Δ Production Calculation with Nuclear Effects

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- Δ Production with Protons and Deuterons

  : Form Factors & Cross Sections :

  * FURUNO, NUINT02, K2K RESULTS NUINT02

- Δ Production from Nuclei

  : Nuclear effects :

- Pion Production (Coherent)

- Summary and Future work

Collab.

M. S. Athar
E. Oset
M. Vicente Vacas
L. Alvarez Ruso

} Valencia Uni.

Llewelyn Smith (1972)

- **Matrix Element $N \to \Delta$ transition:**

- $\langle p' | J^\alpha | p \rangle = \langle p' | V^\alpha | p \rangle - \langle p' | A^\alpha | p \rangle$

- $$\langle p' | V^\alpha | p \rangle = \bar{\Psi}_\mu(p') \Big[\frac{C_3^V(Q^2)}{M} (g^{\mu\alpha} \not{q} - q^\mu \gamma^\alpha) + \frac{C_4^V(Q^2)}{M^2} (g^{\mu\alpha} q\cdot p' - q^\mu p'^\alpha) + \frac{C_5^V(Q^2)}{M^2} (g^{\mu\alpha} q\cdot p - q^\mu p^\alpha) + C_6^V(Q^2) q^\mu q^\alpha \Big] \gamma_5 u(p)$$

- $$\langle p' | A^\alpha | p \rangle = \bar{\Psi}_\mu(p') \Big[(g^{\mu\alpha} \not{q} - q^\mu \gamma^\alpha) \frac{C_3^A(Q^2)}{M} + \frac{C_4^A(Q^2)}{M^2} (g^{\mu\alpha} q\cdot p' - q^\mu p'^\alpha) + C_5^A(Q^2) g^{\mu\alpha} + \frac{C_6^A(Q^2)}{M^2} q^\mu q^\alpha \Big] u(p)$$

- $C_3^V, C_4^V, C_5^V, C_6^V$
  $C_3^A, C_4^A, C_5^A, C_6^A$. F.F.

• CVC and PCAC imply:

$$C_6^V(Q^2) = 0$$

$$C_6^A(Q^2) = C_5^A(Q^2)\,\frac{M^2}{M_\pi^2+Q^2}$$

$$C_5^A(0) = \frac{f_{\Delta N\pi} f_\pi}{2\sqrt{3}M}$$

• C_3^V, C_4^V and C_5^V : Electroproduction of PIONS:

• Preferred values:

VECTOR: $C_5^V = 0$

$$C_4^V = -\frac{M}{M'} C_3^V \; ; \; \left[-\frac{M}{W} C_3^V \right]$$ "M1 DOMINANCE"

$$C_3^V(Q^2) = \frac{2.05}{\left(1+\frac{Q^2}{M_V^2}\right)^2}$$

$$M_V^2 = .54\ \mathrm{GeV}^2$$

New data:

Axial Vector

• Von Hippel and Schreiner (1973)

• Fogli and Nardulli (1979)

• Rein, Sehgal (1981)

Dipole and Deviation from Dipole:

- $$C_i^A(Q^2) = \frac{C_i^A(0)}{\left(1+\frac{Q^2}{M_A^2}\right)^2}\left[1 - \frac{a_i\, Q^2}{b_i + Q^2}\right]$$

$C_3^A(0) = 0$

$C_4^A(0) = -0.3$, $a_4 = -1.2$, $b_4 = 2.0\ \text{GeV}^2$

$C_5^A(0) = 1.2$ $a_5 = -1.2$ $b_5 = 2.0\ \text{GeV}^2$

Adler (68) $Q^2 \lesssim .5\ \text{GeV}^2$

M_A : to be determined. ??

- $C_3^A(0)$, $C_4^A(0)$, $C_5^A(0)$: Model determination

- 1. Quark Model Calculations

- Summary -

- 2. PCAC for $C_5^A(0)$.

depends upon $f_{\Delta N\pi}$ as input.

$= 1.15 \pm .01$ GeV for $f_{\Delta N\pi} = 28.6 \pm .3$

Hemmert et al (1995).

- Cross Sections

$$\frac{d\sigma}{dQ^2} = \frac{G^2 \cos^2\theta_c}{64\pi} \frac{1}{(S-M^2)^2} L_{\alpha\beta} J^{\alpha\beta}$$

$S = (p+k)^2$

$L_{\alpha\beta}$: Leptonic Tensor

$J^{\alpha\beta}$: Hadronic Tensor

- Width of Δ : Γ_Δ

$$\frac{d\sigma}{dQ^2} = \frac{G^2 \cos^2\theta_c}{64\pi^2} \frac{1}{(S-M^2)^2} \frac{M}{M_\Delta} \int dk'^0 L_{\alpha\beta} J^{\alpha\beta} \frac{\Gamma_\Delta(W)/2}{(W-M_\Delta)^2 + \Gamma_\Delta^2(W)/4}$$

- $\Gamma_\Delta(W)$:
  - Constant width Γ_0
  - S-wave:

$$\Gamma(W) = \Gamma_0 \frac{q_{cm}(W)}{q_{cm}(M_\Delta)}.$$

- Schreiner-Von Hippel
- Barish et al
- ANL } data
- BNL

  - P-wave

$$\Gamma(W) = \frac{1}{6\pi} \left(\frac{f^*}{m_\pi}\right)^2 \frac{M}{W} q_{cm}^3$$ Fogli-Nardulli

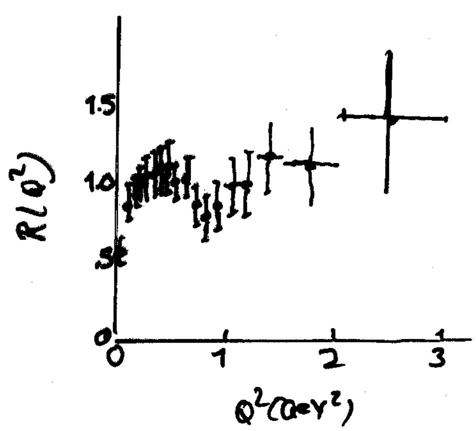
$$q_{cm} = \frac{\sqrt{(W^2 - m_\pi^2 - M^2)^2 - 4 m_\pi^2 M^2}}{2W}$$

$q^2 \approx 0$ limit:

$$\frac{d\sigma}{dq^2}(q^2{=}0) = C_5^A(0)^2 \, \frac{G^2 \cos^2\theta_c}{12\pi^2} \, \frac{\sqrt{s}\,(M+M_\Delta)^2}{M_\Delta^3} \, \frac{(s-M_\Delta^2)^2}{(s-M^2)^2} \int dk'^0 \, \frac{\Gamma(W)/2}{(W-M_\Delta)^2+\Gamma^2(W)/4}$$

$$R(q^2) = \frac{d\sigma}{dq^2}(\mu^-\Delta^{++})_{Exp} \Big/ \frac{d\sigma}{dq^2}(\mu^-\Delta^{++})_{th}$$

$$= \frac{d\sigma}{dq^2}(\mu^-\Delta^{++})_{\nu d} \Big/ \frac{d\sigma}{dq^2}(\mu^-\Delta^{++})_{\nu p}$$



Kitagaki et al PR D34(1986)2554

- data used Q^2 .1 — 3 GeV^2 for M_A
- Analysis in $0 < Q^2 < .1$ GeV^2

yields: $C_5^A(0)$, $C_4^A(0)$ both

Cross Sections in deuteron

$$\frac{d\sigma}{dq^2} = \frac{G^2 \cos^2\theta_c}{64\pi^2} \frac{M_d^2}{(S-M_d^2)^2} \int dk'^0 L_{\alpha\beta} J^{\alpha\beta} \int \frac{d^3\vec{p}_2}{(2\pi)^3 p_2^0}$$

$$\frac{\Gamma_\Delta(W)/2}{(W-M_\Delta)^2 + \Gamma_\Delta^2(W)/4} \; \tilde{\varphi}_d^2(p_2')$$

- $\tilde{\varphi}_d(p_2)$: Fourier Transform of $\varphi_d(\vec{r})$
- $\varphi_d(r) = \frac{1}{\sqrt{4\pi}} \frac{1}{r} \sum_{j=1}^{n} c_j e^{m_j \cdot r}$

$$\varphi_d(p) = 2\sqrt{\pi} \sum_{j=1}^{n} c_j \frac{1}{p^2 + m_j^2}$$

- C_j and m_j for BONN, PARIS.

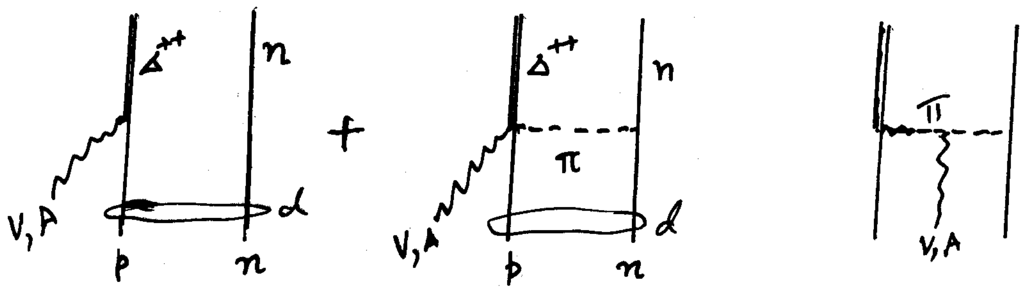
APPROXIMATIONS:

1. Neutron spectator
2. Deuteron S wave
3. NO MEC, NO. F.S.I.
4. Antisymmetrisation, ΔN Phase shifts ??
   COUPLED CHANNEL.

: Exchange currents and F.S.I.

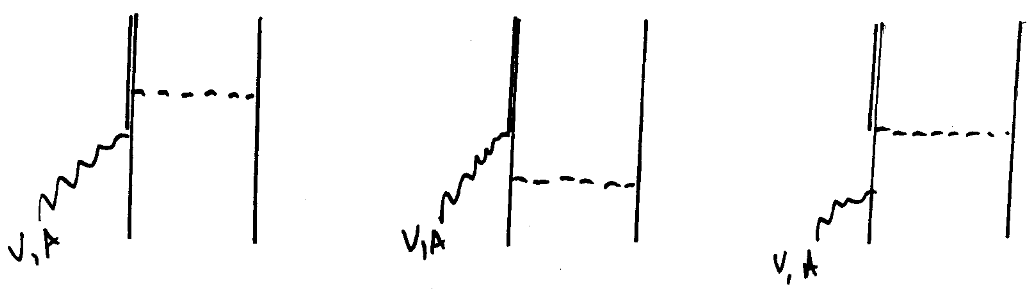
Consider the diagrams : (Beyond Spectator Model).

(i)



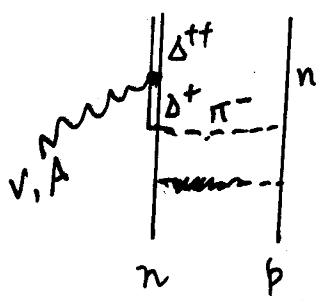
MEC ...

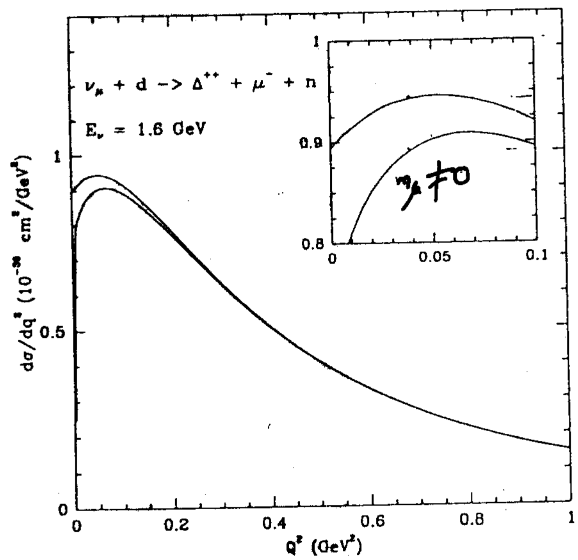
(ii)



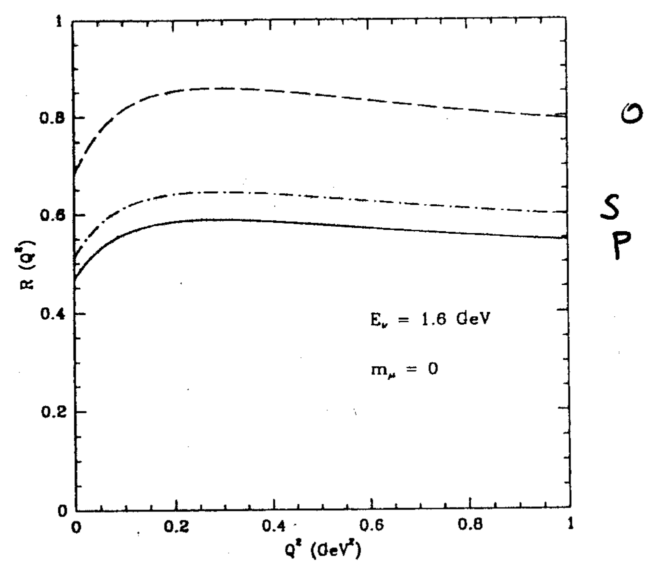
F.S.I.

(iii)





,: Effect of the muon mass on the differential cross section for the $\nu d \rightarrow \mu^- \Delta^{++} n$ reaction. In the upper line, the muon mass is neglected while it is considered in the lower one. Both curves include deuteron effects using the Paris parametrization of the deuteron wave function.



Effect of the Δ width in $R(Q^2)$: the solid line corresponds to a P-wave width, the dash-dotted line, to an S-wave width and the dashed line, to the case of zero width resonance. Deuteron effects have been neglected in all curves.

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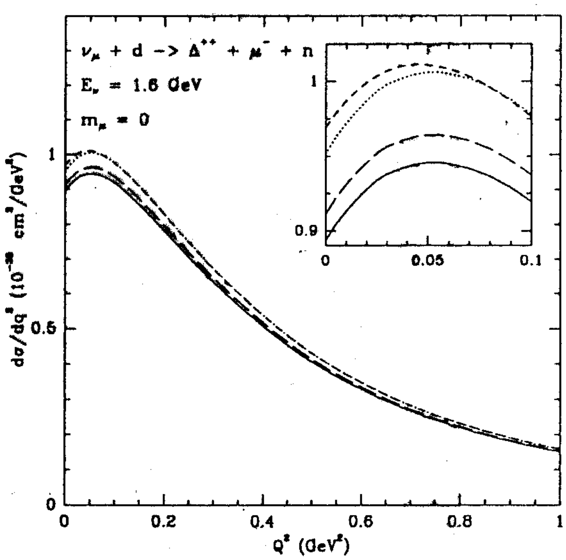
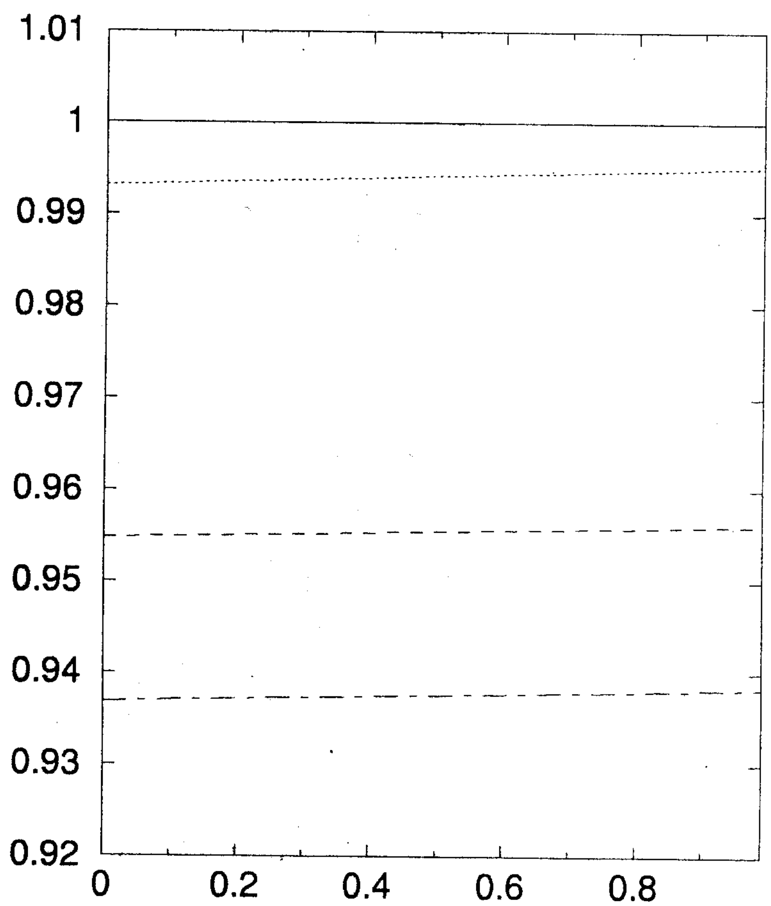


Bonn
Paris



• Δ Excitation IN Nuclei

$$\sigma = \frac{1}{E_\nu} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2E(k')} \cdot \frac{\Gamma/2}{(W-M_\Delta)^2 + \Gamma^2/4} \prod_e 2m_e \Sigma\Sigma |t|^2$$

IN a Nucleus:

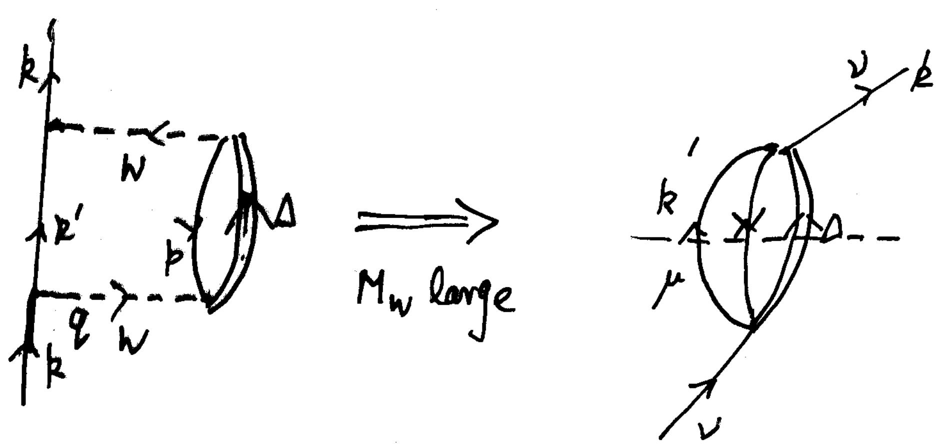
density $\rho(r)$

$$p_N \leq p_{FN}(r) = \left[\frac{3\pi^2}{2}\rho(r)\right]^{1/3}$$

p_Δ : No constraints.

$$\boxed{\sigma_A = \frac{2m_\nu}{k_\nu} \int \mathrm{Im}\, \Sigma_\nu \, d^3r}$$

Σ_ν is self energy of ν in Nucleus



$$\Sigma_\nu = \int \frac{d^4k'}{(2\pi)^4} \bar{u}_r(k) \gamma_\mu \frac{\not{k}' + m_e}{k'^2 - m_e^2 + i\epsilon} \gamma_\nu u_r(k) \Pi_W^{\mu\nu}(q)$$

$$\Pi_W^{\lambda\nu}(q) = -\int \frac{d^4p}{(2\pi)^4} \frac{M}{E_p} \frac{n(p)\, i}{p^0 - E(p) + i\epsilon} \cdot \frac{M_\Delta}{E_\Delta} \frac{i}{p^0 + q^0 - E_\Delta(p+q) + i\epsilon} J^{\lambda\nu}$$

$$\Sigma_\nu = \int \frac{d^4k'}{(2\pi)^4} \Sigma\Sigma|t|^2 \frac{2m_e}{k'^2 - m_e^2 + i\epsilon} \tilde{U}_\Delta$$

$$\tilde{U}_\Delta = \frac{1}{2} \int_p \frac{1}{\sqrt{s} - M_\Delta + i\Gamma/2}$$

$$\int_{(r)} = \int \frac{d^3p}{(2\pi)^3} \cdot n(p, \vec{r}) \quad , \quad \Sigma\Sigma|t|^2 = L_{\lambda\nu} J^{\lambda\nu}$$

$$\frac{M_\Delta}{E_\Delta} \delta(\sqrt{s} - E_e - E_\Delta) = -\frac{1}{\pi} \operatorname{Im} \frac{1}{E_p + E_\nu - E_e - E_\Delta - i\Gamma/2}$$

Thus:

$$\boxed{\Sigma_\nu = \int \frac{d^4k'}{(2\pi)^4} \frac{2m_e}{k'^2 - m_e^2 + i\epsilon} \int_p \frac{1}{\sqrt{s_\Delta} - M_\Delta + i\Gamma/2} \Sigma\Sigma|t|^2}$$

<u>Cutkosky Rules:</u>

$$\Sigma_\nu \rightarrow 2i \operatorname{Im} \Sigma_\nu$$

$$G(k') \rightarrow 2i\, \theta(k_0') \operatorname{Im} G(k')$$

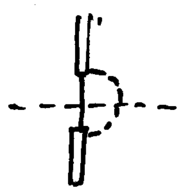
$$\tilde{U}(q) \rightarrow 2i\, \theta(q_0) \operatorname{Im} U$$

$$\boxed{\operatorname{Im} \Sigma_\nu' = -\int \frac{d^3k'}{(2\pi)^3} \frac{2m_e}{2E_e} \int_p \Sigma\Sigma|t|^2 \frac{\Gamma/2}{(\sqrt{s} - M_\Delta)^2 + \Gamma^2/4}}$$

$$\boxed{\sigma_A = \int d^3r \,\frac{1}{E_\nu} \int \frac{d^3k'}{(2\pi)^3}\, \frac{1}{2E_e}\, \rho_{p,n} \prod_e 2m_e \,\bar{\sum}\sum |M|^2 \,\frac{\Gamma/2}{(\sqrt{s}-M_\Delta)^2+\Gamma^2/4}}$$

Medium Modifications:

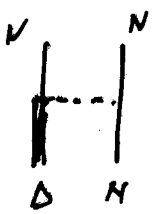
1. Pauli Blocking $\tilde{\Gamma}$

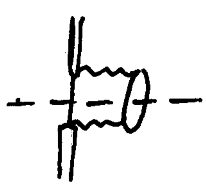


$\Delta \to N\pi$: N is Pauli Blocked.

Pion reduction :

2. $\Delta N \to NN$ open: Δ reduction





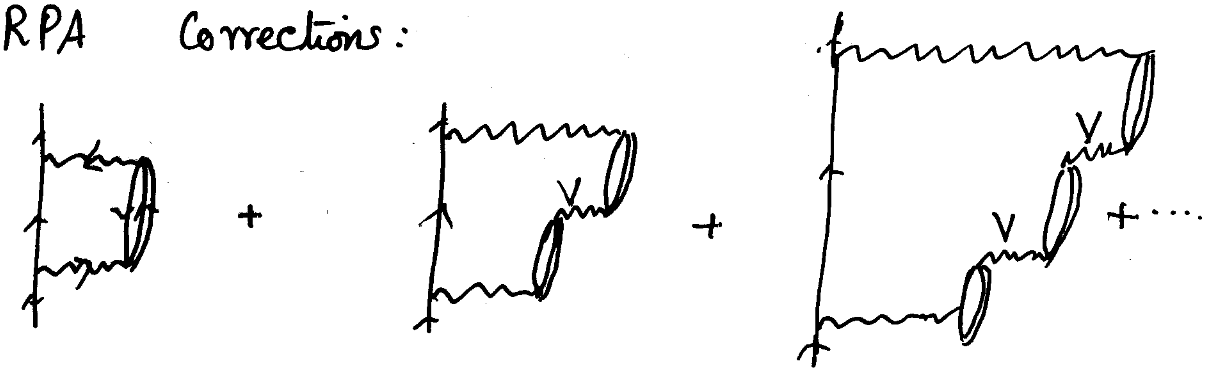
$\mathrm{Im}\,\Sigma_\Delta$ modified:

3.

$$\boxed{\Gamma/2 \longrightarrow \frac{\tilde{\Gamma}}{2} - \mathrm{Im}\,\Sigma_\Delta}$$

$$\boxed{M_\Delta \longrightarrow M_\Delta + \mathrm{Re}\,\Sigma_\Delta}$$

3. RPA Corrections:



$$V = \left(\frac{f^*}{\mu}\right)^2 \left\{ V_L'(q)\, \hat{q}_i \hat{q}_j + V_T'(q)\left(\delta_{ij} - \hat{q}_i \hat{q}_j\right)\right\} S_i S_j^{\dagger} T^{\lambda} T^{\dagger \lambda}$$

- Implement in Nonrelativistic scheme by splitting in Longitudinal and Transverse channels.

  Geometric Series:

  in Both Channels

- $\boxed{\text{Re } \Sigma_\Delta \text{ is Modified.}}$

- V_L and V_T in π, ρ exchange with Migdal parameter.

Parametrisation of $\hat{\Gamma}$ and Σ_Δ:

- $$\tilde{\Gamma} = \frac{1}{6\pi}\left(\frac{f^*}{m_\pi}\right)^2 \frac{M}{W}\, q_N^3 F(k_F, E_\Delta, k_\Delta)$$

$$F(k_F, E_\Delta, k_\Delta) = \frac{K_\Delta q_N + E_\Delta E_N - E_F\sqrt{s}}{2 k_\Delta q_N}$$

$$\bar{s} = M^2 + m_\pi^2 + 2\omega\left(M + \frac{3}{5}\frac{k_F^2}{2M}\right)$$

- $$Re\, \Sigma_\Delta = 40 \frac{\rho}{\rho_0} \ (\mathrm{Mev})$$

- $$-Im\, \Sigma_\Delta = C_Q\left(\frac{\rho}{\rho_0}\right)^\alpha + C_{A2}\left(\frac{\rho}{\rho_0}\right)^\beta + C_{A3}\left(\frac{\rho}{\rho_0}\right)^\gamma$$

$$\gamma = 2\beta$$

- $$C(T_\pi) = ax^2 + bx + C \quad , \quad x = \frac{T_\pi}{m_\pi}$$

- Values of the constant:

	C_Q MeV	C_{A2} MeV	C_{A3} MeV	α	β
a	−5.19	1.06	−13.46	.382	−.038
b	15.35	−6.64	46.17	−1.322	−.204
c	2.06	22.66	−20.34	1.466	.613

- PION ABSORPTION :

{ Oset and Strottman Phys. Rev C42 (1990) 2454
Absorption Factor (r; E_π):

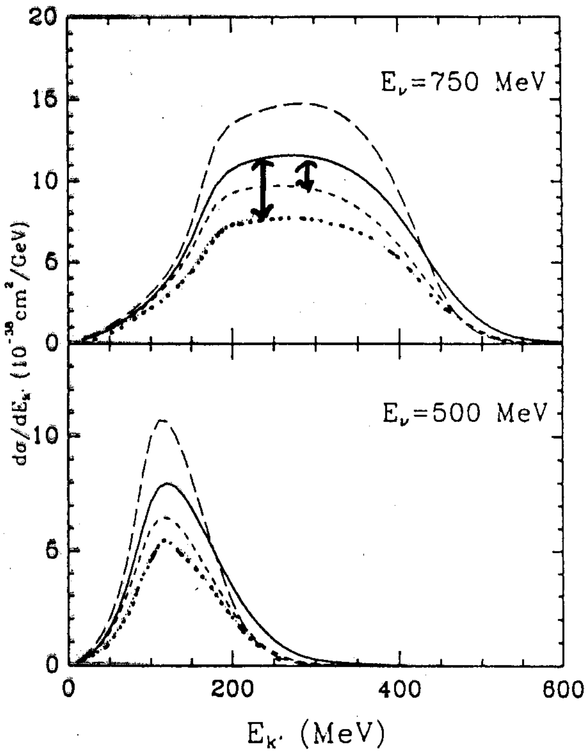
- 3PF Fermi density C.W. De Jager et al (1970)

$$\rho(r) = \rho_0\left(1 + w\frac{r^2}{c^2}\right)\left[1 + e^{(r-c)/z}\right]^{-1}$$

$$\langle r^2\rangle^{-1/2} = 2.30 \text{ fm}$$

$$c = 2.608 \text{ fm}$$

$$z = .513$$



$\Delta N \to NN$

$\Delta N \to NN$:
$\Delta \to N\pi$:

- - - - Free $\Delta \to N\pi$
——— Δ in nuclei
- - - - - $\Delta \to N\pi$
. . . . . . After Absorption of pions

Calculate Pion Spectrum:

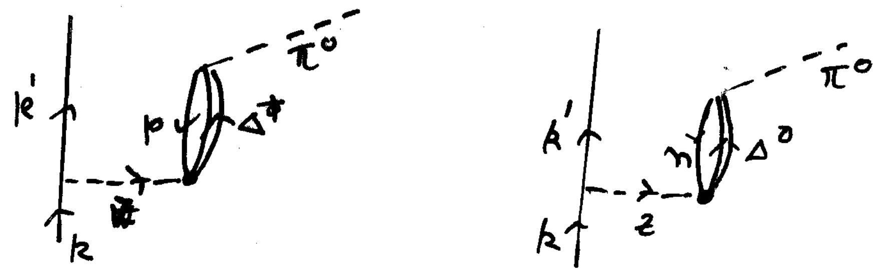
1) Energy

2) Ang. distribution.

Coherent π^0 Production by Neutral Currents.

$$\nu\, p \to \nu\, \Delta^+ \to \nu\, p\, \pi^0$$

$$\nu\, A_{gs} \to \nu + A_{gs} + \pi^0$$



Weak Vertex: $\frac{G}{\sqrt{2}}\, \ell_\mu J_\mu^{NC}$

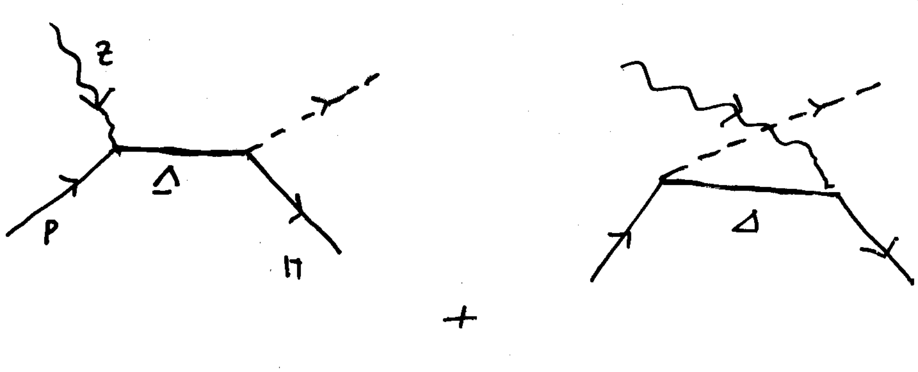
$$J_\mu = \bar{\psi}_\Delta^\lambda J_{\lambda\mu}\, \psi_p$$

Δ dominance

strong Vertex : $\frac{f^*}{\mu_\pi}\, \bar{\psi}_\mu\, \partial_\mu \vec{\phi} \cdot \vec{T}\, \psi$

Direct

Exchange.



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BORN DIAGRAMS :

NOT INCLUDED

e- Production

[W Important]

$$\frac{d\sigma}{d\Omega_\pi\, dE_\pi\, d\Omega'_\nu} = \frac{1}{(2\pi)^5}\,\frac{1}{8}\,\frac{1}{4M^2}\,\frac{p_\pi}{k}\,|t|^2\, F(\theta_\pi, E_\pi)$$

$$\frac{d\sigma}{d\Omega'_\nu\, dE'_\nu\, d\Omega_\pi} = \frac{1}{(2\pi)^5}\,\frac{1}{8}\,\frac{1}{4M^2}\,k'\,\frac{k_\pi}{k}\,|t|^2$$

$$t = \frac{G\cos\theta_c}{\sqrt{2}}\,\frac{f^*}{m_\pi}\,\bar{u}\, p^\pi_\sigma\, \Delta^{\sigma\lambda}\, J_{\lambda\nu}\, u\, \bar{u}(k')\,\gamma^\nu(1-\gamma_5)\,u(k)$$

$$\Delta^{\sigma\lambda} = \frac{i \quad \not{p}_\Delta + M_\Delta}{s - M_\Delta^2 + i\sqrt{s}\,\Gamma(s)}\,\Lambda^{\sigma\lambda}$$

$$\Lambda^{\sigma\lambda} = \left[\left(g^{\sigma\lambda} - \frac{1}{3}\gamma^\sigma\gamma^\lambda - \frac{2}{3M_\Delta^2}\,p^\mu_\Delta p^\nu_\Delta - \frac{1}{3M_\Delta}\left(\gamma^\mu p^\nu_\Delta - \gamma^\nu p^\mu_\Delta\right)\right]\right.$$

IN NUCLEAR MEDIUM

- M_Δ
- Γ MODIFICATION IN MEDIUM.
- t :

$$\frac{1}{s-M_\Delta^2+i\sqrt{s}\,\Gamma(s)} \longrightarrow F(k-q)\,\frac{1}{s-M_\Delta^2+i\sqrt{s}\,\Gamma(s)} = \tilde{F}(k-q)$$

$F(k-q) \equiv$ Nuclear Form Factor

$$= \int d\vec{r}\,\rho(r)\,e^{i(\vec{k}-\vec{q})\cdot\vec{r}}$$

with M_Δ and Γ becoming $\rho(\vec{r})$ dependent in local density approximation:

$$\boxed{\tilde{F}(k-q) = \int d\vec{r}\,\rho(r)\,e^{i(\vec{k}-\vec{q})\cdot r}\,\frac{1}{s-M_\Delta^2+i\sqrt{s}\,\Gamma(s)}}$$

$\tilde{F}$ has dimensions:

$$M_\Delta \longrightarrow M_\Delta' = M_\Delta + \mathrm{Re}\,\Sigma_\Delta$$

ie $\qquad \Gamma \rightarrow \tilde{\Gamma} - \mathrm{Im}\,\Sigma'_\Delta$

PION ABSORPTION.

(i) $e^{i\vec{p}_\pi\cdot\vec{r}} \rightarrow \varphi_\pi(r)$

$\varphi_\pi(r)$: Solution of ~~opt~~ K-G equation with an optical Potential

Oset - Kelkar (1998)

(ii) Eikonal approximation

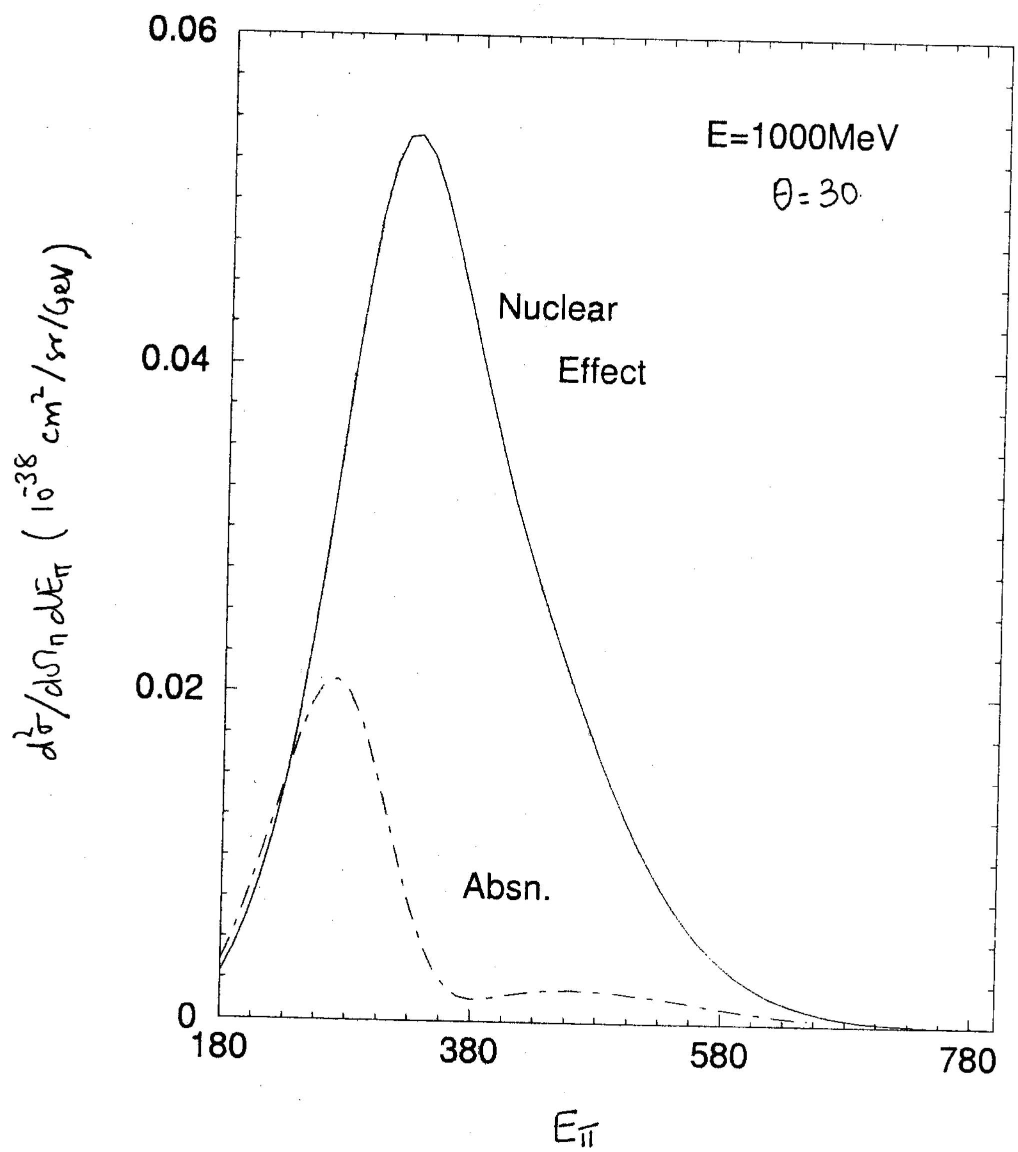
$$\tilde{F}(k-q) = \int d\vec{r}\, \rho(r)\, e^{i(\vec{k}-\vec{q})\cdot r}\, G_{\Delta h}(s, \rho(r))\, e^{-i\int_0^\infty \frac{1}{2q}\Pi(\rho(r'))\,dl}$$

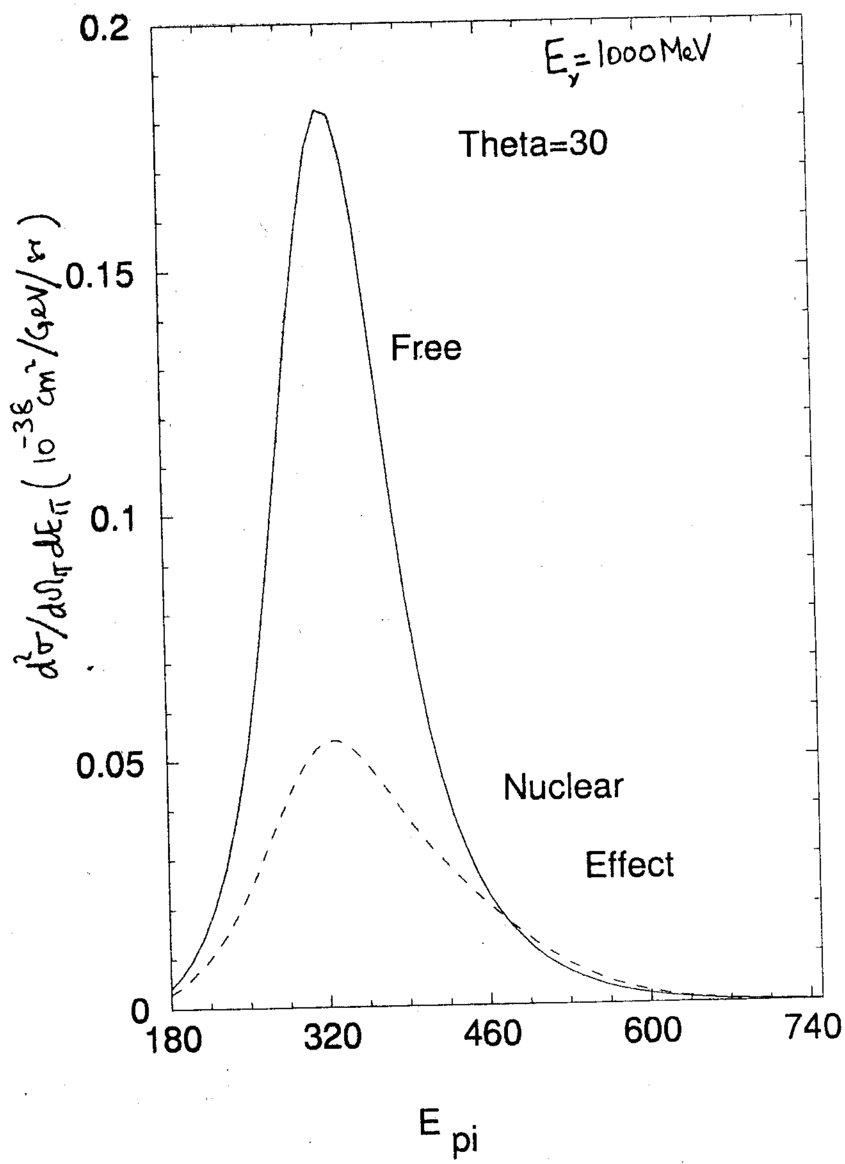
$$G_{\Delta h} = \frac{1}{\sqrt{s} - M_\Delta + i\,\Gamma(s)}$$ {Nonrelativistic expression}

$$\Pi(\rho) = \frac{4}{9}\left(\frac{f^*}{\mu}\right)^2 \frac{M^2}{s}\, q^2 \rho(r)\, G_{\Delta h}(s,\rho)$$

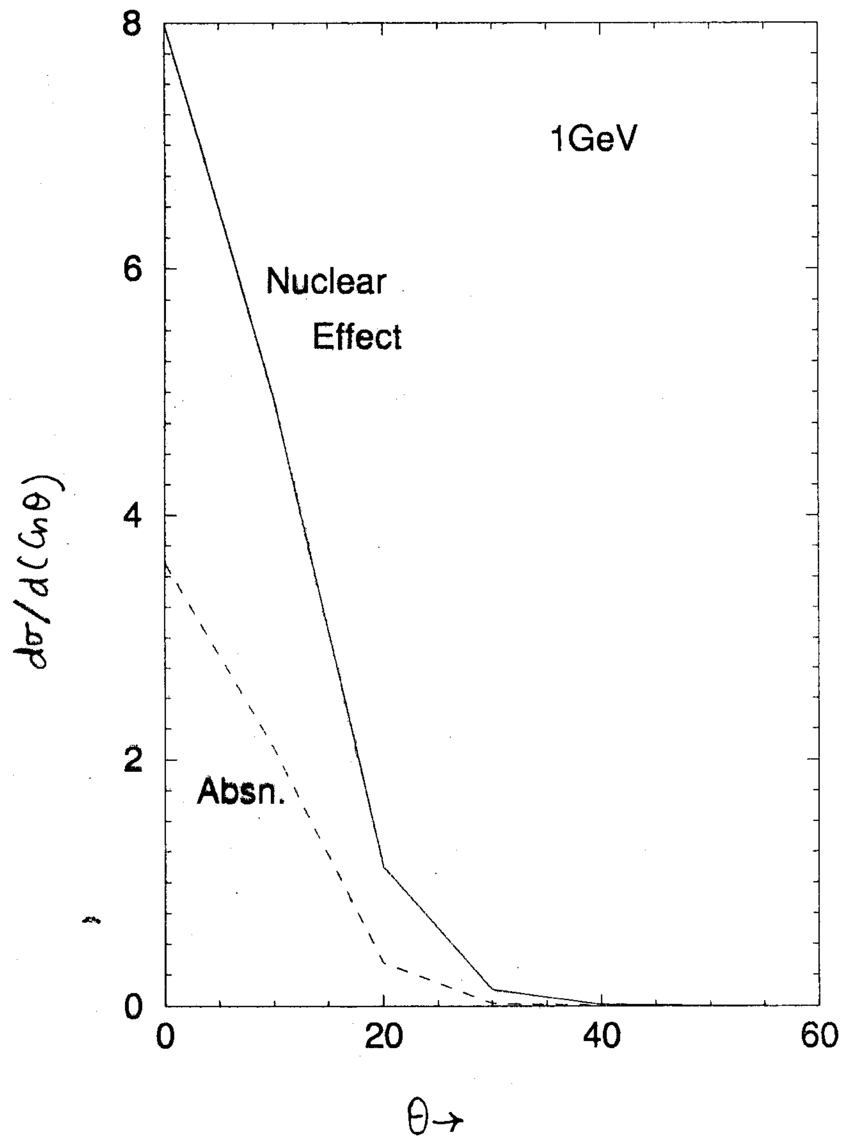
$$\vec{r}\,' = \vec{r} + \frac{\vec{q}}{q}\, l$$

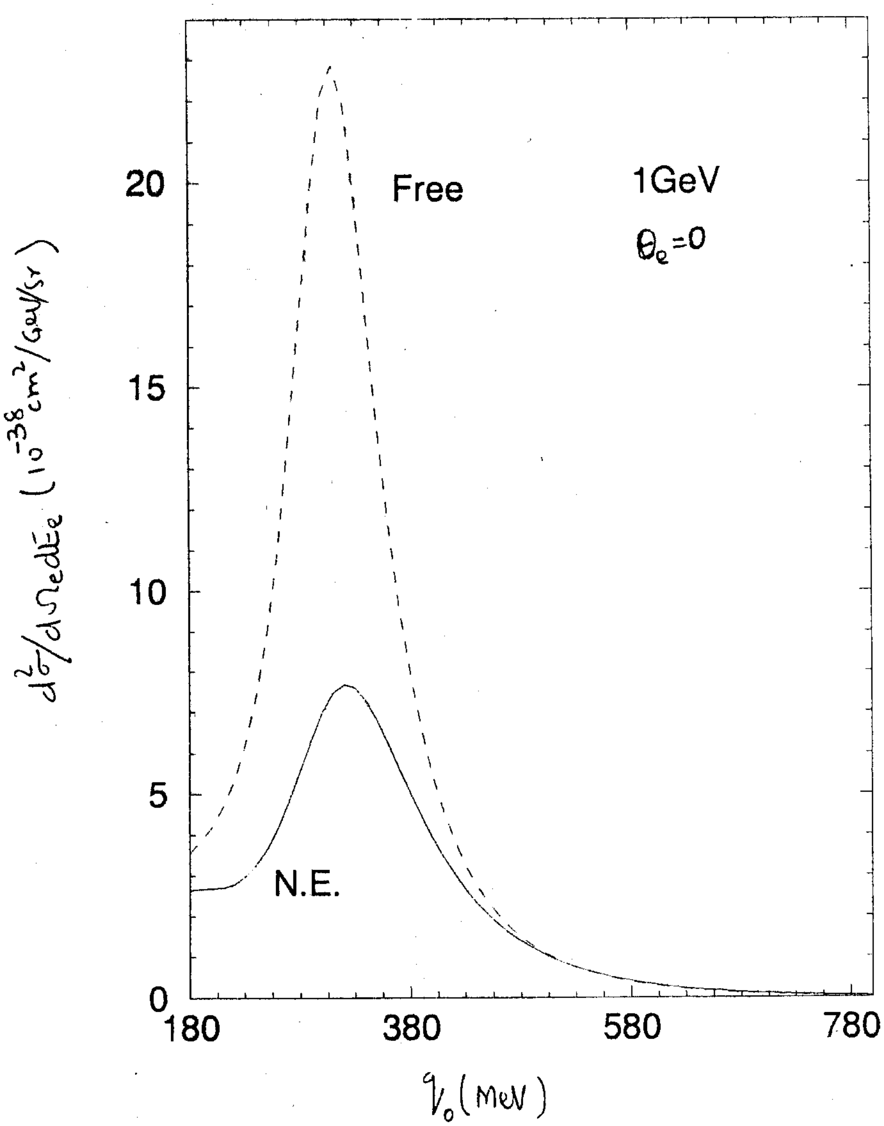
$\Pi(\rho)$ is the Self energy associated with the optical potential


E=1000MeV
θ=30
Nuclear
Effect
Absn.
0.06
0.04
0.02
0
180
380
580
780
$d^2\sigma/d\Omega_\eta dE_\pi$ (10^{-38} cm^2/sr/GeV)
E_π



$E_\gamma = 1000$ MeV
Theta=30
Free
Nuclear
Effect
$d^2\sigma/d\Omega_\pi dE_\pi$ (10^{-38} cm^2/GeV/sr)
0.2
0.15
0.1
0.05
0
180
320
460
600
740
E pi

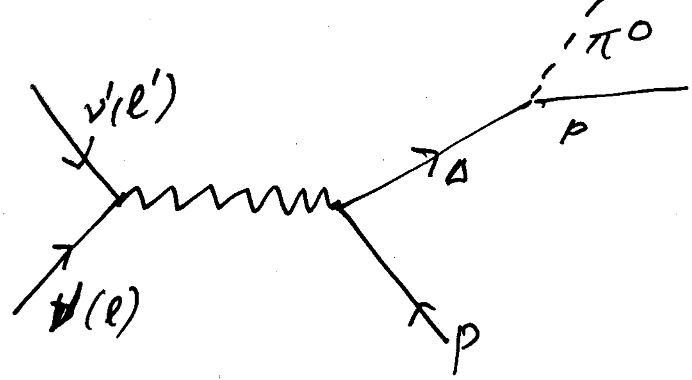

8
6
4
2
0
0
20
40
60
1GeV
Nuclear
Effect
Absn.
dσ/d(Cnθ)
θ→



Free
1GeV
$\theta_e=0$
N.E.
20
15
10
5
0
180
380
580
780
$d^2\sigma/d\Omega_e dE_e$ ($10^{-38}cm^2/GeV/sr$)
q_0 (MeV)

ii) ν- magnetic Moment induced π^0 production:

Kang, Kim, Lee (1999)



Vertex:

$$i f_{\nu'\nu} \mu_B \, \bar{u}(l') \, \sigma_{\mu\nu} q^{\nu} u(l) \, J^{\mu}$$

$$|A_M|^2 = \frac{f'^2 \mu_B^2}{q^4} M^{\mu\nu} J_{\mu\nu}^{em}$$

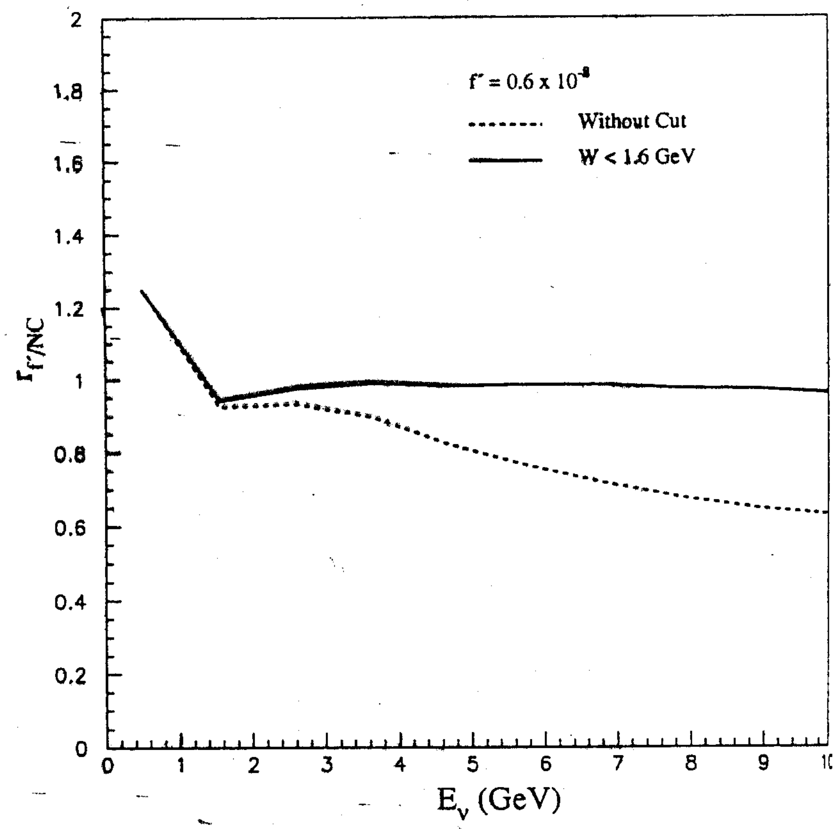
$$\frac{\sigma_f'}{\sigma_0^{NC}} = R$$

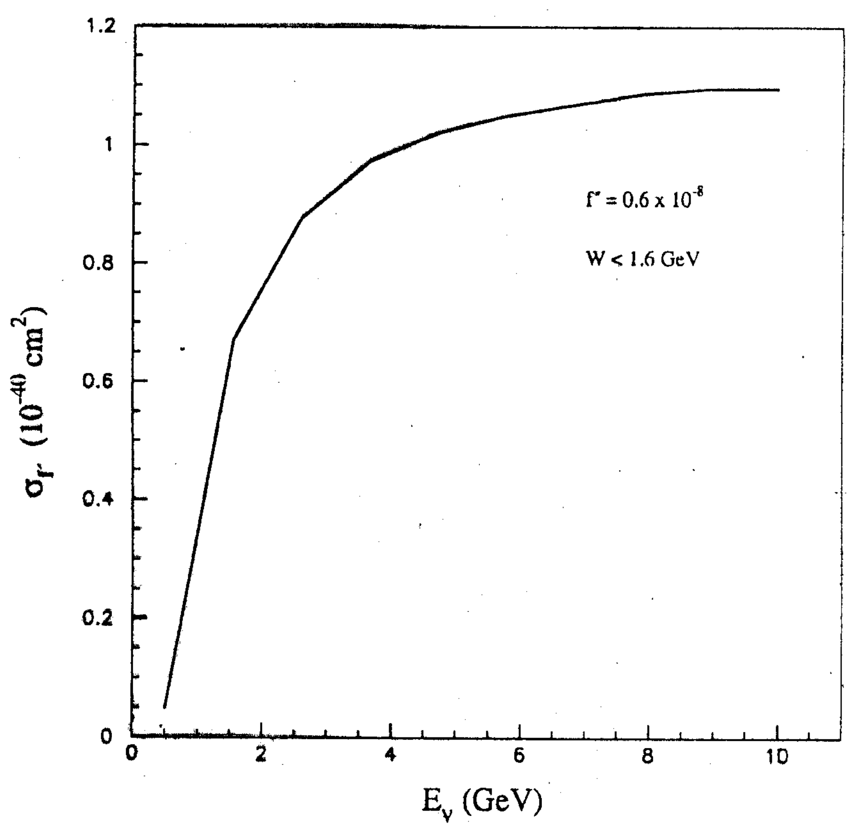
$$f' = 6 \times 10^{-9}$$ Present limit $< 10^{-9}$ PDG. 1996.

π^0: Energy distn and Angular distn with ν- magnetic Moment

IN NUCLEUS: MODIFICATIONS.

IS IT POSSIBLE TO SET A LIMIT ON ν Magnetic Moment from Atmos. ν ??


f' = 0.6 x 10^-8
Without Cut
W < 1.6 GeV
r_f'/NC
E_ν (GeV)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
1
2
3
4
5
6
7
8
9
10


f' = 0.6 x 10^-8
W < 1.6 GeV
σ_f' (10^-40 cm^2)
E_ν (GeV)
0
0.2
0.4
0.6
0.8
1
1.2
2
4
6
8
10

SUMMARY :

A. • FORM FACTOR DETERMINATION

i) Reanalysis of BNL data
G^P

ii) Parity violation $\vec{e}p \to e\Delta$ experiments

iii) Deuteron Effects / Nuclear effects in F.F.

B. i) Reasonable Model for Δ production in Nucleus

ii) Inputs to be checked in electron Scattering

iii) Incoherent pion production to be calculated

C. Coherent pion production
comparision with experiments ??

d. Pion Absorption and Rescattering Models to be developed relevant to specific ν-exp.

e. $\sigma_{AB} / \sigma_{NC}$ in Nuclei

$\nu_{AB} \equiv$?? Limit Possible from Atmo. Neutrino exp. data.