{1}{8\pi M}; \; A = 4\pi(3\sqrt{3}M)^2 \tag{16}$$

- Conservation of energy flux at the BH $\longrightarrow$ negative energy flux equal in magnitude to $F = \alpha/M^2$ must flow into the BH.

- This is the so-called Back-Reaction Effect

- BH lose mass

- Calculate this effect $\longrightarrow$ solve the semi-classical Einstein equation.

- BH mass $M_{BH} >> M_P = (c\hbar/G)^{1/2} \approx 10^{-5} gm \longrightarrow$ M decreases slowly with time according to $F = \alpha/M^2$:

$$\frac{dM}{dT} = -F = -\frac{\alpha}{M^2} M(t) = (M_0^3 - 3\alpha t)^{1/3} \qquad (17)$$

- BH should completely evaporate in a finite time $t_f = M_0^3/3\alpha$

- Rate emission becomes very large just before t_f

- BH lifetime

$$\tau = 8.3\text{x}10^{-26} \left(\frac{\text{M}}{1\text{g}}\right)^3 \text{sec} \qquad (18)$$

- The lifetime of BH of mass $\approx M_\odot$ is $\approx 10^{66}$ yr

- BH formed in the early universe with $M_{BH} \approx 10^{14} g$ would be exploding today

- smaller BH would have already evaporated!

BARYON NUMBER VIOLATION

- ramification of BH evaporation
- BH of $M_{\odot}$ by collapse of neutron star $\longrightarrow$ baryon number $\approx 10^{57}$
- BH evaporate completely $\longrightarrow$ in order not to violate baryon number BH should evaporate in a baryon-antibaryon symmetric manner.
- so expected only radiation carrying zero baryon number $\longrightarrow$ which is implausible
- Baryon number can be grossly violated!

LOSS OF QUANTUM COHERENCE

- Hawking Effect $\longrightarrow$ loss of quantum coherence
- Back-Reaction $\longrightarrow$ BH evaporates
- S-matrix: particles propagating to infinity must be correlated with particles entering the BH
- However: BH disappear from the spacetime $\longrightarrow$ at late time entire state of the field is mixed
- Ordinary S-matrix cannot describe the particle creation, since PURE QM state evolves to density matrix of MIXED state.
- This is the phenomenon of loss of quantum coherence

CONCLUSION AND OUTLOOK

- Gravity + General Relativity: completely different from other field theory
- Gravity causes spacetime to have a beginning and end
- Discovery of intrinsic gravitational entropy
- Some people say "those are just artifact predictions of semi-classical approximation"
- Other people claim "string theory should be the true quantum theory of gravity"
- BUT: so far string theory is incapable of making any predictions except by appealling to GR as low energy theory
- Black Hole radiation is testable. All to do is to find a primordial black hole!
- If there had been we would know how to quantize gravity
- Gravitation is different to other field theory, since it shapes the arena in which it acts

- Gravitation shapes its spacetime leading to regions of the universe which one can't observe

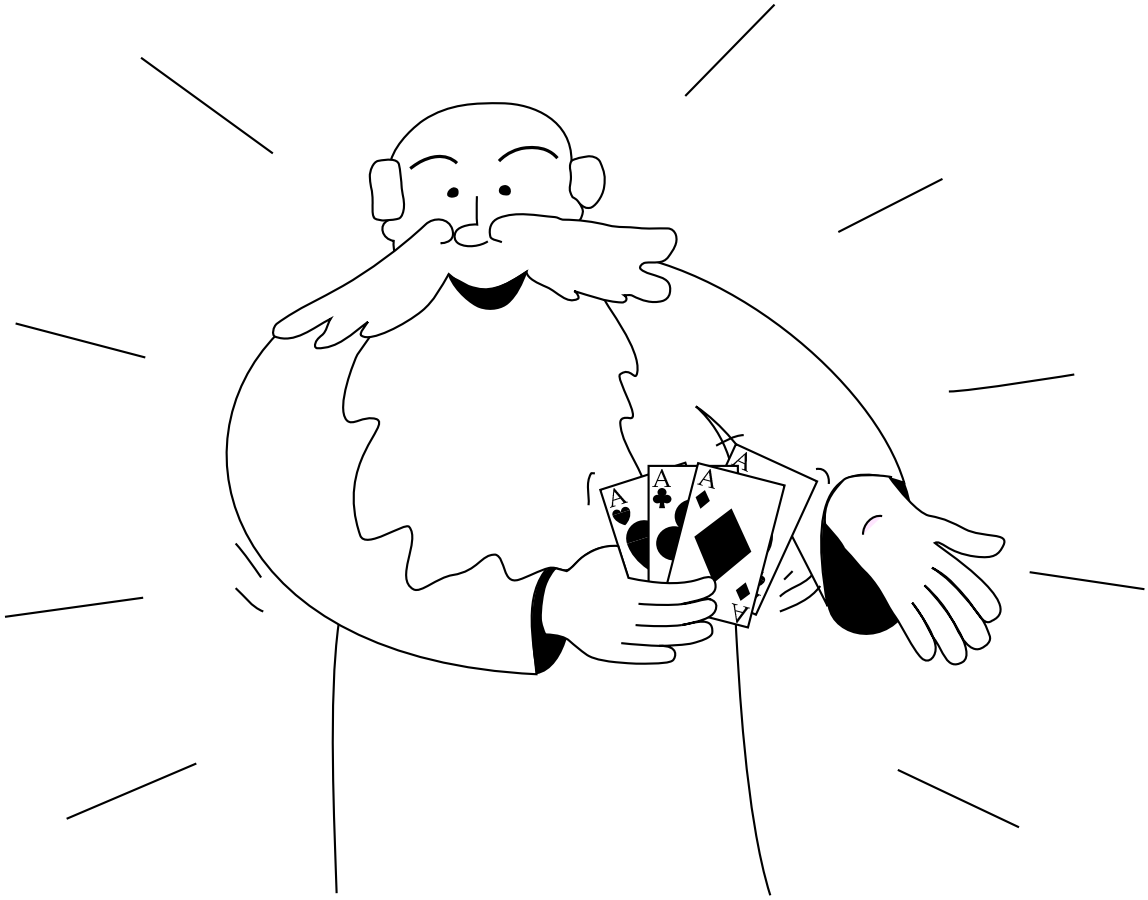
- gravitational entropy as measure of what we can't know

- So Einstein might be wrong when he said: "God does not play dice"

- Black Holes tell us: "Not only God does play dice, but He sometimes confuse us by throwing them whew they can't be seen"

ARTIFICIAL BLACK HOLES

- Condensed-matter analogue of a black hole: A SONIC HOLE
- Quantum fluid $\longrightarrow$ super-sonic horizon
- Pair of phonons: before and behind the horizon
- Bose-Einstein condensation
- Quantum gravity ??? BUT dilute quantum gases !!
- Theory needed which bridges Hawking radiation to super-sonic transition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