NUINT02, 12-15/12/2002

Pion absorption and rescattering

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Outline

- Introduction
  - Factorization and charge exchange matrix M
  - ANP model
- Experimental information on M
- Dynamics of the ANP model
  - Absorption
  - Multiple Scattering
- Conclusions

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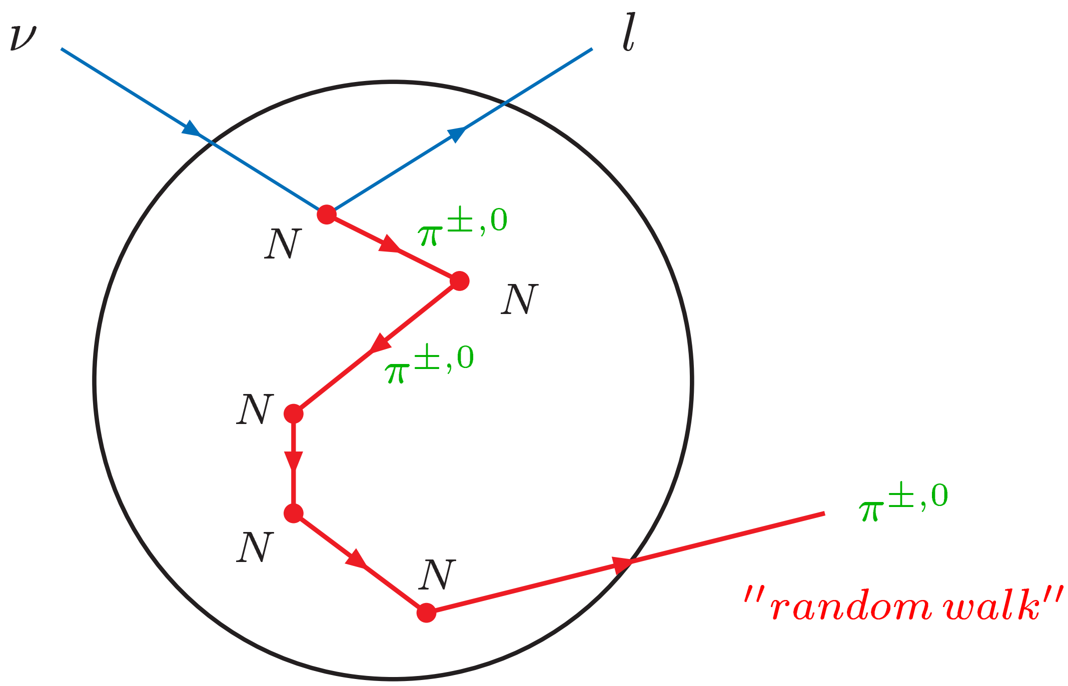
Factorization

- Reactions

$$\nu + T \rightarrow l + T' + \pi^{\pm,0}$$

  - T: nuclear target (${}_8O^{16}$, ${}_{18}Ar^{40}$, ${}_{26}Fe^{56}$)
  - T': final nuclear state

- Two step process



1. single pion production in νN scattering
   $\rightarrow$ Pauli Principle, Fermi motion
2. multiple scattering of pions
   $\rightarrow$ Charge exchange, absorption, Pauli Principle

- step 2 is described by the charge exchange matrix M
  - only depends on properties of the target
    $\rightarrow$ charge density profile $\rho(r)$
- basic assumption: two steps independent $\rightarrow$ predictive power

The charge exchange matrix M

- Differential cross sections for leptonic pion production on free nucleon targets $\longrightarrow$ cross sections on nuclear targets:

$$\underbrace{\begin{pmatrix} \frac{d\sigma({}_Z T^A;\pi^+)}{dQ^2 dW} \\ \frac{d\sigma({}_Z T^A;\pi^0)}{dQ^2 dW} \\ \frac{d\sigma({}_Z T^A;\pi^-)}{dQ^2 dW} \end{pmatrix}}_{\text{nuclear target}} = M \underbrace{\begin{pmatrix} \frac{d\sigma(N_T;\pi^+)}{dQ^2 dW} \\ \frac{d\sigma(N_T;\pi^0)}{dQ^2 dW} \\ \frac{d\sigma(N_T;\pi^-)}{dQ^2 dW} \end{pmatrix}}_{\text{free nucleon}}$$

where

$$\frac{d\sigma(N_T;\pm 0)}{dQ^2 dW} = Z \frac{d\sigma(p;\pm 0)}{dQ^2 dW} + (A - Z) \frac{d\sigma(n;\pm 0)}{dQ^2 dW}$$

- charge exchange matrix M for isoscalar targets ($M = M^T, \sum_j M_{ij} = A_p, M_{+0} = M_{-0} \rightarrow$ 3 param. A_p, d, c)

$$M = A_p \begin{pmatrix} 1-c-d & d & c \\ d & 1-2d & d \\ c & d & 1-c-d \end{pmatrix}$$

- Factorization assumption (two step process) $\longrightarrow$ predicitve power
- $M(T;Q^2,W)$ (i.e. A_p, d, c) to be measured experimentally neutrino-prod., electro-prod. $\longrightarrow$ Test of Factorization
- . . . to be predicted theoretically $\longrightarrow$ for example ANP model [1]

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[1] Adler, Nussinov, Paschos, Phys. Rev. **D9**, 2125 (1974)

ANP model

ANP model: 'Isospin C.G. analysis $\oplus$ transport integral equation for pions'

πN reactions $\longrightarrow$ charge exchange:

$$\pi^+ + N \to \pi^+ + N, \quad \pi^+ + n \to \pi^0 + p$$

$$\pi^0 + N \to \pi^0 + N, \quad \pi^0 + p \to \pi^+ + n$$

$$\pi^0 + n \to \pi^- + p, \quad \pi^- + N \to \pi^- + N$$

$$\pi^- + p \to \pi^0 + n,$$

$$\pi^{\pm,0} + N \to X, \quad (\pi \not\in X)(\to absorption)$$

- Charge exchange in a single scattering:

$$\vec{q}_f = Q\,\vec{q}_i, \qquad \vec{q}_{i,f} = \big(n(\pi^+), n(\pi^0), n(\pi^-)\big)^T_{i,f}$$

  3×3 matrix Q from C.G. analysis of isospin

- Charge exchange in multiple scattering:

$$\vec{q}_f = M\,\vec{q}_i, \qquad M = \sum_{n=0}^{\infty} P_n Q^n$$

  P_n: prob. that pion exits medium after exactly n πN-scatterings
  Note: Absorption $\longrightarrow \sum_{n=0}^{\infty} P_n < 1$
  (The matrix Q does not include any absorption)

- eigenvalues/vectors of Q:

$$\lambda_1 = 1,\ q_1 = (1,1,1)^T,$$
$$\lambda_2 = \tfrac{5}{6},\ q_2 = (1,0,-1)^T,$$
$$\lambda_3 = \tfrac{1}{2},\ q_3 = (1,-2,1)^T,$$

- eigenvalues of M:

$$\boxed{f(\lambda_k) = \sum_{n=0}^{\infty} P_n \lambda_k^n}$$

- Connection between eigenbasis and canonical basis:

$$
\begin{aligned}
A_p\,(1-c-d) &= \tfrac{1}{3}f(1) + \tfrac{1}{2}f(\tfrac{5}{6}) + \tfrac{1}{2}f(\tfrac{1}{2}) \\
A_p\,d &= \tfrac{1}{3}f(1) - \tfrac{1}{3}f(\tfrac{1}{2}) \\
A_p\,c &= \tfrac{1}{3}f(1) - \tfrac{1}{2}f(\tfrac{5}{6}) + \tfrac{1}{6}f(\tfrac{1}{2})
\end{aligned}
$$

$$
\begin{aligned}
A_p &= g(W,Q^2)\,f(1) \\
c &= \tfrac{1}{3} - \tfrac{1}{2}f(\tfrac{5}{6})/f(1) + \tfrac{1}{6}f(\tfrac{1}{2})/f(1) \\
d &= \tfrac{1}{3}\left[1 - f(\tfrac{1}{2})/f(1)\right]
\end{aligned}
$$

- the function $f(\lambda)$ contains the dynamical details of pion multiple scattering in the nucleus

Calculation of $f(\lambda)$ (Outline)

- 1-D transport problem in a nucleus:

  probability for a pion density to propagate from y to x and interact (scattering,absorption) in $[x, x+dx]$:

$$\langle x|\ P_{\rm tot}\ |y\rangle = \kappa e^{-\kappa|y-x|}\Theta(y-x)$$

with $\kappa = \rho(0)\sigma_{\rm tot}$ ('inverse interaction length')

probability that π is scattered: $\mu = \sigma_{\rm cex}/\sigma_{\rm tot}$

probability that π is absorbed: $a = \sigma_{\rm abs}/\sigma_{\rm tot}$, $\mu + a = 1$

$$\begin{aligned} \langle x|\ P_{\rm cex}\ |y\rangle &= \mu\,\langle x|\ P_{\rm tot}\ |y\rangle \\ \langle x|\ P_{\rm abs}\ |y\rangle &= a\,\langle x|\ P_{\rm tot}\ |y\rangle \end{aligned}$$

- calculation of dynamical function $f(\lambda)$:

  density of pions in medium after n scatterings: $\left|\rho^{(n)}_{\rm in}\right\rangle$

  density of pions leaving the medium after n scatterings: $\left|\rho^{(n)}_{\rm out}\right\rangle$

$$\begin{aligned} \left|\rho^{(n)}_{\rm in}\right\rangle &= P^n_{\rm cex}\ \left|\rho^{(0)}_{\rm in}\right\rangle \\ \left|\rho^{(n)}_{\rm out}\right\rangle &= (1-P_{\rm tot})\ \left|\rho^{(n)}_{\rm in}\right\rangle \\ &= (1-P_{\rm tot})P^n_{\rm cex}\ \left|\rho^{(0)}_{\rm in}\right\rangle \end{aligned}$$

$P_n \equiv N^{(n)}_{\rm out} = \int_0^L\ dx\ \left\langle x|\rho^{(n)}_{\rm out}\right\rangle$

$|\psi_{\rm out}\rangle = \sum_{n=0}^{\infty} \lambda^n\ \left|\rho^{(n)}_{\rm out}\right\rangle$

$f(\lambda) = \sum P_n\lambda^n = \int_0^L\ dx\ \langle x|\psi_{\rm out}\rangle$

$\longrightarrow$ transport integral equation

$\longrightarrow$ differential equation

$\longrightarrow$ solution by ansatz . . .

ANP model: Input

✓ $\rho(r) \to L(b) = \frac{1}{\rho(0)} \int \mathrm{d}z\, \rho[r(z,b)]$ 'effective length' at impact b

✓ elastic πN scattering in the $(3,3)$ region ($I = \frac{3}{2}$ dominance):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \sigma_{\pi^+ p}(W)\;(1 + 3\cos^2\phi)\quad \underbrace{h(W,\phi)}_{Pauli\ fac.}$$

$$\sigma_{\pi^+ p}(W) = \underbrace{\sigma_{(3,3)}(W)}_{resonant} + 20\ \mathrm{mb}$$

$$\boxed{\sigma_{\mathrm{cex}} = \frac{1}{3}\sigma_{\pi^+ p}(W)\;[h_+(W) + h_-(W)]}$$

? absorption:
→ two parametrizations for $\sigma_{\mathrm{abs}}(W)$ by Sternheim, Silbar ('72), ('73)
extracted from data on single pion production in pA scattering
→ model (A) and (B) for σ_{abs} different in shape and normalization!

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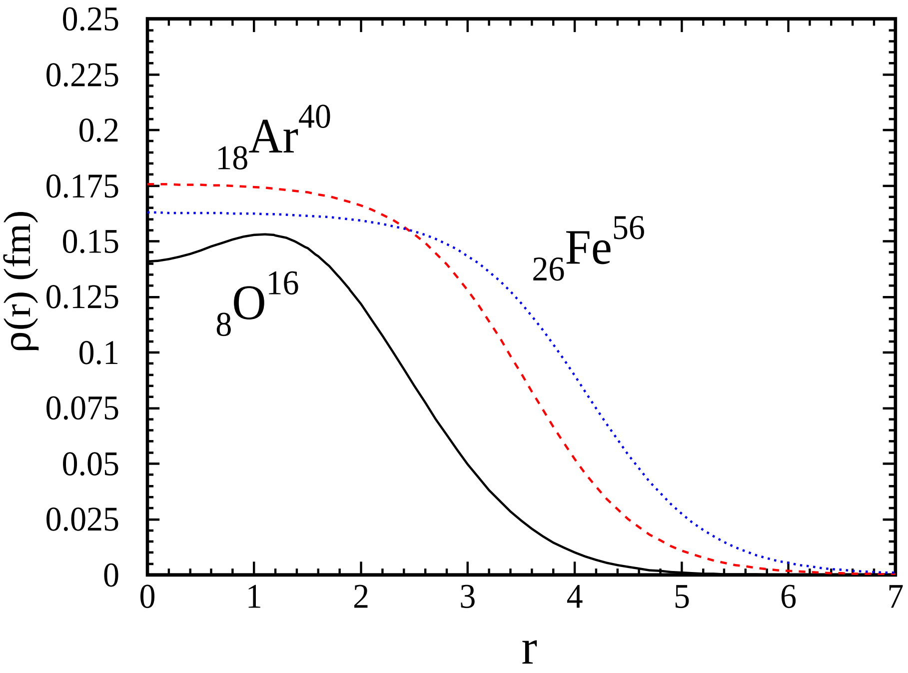
dependent quantities:

- $\sigma_{\mathrm{tot}}(W) = \sigma_{\mathrm{abs}}(W) + \sigma_{\mathrm{cex}}(W)$
- $\kappa = \rho(0)\sigma_{\mathrm{tot}}$: inverse free path length
- $\mu_\pm = \frac{1}{3}\sigma_{\pi^+ p}(W)\; h_\pm(W)/\sigma_{\mathrm{tot}}$:
  probability for charge exchange in a single πN scattering
- $a = \sigma_{\mathrm{abs}}(W)/\sigma_{\mathrm{tot}}(W)$:
  probability for absorption in a single πN scattering
- $\mu_+ + \mu_- + a = 1$

Charge density profiles



- 'Harmonic Oscillator Model' (HOM) for ${}_8O^{16}$:

$$\rho(r) = \rho(0)\exp(-r^2/R^2)\left(1 + C\frac{r^2}{R^2} + C_1\left(\frac{r^2}{R^2}\right)^2\right)$$

- 'Two Parameters Fermi Model' (2PFM) for ${}_{18}Ar^{40}$ and ${}_{26}Fe^{56}$:

$$\rho(r) = \rho(0)\left[1 + \exp\left((r - C)/C_1\right)\right]^{-1}$$

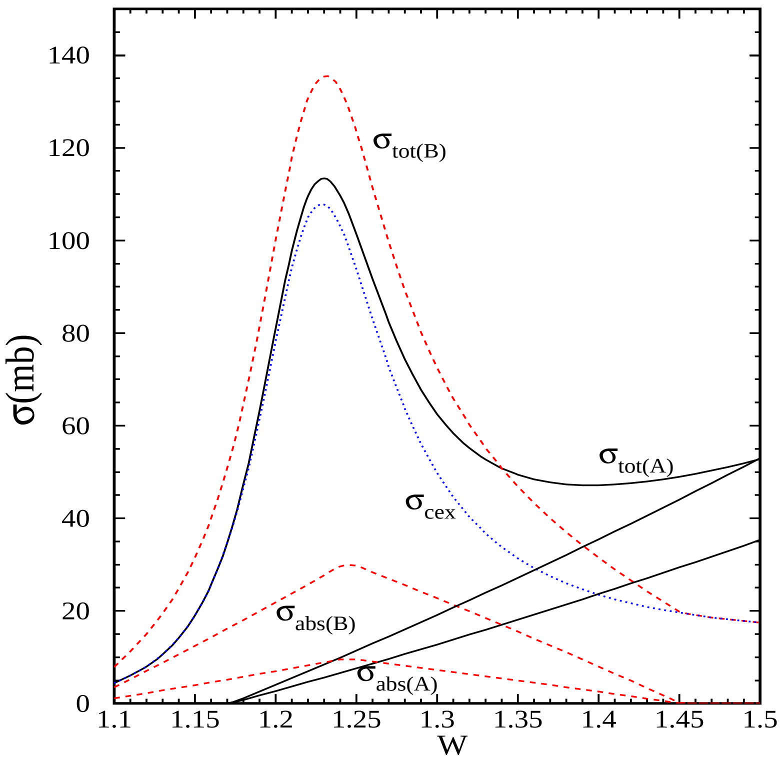
Parameters: Atom. Data Nucl. Data Tabl. **36** ('87) 495

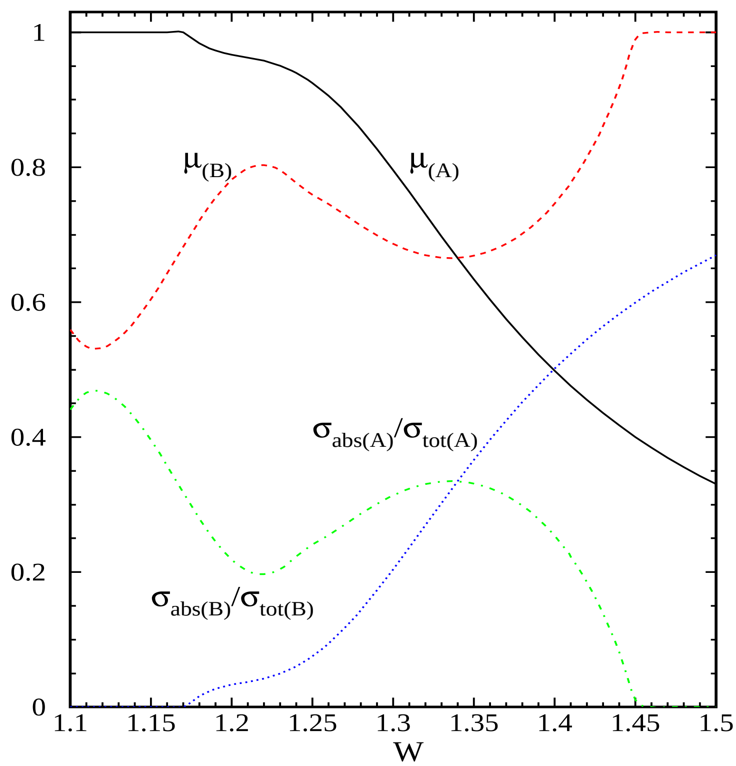
${}_Z T^A$	$a[fm]$	$C[fm]$	$C_1[fm]$	$R[fm]$	$\rho(0)[fm^{-3}]$
${}_8O^{16}$	2.718	1.544	0	1.833	0.141
${}_{18}Ar^{40}$	3.393	3.530	0.542	4.380	0.176
${}_{26}Fe^{56}$	3.801	4.111	0.558	4.907	0.163

Normalization: $\int d^3x\, \rho(r) = A$

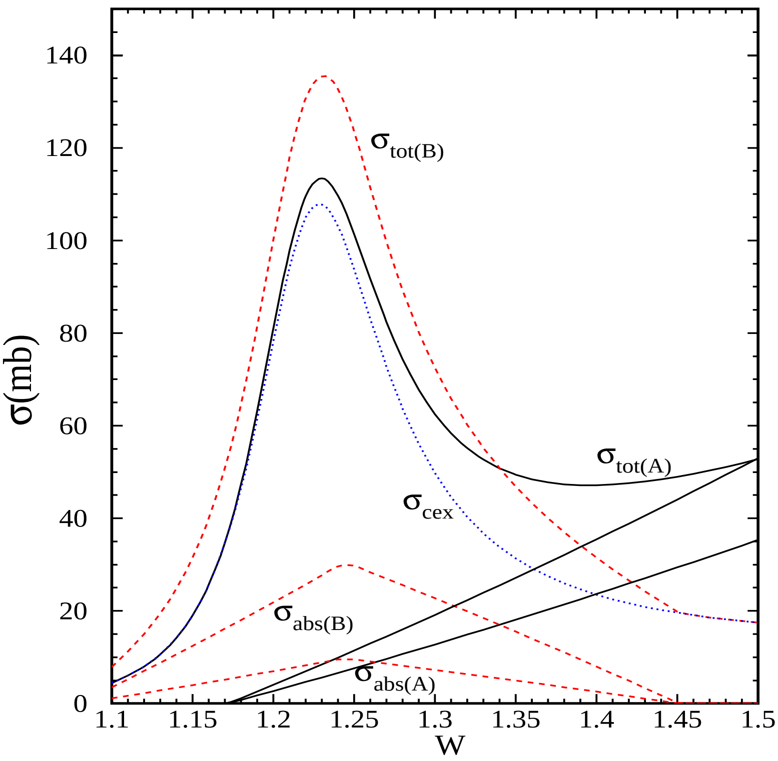
$a = \sqrt{< r^2 >}$ (root-mean-square radius), 2PFM: $R^2 = \frac{5}{3} < r^2 >$

πN cross sections





πN cross sections



ANP model: final results

$$M_\pm = A_\pm \begin{pmatrix} 1-c_\pm-d_\pm & d_\pm & c_\pm \\ d_\pm & 1-2d_\pm & d_\pm \\ c_\pm & d_\pm & 1-c_\pm-d_\pm \end{pmatrix}$$

$A_\pm = g(W,Q^2) f_\pm(1)$,
$c_\pm = \frac{1}{3} - \frac{1}{2} f_\pm(\frac{5}{6})/f_\pm(1) + \frac{1}{6} f_\pm(\frac{1}{2})/f_\pm(1)$,
$d_\pm = \frac{1}{3}[1 - f_\pm(\frac{1}{2})/f_\pm(1)]$
with

$$f_\pm(\lambda) = \frac{\int_0^\infty b\,db\,L(b) f_\pm(\lambda, L(b))}{\int_0^\infty b\,db\,L(b)}$$

Final result for the dynamical functions $f(\lambda, L)$:

$$\begin{aligned} f &= \frac{e^{\kappa\sigma L}-1}{\kappa\sigma L}\frac{1+\mu e^{-\kappa\sigma L}}{1+\mu} \\ &= f_+ + f_- \\ f_+ &= \frac{e^{\kappa\sigma L}-1}{\kappa\sigma L}\frac{\mu^2 e^{-\kappa\sigma L}-1}{\mu^2-1} \\ f_- &= \frac{e^{\kappa\sigma L}-1}{\kappa\sigma L}\frac{\mu(1-e^{-\kappa\sigma L})}{\mu^2-1} \end{aligned}$$

with

$$\sigma = \sqrt{(1-\sigma_+)^2 - \sigma_-^2}, \qquad \mu = \frac{\sigma+1-\sigma_+}{\sigma_-} e^{\kappa\sigma L}, \qquad \sigma_\pm = \lambda\mu_\pm$$

Limiting case: only forward scattering ($\mu_- = 0$)

$$f_- = 0, \qquad f_+ = \frac{1-e^{-\kappa L(1-\lambda\mu_+)}}{\kappa L(1-\lambda\mu_+)}$$

Fixing the normalization of $\sigma_{\rm abs}(W)$

- Parametrizations for $\sigma_{\rm abs}(W)$ by Sternheim, Silbar [1]
  different in shape and normalization
  $\rightarrow$ model (A): about 19% absorption for Oxygen
  $\rightarrow$ model (B): about 43% absorption for Oxygen

  However, averaging approximation ($\leftrightarrow$ total cross sections)

  $$\bar{f}(\lambda) = \frac{\int dW\ q^{-1}(W)\sigma_{(3,3)}(W)\ f(\lambda,W)}{\int dW\ q^{-1}(W)\sigma_{(3,3)}(W)}$$

  mainly sensitive to region around $W \simeq m_\Delta$
  $\rightarrow$ mainly sensitive to normalization of $\sigma_{\rm abs}(W \simeq m_\Delta)$

- Use data for D ($\leftrightarrow$ free) and Ne normalized to the <u>same</u> atmospheric E_ν spectrum given in the paper by Merenyi et al [2]

  (D): $\vec{q}_i = \left(n(\pi^+), n(\pi^0), n(\pi^-)\right)_i^T = (0.165, 0.09, 0)^T$
  (Ne): $\vec{q}_f = (0.11, 0.05, 0.01)^T \pm (0.014, 0.02, 0.01)^T$

  It is <u>not viable</u> to solve $\vec{q}_f \overset{!}{=} M[A_p, d, c]\ \vec{q}_i$ for the three parameters A_p, d, c since the solution <u>strongly</u> varies within the errors of $\vec{q}_f$

  <u>Instead:</u> reasonable assumption $0 \leq c < d$
  (prob. for $\pi^- \rightarrow \pi^+$ smaller than prob. for $\pi^0 \rightarrow \pi^+$)
  $\rightarrow c = 0$ and fit $A_p = 0.695, d = 0.147$
  $\rightarrow c = 0.01$ and fit $A_p = 0.696, d = 0.128$
  $\rightarrow c = 0.02$ and fit $A_p = 0.696, d = 0.109$
  $\rightarrow c = 0.03$ and fit $A_p = 0.696, d = 0.091$
  $\rightarrow c = 0.04$ and fit $A_p = 0.697, d = 0.072$
  $\rightarrow c = 0.05$ and fit $A_p = 0.698, d = 0.053$
  ($\chi^2/d.o.f \simeq 0.4$)

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[1] Sternheim, Silbar, PRD6(1972)3117; PRC8(1973)492
[2] Merenyi et al, PRD45(1992)743

- Remarks:
  The largest contribution to the χ^2 is from the π^0 which tends to adopt values about $\gtrsim 0.06$ ($\leftrightarrow$ 0.05 ± 0.02 (exp))

  Of course it will be necessary to include more data to obtain a more reliable result!

- Conclusions:
  The parameter A_p is well constrained: $A_p = 0.696 \pm 0.002$
  More conservatively:

  $$\boxed{A_p = 0.70 \pm 0.02}$$

  The parameters d and c are correlated:

  $$\boxed{c \in [0, 0.05], \qquad d \in [0.15, 0.05]}$$

- Fraction A of absorbed pions:
  $A = 1 - \bar{f}(\lambda = 1)$, $A_p = g(\bar{W}, \bar{Q}^2)\bar{f}(1)$ (g: Pauli factor in step 1!)

  Taking $\bar{W} \simeq m_\Delta, \bar{Q}^2 \simeq 0.1\ \mathrm{GeV}^2 \rightsquigarrow g(\bar{W}, \bar{Q}^2) = 0.93 \pm 0.05$
  $\rightsquigarrow \bar{f}(1) = 0.75 \pm 0.05 \rightsquigarrow \boxed{A = 0.25 \pm 0.05}$

- Fixing the normalization of $\sigma_{\mathrm{abs}}(W)$

  $\longrightarrow$ 25% absorption can be obtained by renormalizing absorption model (B) by a factor $\simeq 0.3$

- with this renormalization we can compare the above results with the results of theANP model:
  $\bar{f}(1) = 0.75$ (by construction), $d = 0.15$, $c = 0.05$

  $\longrightarrow$ compares favourably

Linearisations

Take forward solution with $\lambda = 1$:

$$f(\lambda = 1, L, W) = \frac{1 - e^{-\rho_0 L \sigma_{\rm abs}}}{\rho_0 L \sigma_{\rm abs}}$$

In the limit $\rho_0 L \sigma_{\rm abs} \ll 1$:
Absorption: $A(L, W) = 1 - f(1, L, W) \simeq \frac{1}{2} \rho_0 L \sigma_{\rm abs}(W)$

$\rightarrow$ estimate of fraction of absorbed pions from $\sigma_{\rm abs}(W)$

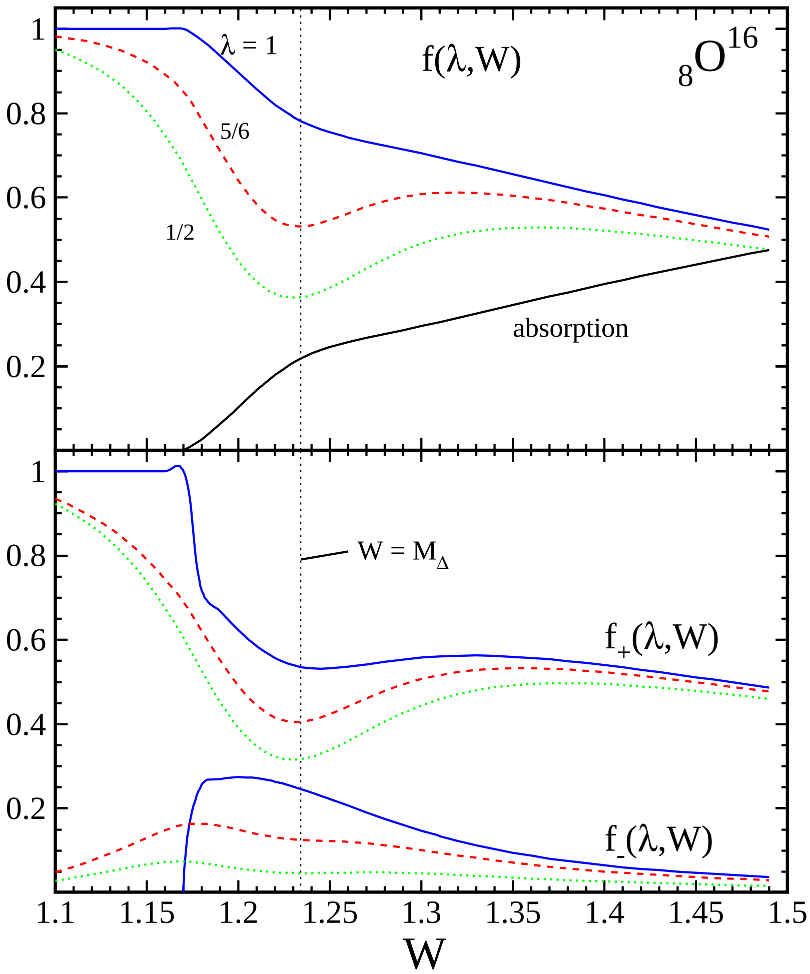
For Oxygen:
Averaging $L(b)$ over impact parameters b:
$\bar{L} \simeq 1.9R$ with radius $R \simeq 1.833$ fm
$\rho_0 = 0.141\ {\rm fm}^{-3} \rightarrow \rho_0 \bar{L} \simeq 0.05\ {\rm mb}^{-1}$

$\rightarrow$ $\boxed{A(W) \simeq 0.025\, \sigma_{\rm abs}(W)[{\rm mb}]}$ (Oxygen)

model (A): $\sigma_{\rm abs}(W = m_\Delta) \simeq 6.0$ mb $\rightarrow A \simeq 15\%$
model (B): $\sigma_{\rm abs}(W = m_\Delta) \simeq 28.4$ mb $\rightarrow A \simeq 71\%$ ($\rho_0 \bar{L} \sigma_{\rm abs} \not\ll 1$)

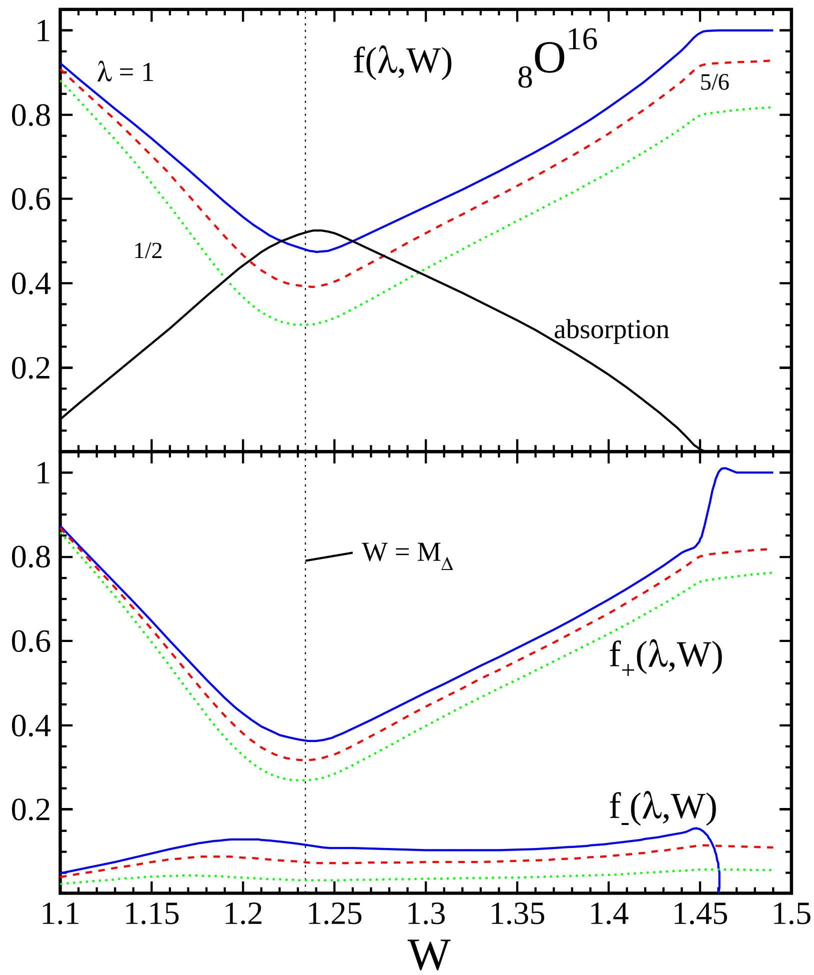
Renormalization factors from previous analysis:
model (A): 1.4, model (B): 0.3
$\rightarrow$ $\boxed{\sigma_{\rm abs}(W = m_\Delta) \simeq 8.5 \text{ mb} \rightarrow A \simeq 21\%}$

W-dependence of $f(\lambda, W)$ for Oxygen



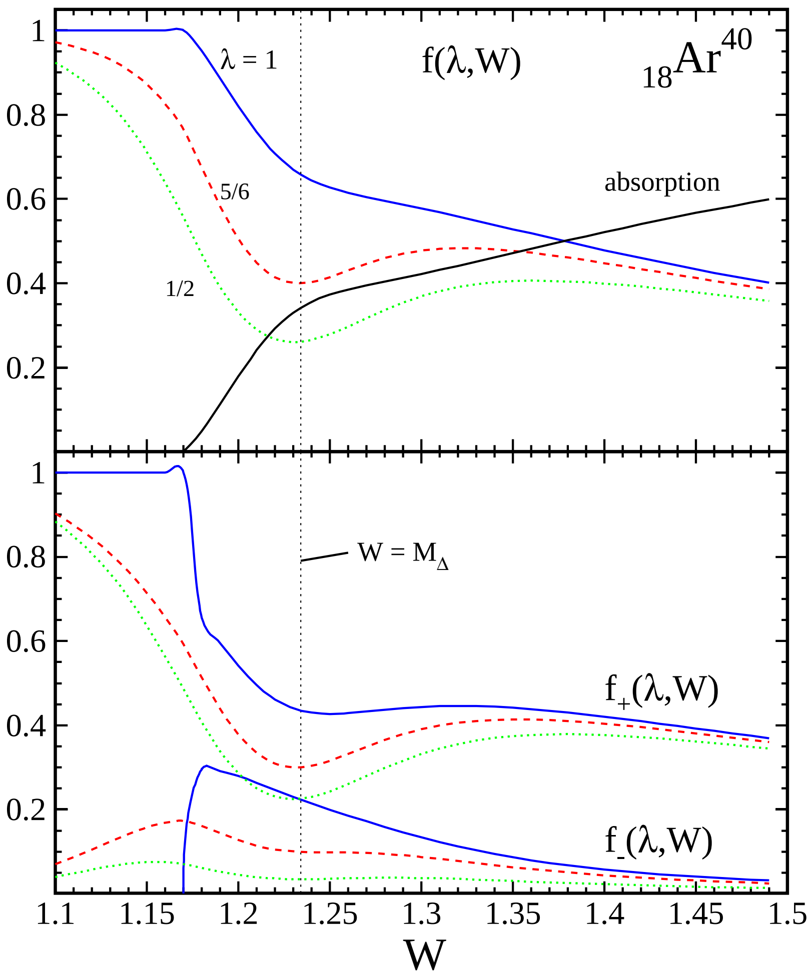
- Absorption model (A): $A = 1 - f(\lambda = 1)$
- region around $W \simeq M_\Delta$:
  - most relevant $\leftrightarrow$ cross section peaks $\leftrightarrow$ averaging approximation
  - Absorption: about 20%
  - strong dependence on $\lambda \longrightarrow$ multiple scattering
- $W < 1.15$ GeV:
  - no absorption;
  - weak dependence on $\lambda \longrightarrow$ little charge exchange
- $W > 1.35$ GeV:
  - $35 \ldots 50\%$ absorption (however, cross section small)
  - weak dependence on $\lambda \longrightarrow$ little charge exchange
- lower figure: $f = f_+ + f_-$; $f_-(\lambda, W)$ relevant for proton decay

W-dependence of $f(\lambda, W)$ for Oxygen

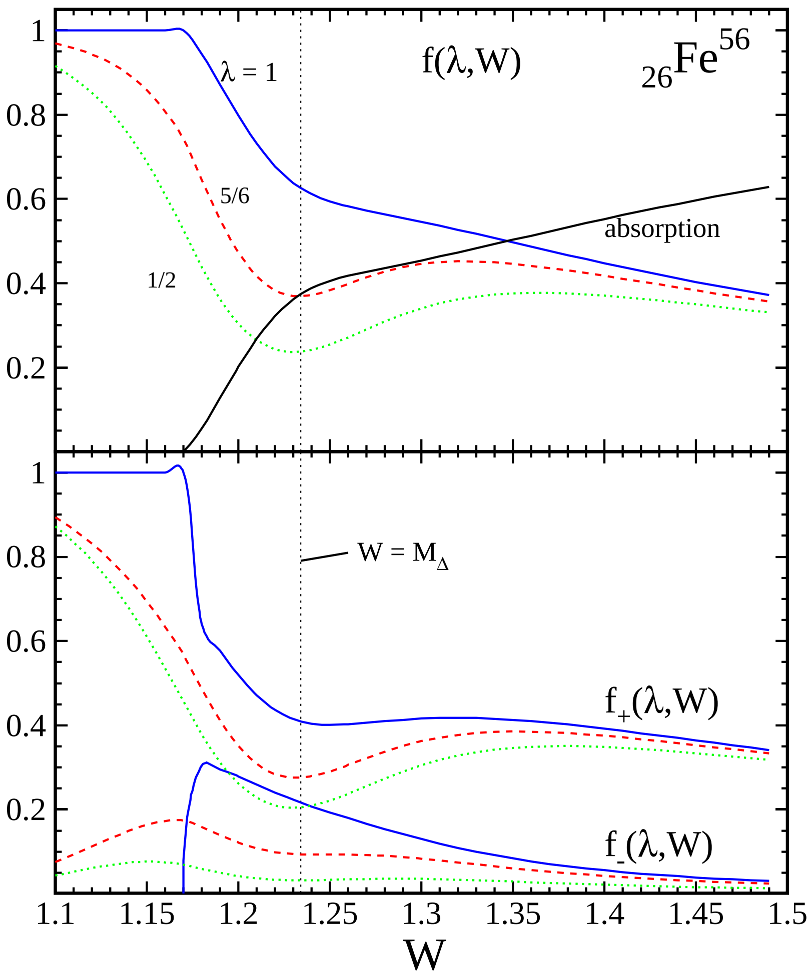


- Absorption model (B): $A = 1 - f(\lambda = 1)$
- quite different shape compared to model (A)!
  $\leftrightarrow$ different shapes of $\sigma_{\rm abs}(W)$

W-dependence of $f(\lambda, W)$ for Argon



W-dependence of $f(\lambda, W)$ for Iron



’How many’ multiple scatterings?

Probabilities:

A : prob. for pion absorption

P_0 : pion observed after 0 πN scatterings

P_1 : pion observed after 1 πN scattering

P_k : pion observed after k πN scattering

$\sum_{k=0}^{\infty} P_k + A = 1$

dynamical function:

$f(\lambda) = \sum_{n=0}^{\infty} P_n \lambda^n$

$f(\lambda = 0) = P_0$, $f(\lambda = 1) = 1 - A$

Assumption: (only approx. correct!)

Energy of pions remains constant in multiple scattering ($\leftrightarrow$ nucleons fixed)

$\rightarrow$ $\boxed{P_k \simeq P_1 r^{k-1} \text{ for } k \geq 1}$ (r to be determined)

$\rightarrow f(\lambda) \simeq \sum_{n=0}^{\infty} P_n \lambda^n$ geometrical series

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Simple solution:

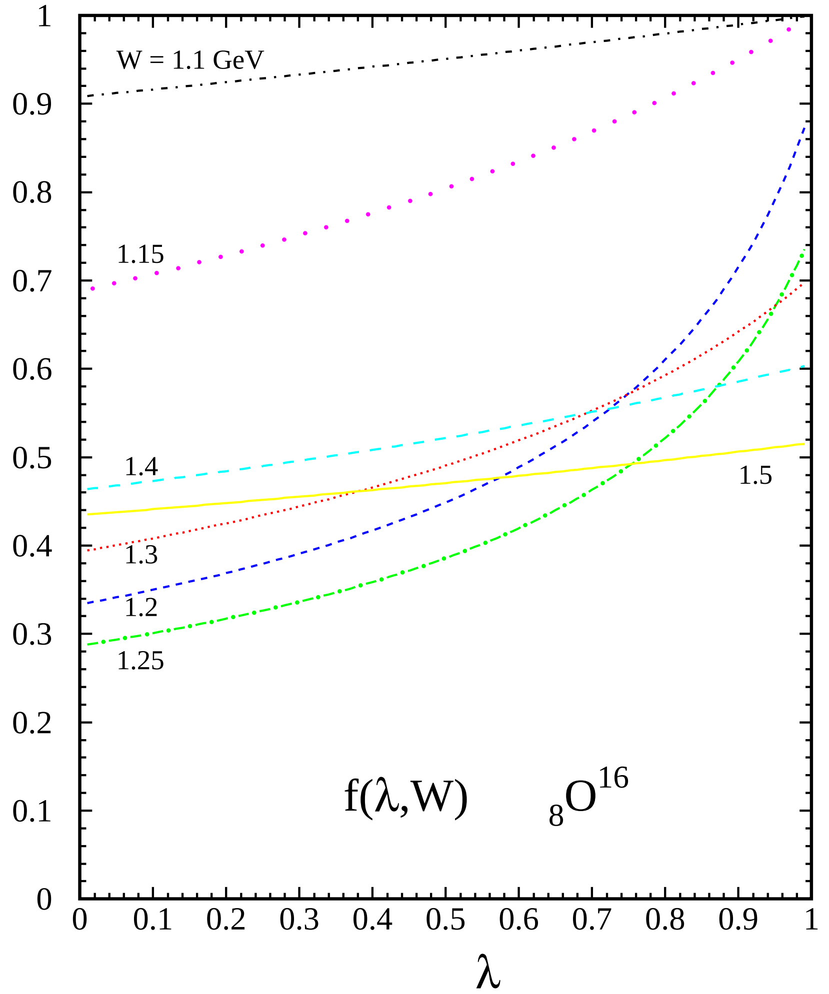
$$f(\lambda) = \sum_{n=0}^{\infty} P_n \lambda^n = P_0 + \frac{P_1 \lambda}{1 - r\lambda}$$

The constant r can be fixed from $f(\lambda = 1) = 1 - A$:

$\rightarrow$ $\boxed{r = 1 - \frac{P_1}{1 - A - P_0}}$

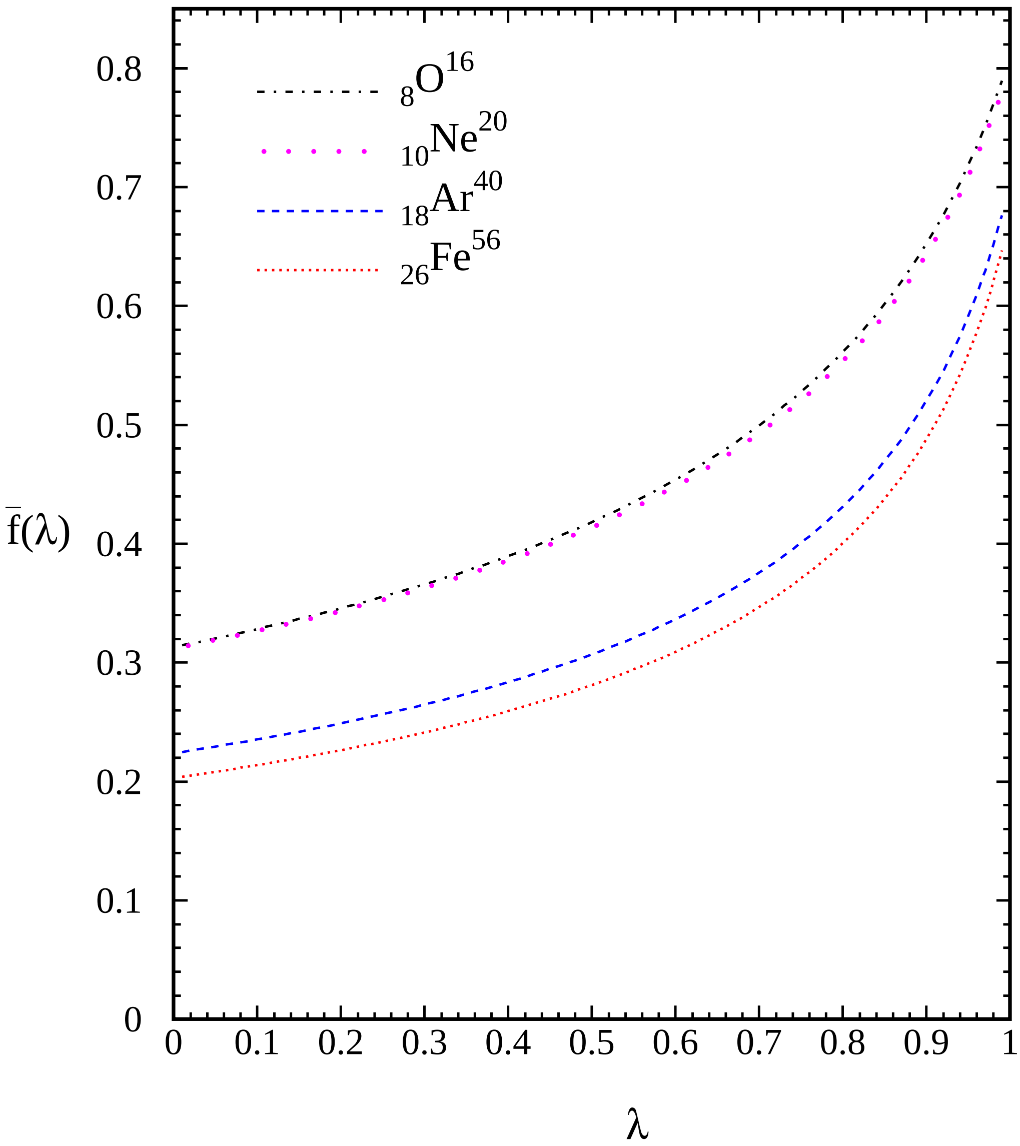
- All probabilities P_k and $f(\lambda)$ can be obtained from A, P_0, P_1; this is a direct consequence of the assumption above

λ-dependence of $f(\lambda, W)$ for Oxygen



$W\,[\mathrm{GeV}]$	1.1	1.2	1.25	1.5
A	0.0	11.8	25.7	48.4
P_0	90.8	33.6	28.8	43.4
P_1	8.0	14.1	12.2	6.5
P_2	1.0	10.5	8.9	1.3
P_3	0.13	7.8	6.5	0.26
P_4		5.8	4.8	
P_5		4.3	3.5	
P_6		3.2	2.6	
P_7		2.4	1.9	
P_8		1.8	1.4	
P_9		1.3	1.0	
P_{10}		1.0	0.7	

Averaging approximation: $\bar{f}(\lambda)$ for relevant targets



Conclusions

Have discussed absorption and rescattering of pions in the ANP model

The main conclusions:

- The charge exchange matrix M is an experimentally observable quantity
  $\rightarrow$ to make progress it is essential to fix M experimentally!
  This would allow to:
  - test and improve ANP model $\rightsquigarrow$ better predictions for other targets
  - test factorization assumption (neutrino-prod., electro-prod.)
- We have estimated that multiple scattering is quite probable!
- Using data for Ne we find the fraction of absorbed pions to be $(25 \pm 5)\%$;
  The parameters A_p, d, c are determined to be
  $A_p = 0.7 \pm 0.02$, $d \in [0.15, 0.05]$, $c \in [0, 0.05]$
  This compares quite favourably with the predicitons of the ANP model