MODELING NUCLEAR EFFECTS IN NEUTRINO-NUCLEUS INTERACTIONS

(DELORME-MARTEAU MODEL REVISITED)

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MOTIVATION

o good description of Δ excitation region required by MC

o candidate: Delorme-Marteau model

o original (Marteau's PhD thesis) very complicated - monopole expansions etc, but can be simplified

DESCRIPTION OF THE MODEL

o production of Δ and quasielastic process

o Fermi gas & RPA computations

o Δ width with nuclear effects after Oset

o inclusive cross section & exclusive contributions

MODIFICATIONS

o relativistic Lindhard function

o careful treatment of Δ width

o no density profile (to be done by MC)

o no genuine $2p-2h$ part (can be added; originally not put into RPA framework)

TECHNICAL DETAILS

basic formula

$$\frac{d^2\sigma}{dqd\omega} = \frac{G^2 \cos\theta_c^2}{32\pi} \cdot \frac{q}{E^2} \cdot (-\frac{Vol}{\pi}) L_{\mu\nu} H^{\mu\nu}$$

$$L_{\mu\nu} = 8(k'_\mu k_\nu + k_\mu k'_\nu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\alpha\beta} k'^\alpha k^\beta),$$

k, k' - lepton 4-momenta; $(\omega,\ \vec{q})$ energy and momentum transfer; frame $\vec{q} = (q,0,0)$, $Vol = \frac{3\pi^2 A}{2k_F^3}$, $x, y \in \{N, \Delta\}$, $E_q^x = \sqrt{M_x^2 + q^2}$

$$H^{\mu\nu} = H^{\mu\nu}_{NN} + H^{\mu\nu}_{N\Delta} + H^{\mu\nu}_{\Delta N} + H^{\mu\nu}_{\Delta\Delta}$$

$$H_{xy}^{00} = \sqrt{\frac{M_x + E_q^x}{2M_x}} \sqrt{\frac{M_y + E_q^y}{2M_y}} (\alpha_{0x}^0 \alpha_{0y}^0 R_{xy}^c + \beta_{0x}^0 \beta_{0y}^0 R_{xy}^l)$$

$$\alpha_{0x}^0 = F_1(\omega, q) - F_2(\omega, q) \frac{q^2}{2M(M_x + E_q^x)}$$

$$\beta_{0x}^0 = q\Big(\frac{G_A(\omega, q)}{M_x + E_q^x} - \frac{\omega}{2M} \cdot \frac{G_P(\omega, q)}{M_x + E_q^x}\Big) \cdots$$

Fermi gas

$$R_{N\Delta} = R_{\Delta N} = 0$$

$$R_{NN}^{c,l,t} = Im(\Pi_{NN}^0)$$

$$R_{\Delta\Delta}^{l,t} = 4.78 \cdot Im(\Pi_{\Delta\Delta}^0)$$

$$Im(\Pi_{NN}^0(\omega, q)) = -\frac{M^2}{2\pi^2} \cdot$$

$$\int d^3p \frac{\delta(\omega + E_{\vec{p}} - E_{\vec{q}+\vec{p}})}{E_{\vec{p}}E_{\vec{q}+\vec{p}}}\theta(k_F - |\vec{p}|)\theta(|\vec{q}+\vec{p}| - k_F)$$

$$Im(\Pi^0_{\Delta\Delta}(\omega, q)) = -\frac{4M_\Delta^2}{9\pi^3}.$$

$$\int d^3p \frac{\Gamma_\Delta \cdot \theta(k_F - |\vec{p}|)}{(s - M_\Delta^2)^2 + M_\Delta^2\Gamma_\Delta^2}$$

$s = (p+q)^2$, p nucleon 4−momentum.

Pauli blocking

$$\Gamma_\Delta = PBL \cdot \Gamma_{\pi N} - 2Im(\Sigma_\Delta)$$

$PBL \in [0, 1]$

$$PB = \frac{|\vec{p}_\Delta||\vec{q}_{cmf}| - \sqrt{s}E_F + E_\Delta E_{cmf}}{2|\vec{p}_\Delta||\vec{q}_{cmf}|}$$

$PBL = 1$ if $PB > 1$; $PBL = 0$ if $PB < 0$; otherwise $PBL = PB$. $(E_\Delta, \vec{p}_\Delta)$

Δ 4−momentum, $(E_{cmf}, \vec{q}_{cmf})$ pion (from Δ decay) 4−momentum in the cmf.

$$\Gamma_{\pi N} = q_{cm}^3 \frac{f^2}{6\pi m_\pi^2}\frac{M}{\sqrt{s}},$$

In $Im(\Sigma_\Delta)$ contributions from $\Delta \to \pi N$, $N\Delta \to NN$, $NN\Delta \to NNN$ after Oset &Salcedo.

$$Im(\Sigma_\Delta) = Im(\Sigma_\Delta^\pi) + Im(\Sigma_\Delta^{NN,NNN})$$

Because of different kinematics ($\omega^2 - q^2 < 0$) an approximate prescription

$$Im(\Sigma_\Delta)_\nu = Im(\Sigma_\Delta)_\gamma + (Im(\Sigma_\Delta)_\gamma - Im(\Sigma_\Delta)_\pi)$$

$Im(\Sigma_\Delta)_\nu$ treated as a function of s.

$$Re(\Pi^0_{\Delta\Delta}(\omega, q)) = \frac{4M_\Delta}{9\pi^3}\times$$

$$\int d^3p\left(\frac{s - M_\Delta^2}{(s - M_\Delta^2)^2 + M_\Delta^2\Gamma_\Delta^2} - \frac{u - M_\Delta^2}{(u - M_\Delta^2)^2 + \epsilon^2}\right)$$

ϵ - small regularization to avoid a pole.

$$u = M^2 + \omega^2 - q^2 - \frac{(s + M^2 - m_\pi^2)(s - M^2 - \omega^2 + q^2)}{2s}$$

RPA equations (separately for c,l,t)

$$\Pi_{NN} = \Pi^0_{NN} + \Pi^0_{NN} V_{NN} \Pi_{NN} + \Pi^0_{NN} V_{N\Delta} \Pi_{\Delta N},$$

$$\Pi_{\Delta\Delta} = \Pi^0_{\Delta\Delta} + \Pi^0_{\Delta\Delta} V_{\Delta N} \Pi_{N\Delta} + \Pi^0_{\Delta\Delta} V_{\Delta\Delta} \Pi_{\Delta\Delta},$$

$$\Pi_{N\Delta} = \Pi^0_{NN} V_{NN} \Pi_{N\Delta} + \Pi^0_{NN} V_{N\Delta} \Pi_{\Delta\Delta},$$

$$\Pi_{\Delta N} = \Pi^0_{\Delta\Delta} V_{\Delta N} \Pi_{NN} + \Pi^0_{\Delta\Delta} V_{\Delta\Delta} \Pi_{\Delta N}.$$

solutions

$$D \equiv (1 - V_{NN}\Pi^0_{NN})(1 - V_{\Delta\Delta}\Pi^0_{NN}) - V^2_{N\Delta}\Pi^0_{NN}\Pi^0_{\Delta\Delta}$$

$$\Pi_{NN} = \frac{\Pi^0_{NN}(1 - V_{\Delta\Delta}\Pi^0_{\Delta\Delta})}{D}$$

$$\Pi_{\Delta\Delta} = \frac{\Pi^0_{\Delta\Delta}(1 - V_{NN}\Pi^0_{NN})}{D}$$

$$\Pi_{N\Delta} = \Pi_{\Delta N} = \frac{V_{N\Delta}\Pi^0_{\Delta\Delta}\Pi^0_{NN}}{D}$$

$$R_{NN} = Im(\Pi_{NN}),$$

$$R_{N\Delta} = \sqrt{4.78} \cdot Im(\Pi_{N\Delta}),$$

$$R_{\Delta\Delta} = 4.78 \cdot Im(\Pi_{\Delta\Delta}).$$

interactions (exchange of π, ρ and contact terms)

$$V_c^{NN} = \frac{0.6}{m_\pi^2},$$

$$V_l^{NN} = \frac{4\pi \cdot 0.08}{m_\pi^2}(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \omega^2 + q^2})^2(0.7 + \frac{q^2}{\omega^2 - q^2 - m_\pi^2})$$

$$V_t^{NN} = \frac{4\pi \cdot 0.08}{m_\pi^2}\Big(0.7 \cdot (\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \omega^2 + q^2})^2 +$$

$$C_\rho^2 \frac{q^2}{\omega^2 - q^2 - m_\varrho^2} \cdot (\frac{\Lambda_\varrho^2 - m_\varrho^2}{\Lambda_\varrho^2 - \omega^2 + q^2})^2\Big)$$

$$V_l^{\Delta\Delta} = 4.78 \cdot \frac{4\pi \cdot 0.08}{m_\pi^2}(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \omega^2 + q^2})^2$$

$$(0.5 + \frac{q^2}{\omega^2 - q^2 - m_\pi^2})$$

$$V_t^{\Delta\Delta} = 4.78 \cdot \frac{4\pi \cdot 0.08}{m_\pi^2}\Big(g_\Delta \cdot (\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \omega^2 + q^2})^2 +$$

$$C_\rho^2 \frac{q^2}{\omega^2 - q^2 - m_\varrho^2} \cdot (\frac{\Lambda_\varrho^2 - m_\varrho^2}{\Lambda_\varrho^2 - \omega^2 + q^2})^2),$$

$$V_l^{N\Delta} = V_l^{\Delta N} = \sqrt{4.78} \cdot \frac{4\pi \cdot 0.08}{m_\pi^2} \times$$

$$(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \omega^2 + q^2})^2 (0.5 + \frac{q^2}{\omega^2 - q^2 - m_\pi^2})$$

$$V_t^{N\Delta} = V_t^{\Delta N} = \sqrt{4.78} \cdot \frac{4\pi \cdot 0.08}{m_\pi^2} \Big(g_\Delta \cdot (\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \omega^2 + q^2})^2 +$$

$$C_\rho^2 \frac{q^2}{\omega^2 - q^2 - m_\varrho^2} \cdot (\frac{\Lambda_\varrho^2 - m_\varrho^2}{\Lambda_\varrho^2 - \omega^2 + q^2})^2 \Big),$$

$C_\rho^2 = 2$, $g_\Delta = 0.5$, $\Lambda_\pi = 1000 MeV$, $\Lambda_\varrho = 1500 MeV$.

pion production

Δ width and response function are split into two parts

$$\Gamma_\Delta^\pi = PBL \cdot \Gamma_{\pi N} - 2Im(\Sigma_\Delta^\pi)$$

$$Im(\Pi^0_{\Delta\Delta}) = Im(\Pi^0_{\Delta\Delta})_\pi + Im(\Pi^0_{\Delta\Delta})_{NN,NNN}$$

$$Im(\Pi^0_{\Delta\Delta})_\pi = -\frac{4M^2_\Delta}{9\pi^3}\times$$

$$\int d^3p \frac{\Gamma^\pi_\Delta \cdot \theta(k_F - |\vec{p}|)}{(s - M^2_\Delta)^2 + M^2_\Delta \Gamma^2_\Delta}$$

Pions are produced in different *channels*

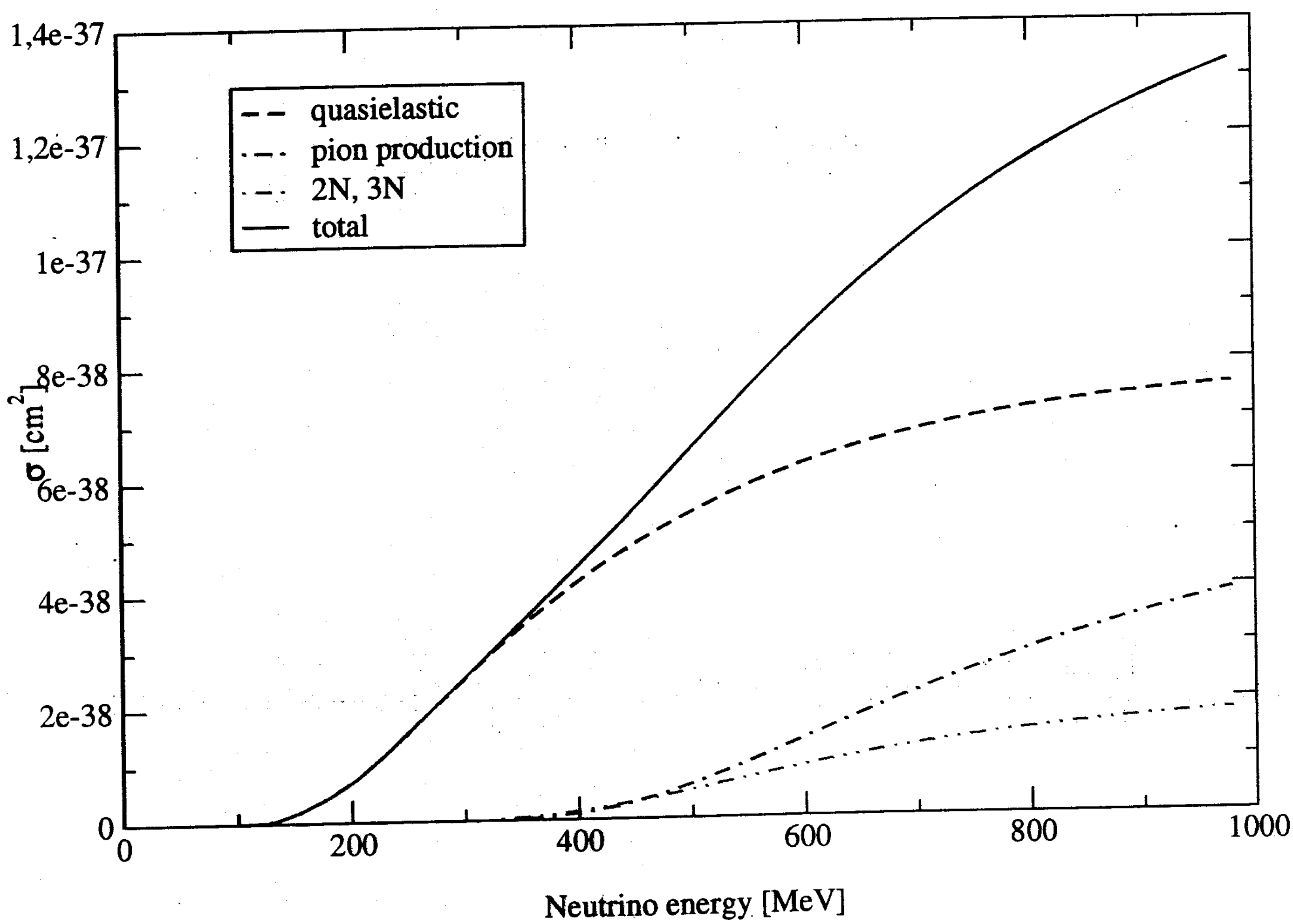
$$Im(\Pi_{NN})_\pi = \frac{|\Pi^0_{NN}|^2 (V_{N\Delta})^2 Im(\Pi^0_{\Delta\Delta})_\pi}{|D|^2},$$

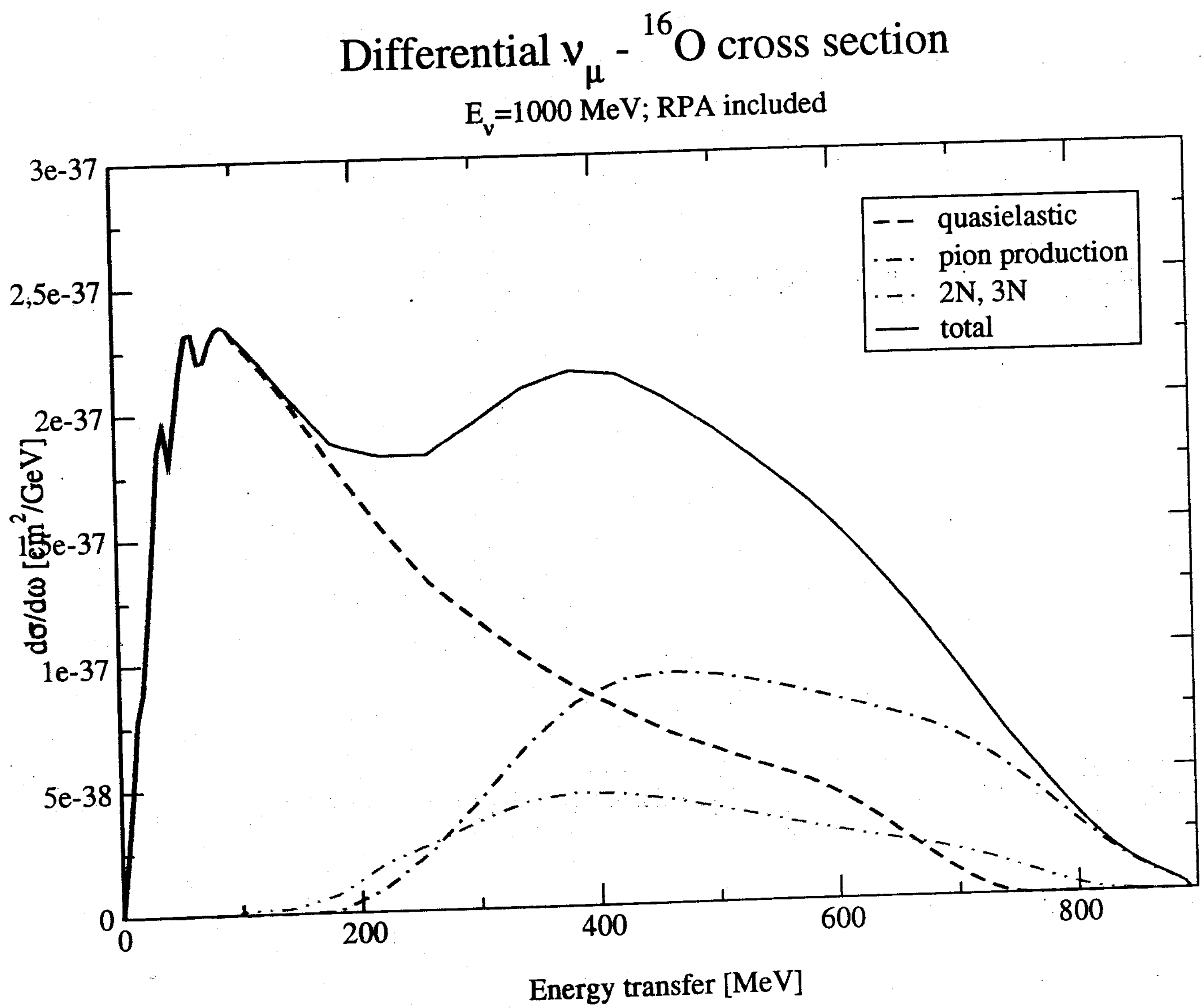
$$Im(\Pi_{N\Delta})_\pi + Im(\Pi_{\Delta N})_\pi =$$

$$\frac{(2V_{N\Delta} Re(\Pi^0_{NN}) - 2V_{N\Delta} V_{NN} |\Pi^0_{NN}|^2) Im(\Pi^0_{\Delta\Delta})_\pi}{|D|^2},$$

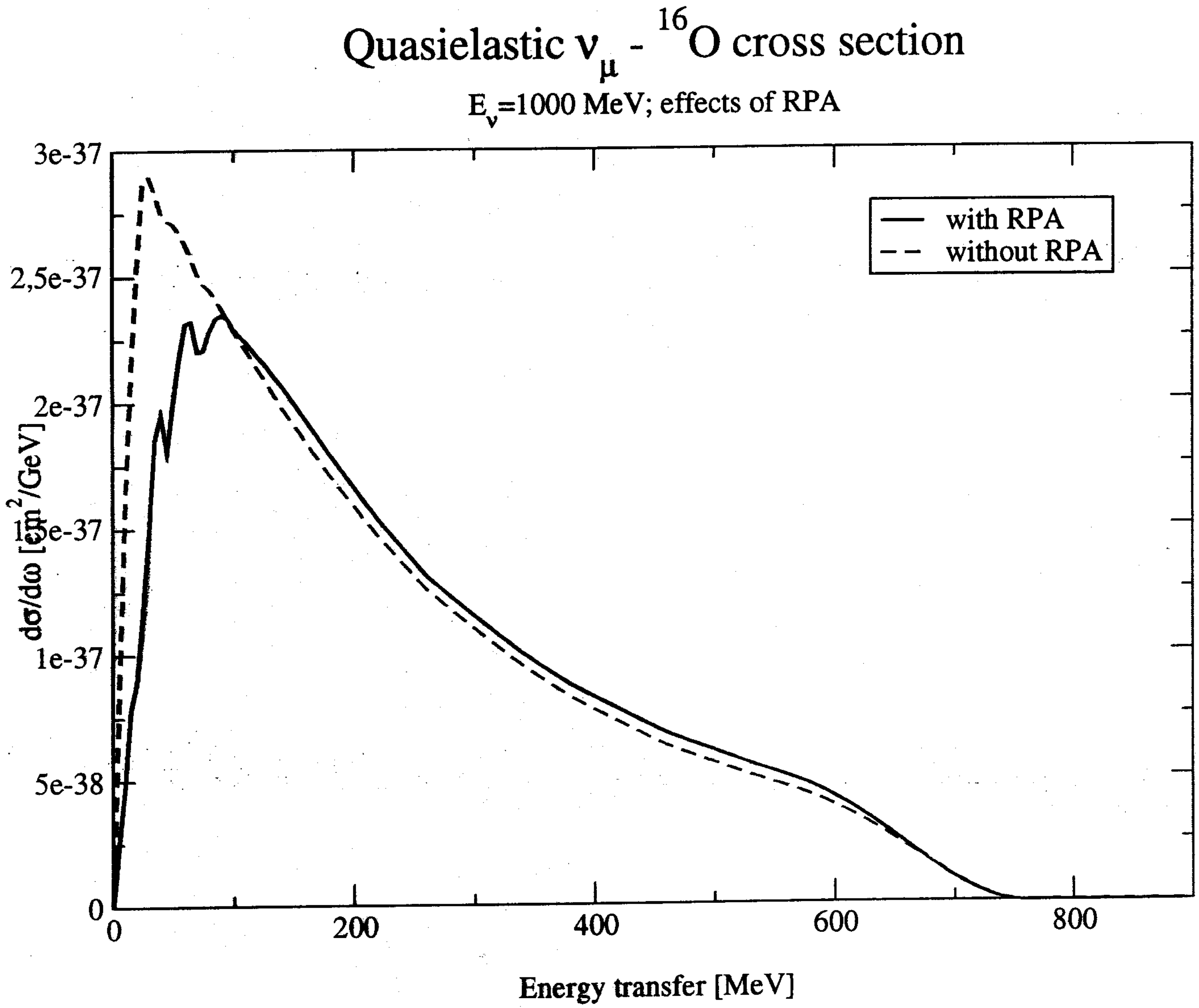
$$Im(\Pi_{\Delta\Delta})_\pi = \frac{|(1 - V_{NN}\Pi^0_{NN})|^2 Im(\Pi^0_{\Delta\Delta})_\pi}{|D|^2}$$

ν_μ - ^{16}O cross section (with RPA)

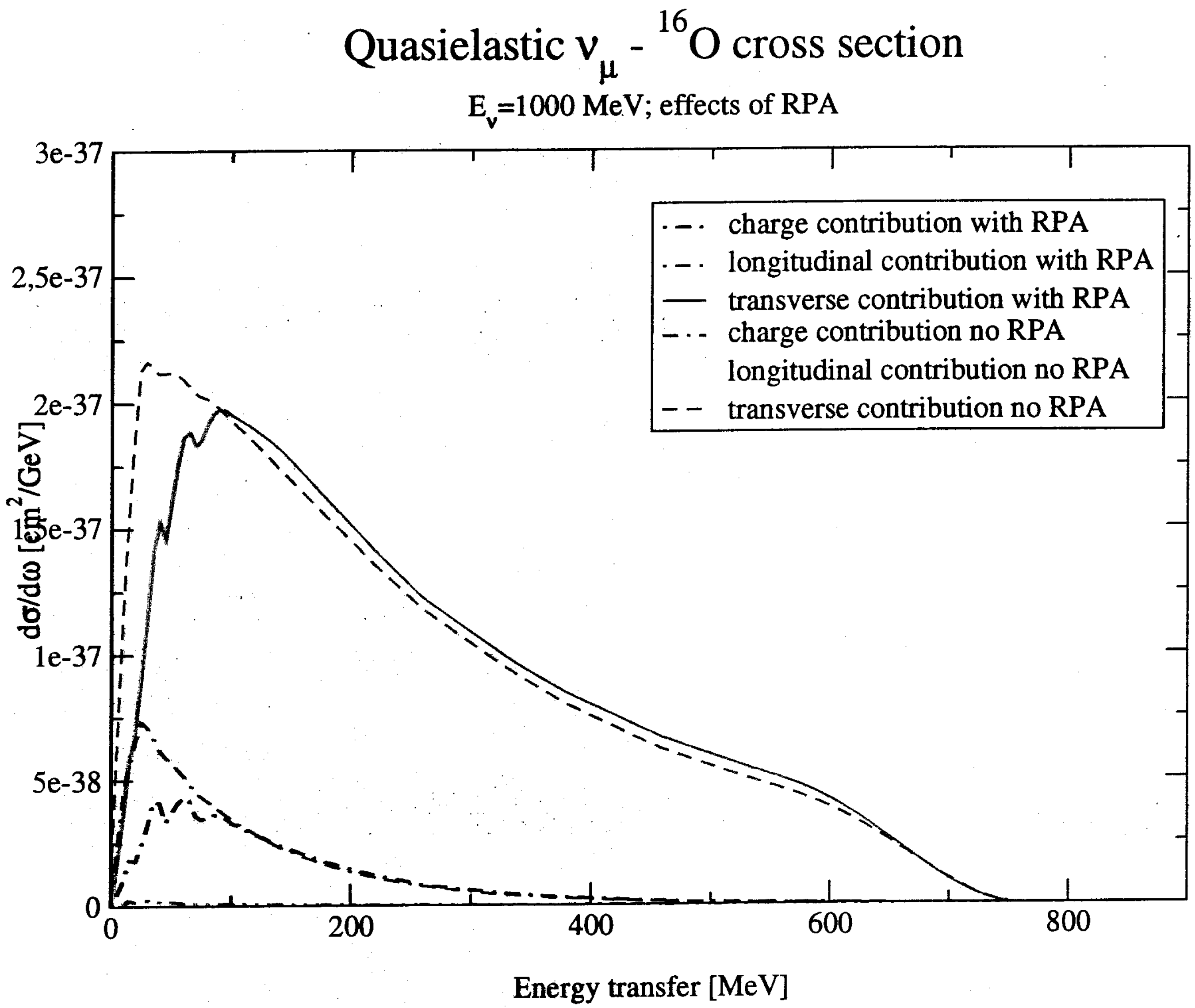




Differential ν_μ - ^{16}O cross section
E_ν=1000 MeV; RPA included
3e-37
2,5e-37
2e-37
1,5e-37
1e-37
5e-38
0
dσ/dω [cm^2/GeV]
quasielastic
pion production
2N, 3N
total
0
200
400
600
800
Energy transfer [MeV]

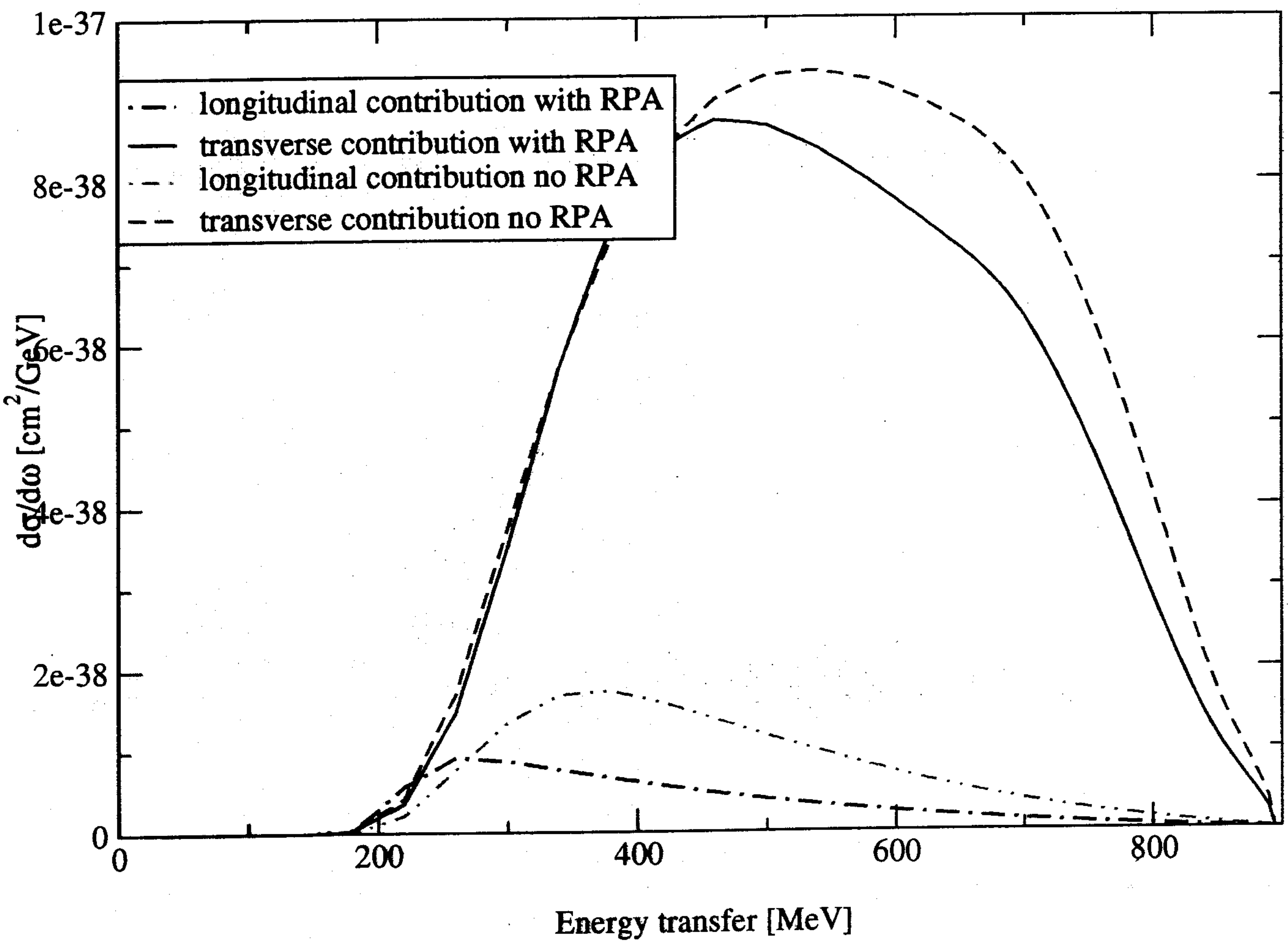


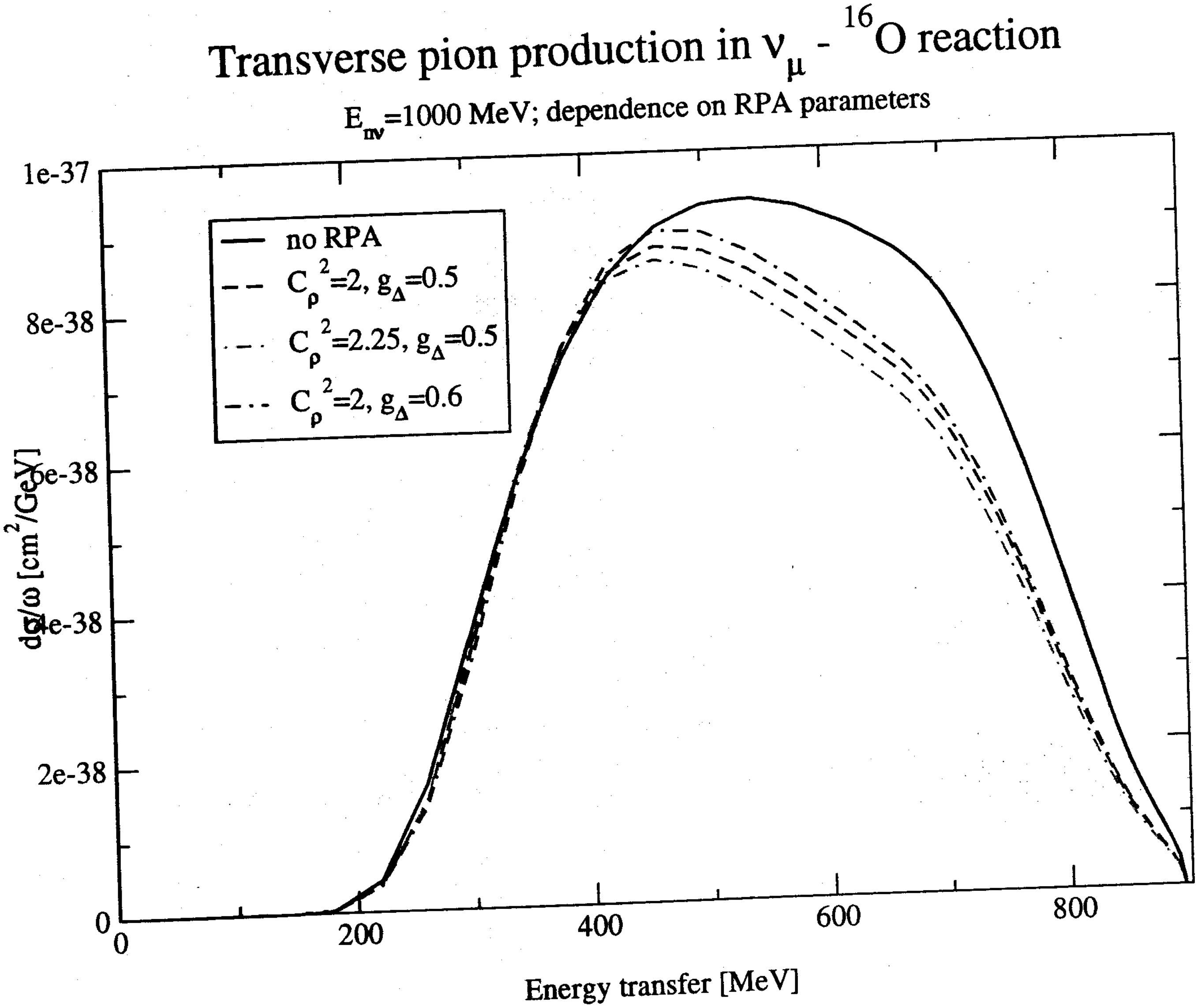
Quasielastic ν_μ - ^{16}O cross section
E_ν=1000 MeV; effects of RPA
with RPA
without RPA
3e-37
2,5e-37
2e-37
1,5e-37
1e-37
5e-38
0
0
200
400
600
800
dσ/dω [cm^2/GeV]
Energy transfer [MeV]


Quasielastic ν_μ - ^{16}O cross section
E_ν=1000 MeV; effects of RPA
3e-37
2,5e-37
2e-37
1,5e-37
1e-37
5e-38
0
0
200
400
600
800
$d\sigma/d\omega$ [cm^2/GeV]
Energy transfer [MeV]
charge contribution with RPA
longitudinal contribution with RPA
transverse contribution with RPA
charge contribution no RPA
longitudinal contribution no RPA
transverse contribution no RPA

Pion production in ν_μ - ^{16}O reaction

E_n=1000 MeV; RPA effects

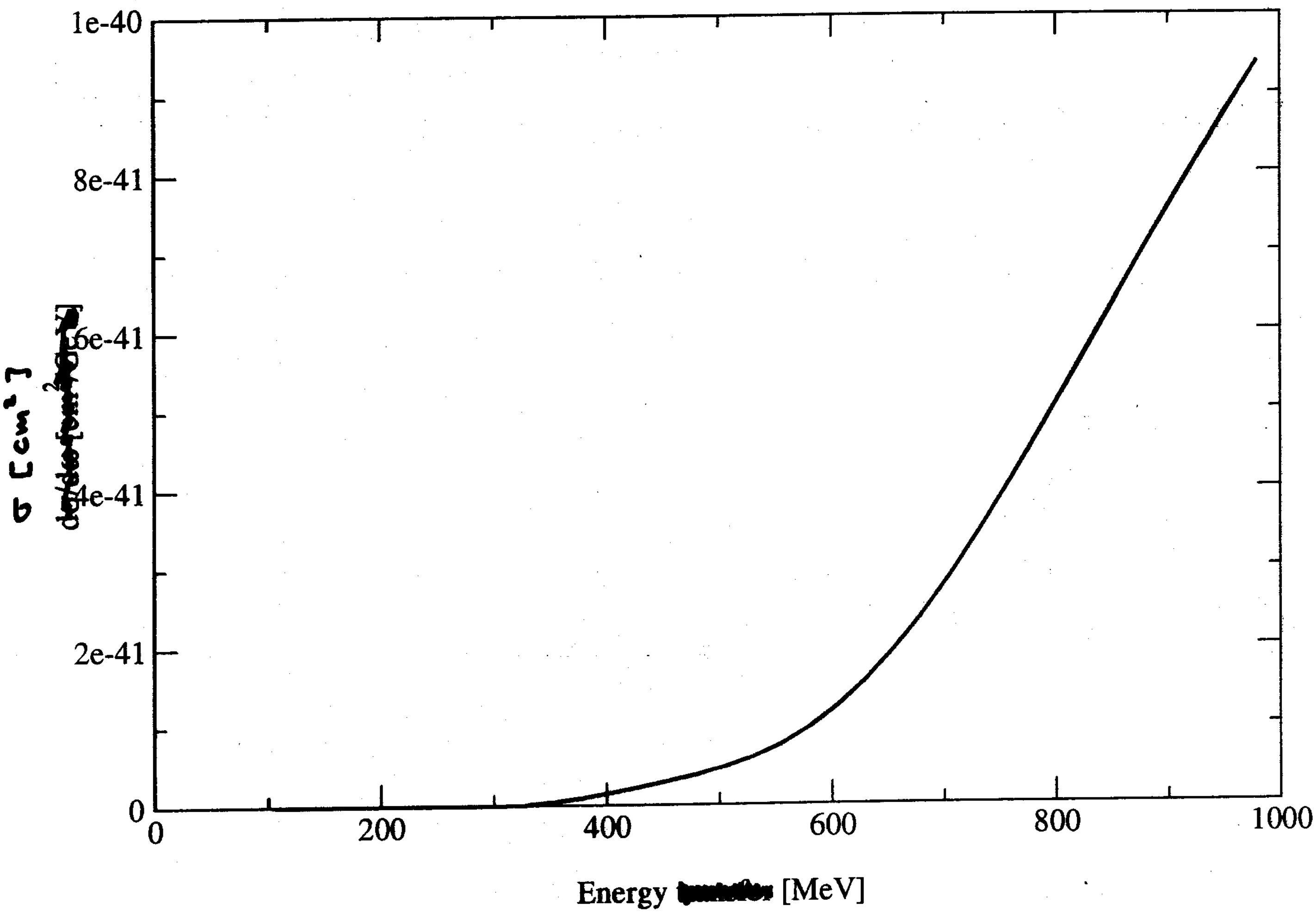




Transverse pion production in ν_μ - ^{16}O reaction
E_{nv}=1000 MeV; dependence on RPA parameters
no RPA
C_ρ^2=2, g_Δ=0.5
C_ρ^2=2.25, g_Δ=0.5
C_ρ^2=2, g_Δ=0.6
dσ/ω [cm²/GeV]
Energy transfer [MeV]
1e-37
8e-38
6e-38
4e-38
2e-38
0
0
200
400
600
800

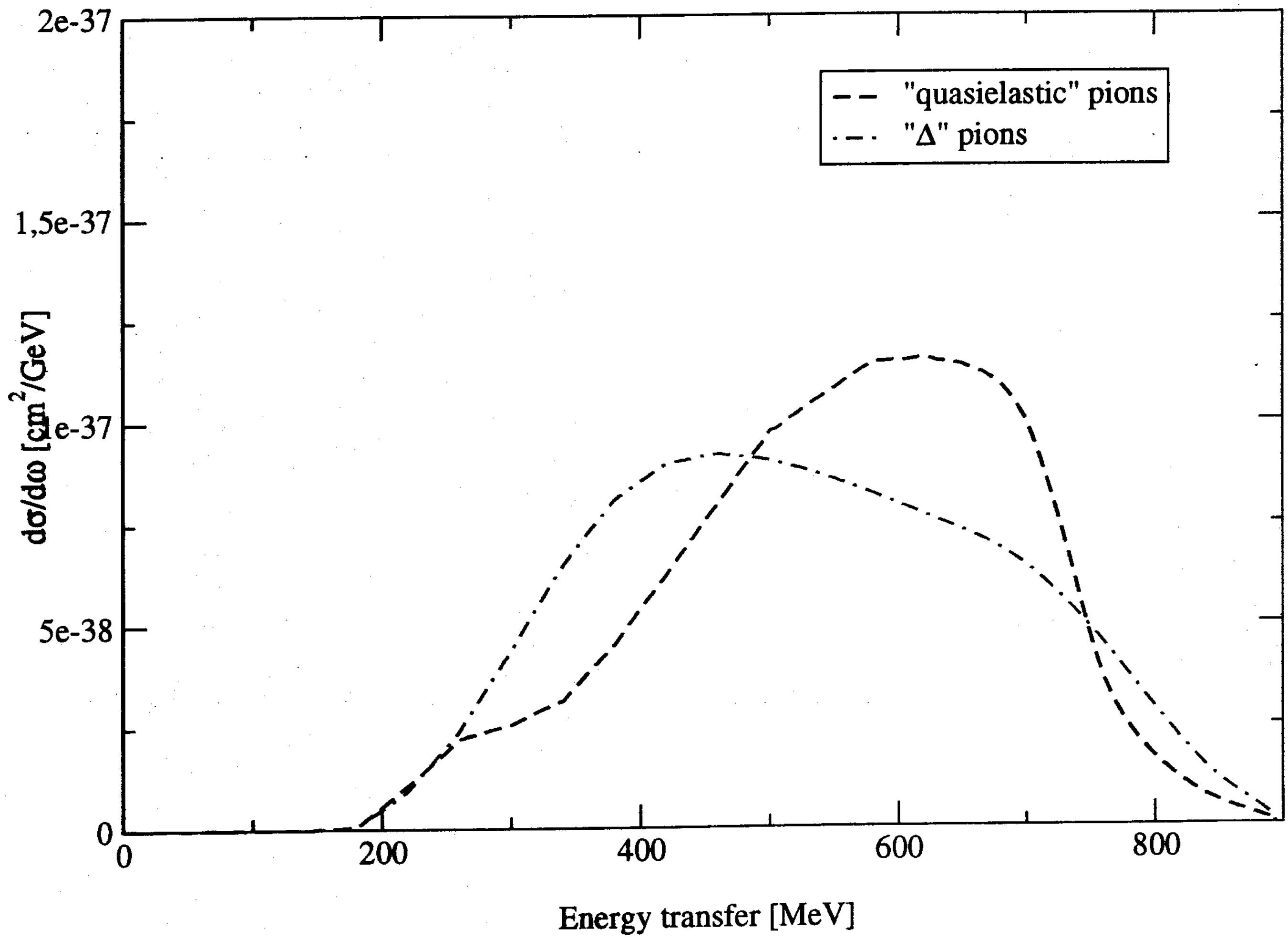
ν_μ-^{16}O "quasielastic" pion production

(effect of nucleons reinteractions)



Two sources of pions in ν_μ-^{16}O reaction

("quasielastic" part scaled by 400!) E_ν = 1000 MeV



o most of pions produced via transverse spin-isospin operator

o total cross section for pion production seems to be OK

o isospin states (charge pion states) are integrated

o it is possible to evaluate pions produced by initially quasielastic vertex via nuclear reinteractions of nucleons

CONCLUSIONS

o one model includes quasielastic and Δ production

o nuclear effects beyond Fermi gas: RPA (exchange of π, ρ), Δ width

o $k_F = 225 MeV$ but local density can be implemented

o $2p - 2h$ can be added but more systematic evaluation os required

o to be compared with data and other models.