COHERENT SCATTERING

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(Irvine, Dec. 14, 2002)

The theory of neutrino interactions at various energies has been very useful. Analyses of data are becoming accurate and reliable.

It has become evident that some earlier calculations make ad hoc assumptions, which can be eliminated.

The work in Germany was done in collaboration with :
J.-Y. Yu, I. Schienbein, F. van Horsten, and A. Kartavtsev;
I also worked closely with Dr. M. Sakuda in Japan.

1. INTRODUCTION

Small Q^2 data have uncertainties to be clarified. One must explain

$$\sigma(E_\nu),\ \frac{d\sigma}{dQ^2},\ \frac{d\sigma}{dW\,dQ^2},\ \cdots$$

Among Nuclear Effects

- Fermi motion has small effects at σ_{tot} but changes width of Δ
- Pauli Factor at $Q^2 \lesssim 0.1\ \mathrm{GeV}^2$
- Coherent Scattering
- Absorption
- Dependence of Form Factors $g_A(Q^2), \cdots$

1) Coherent scattering means that the entire region within the wavelength of incoming particles, or wavelength of momentum transfer to the Nucleus, contributes to the amplitude.

2) Radii of Nuclei determine the Q^2 – range for coherence

$$R_{nucleus} = 2\,fm \approx \frac{1}{0.10\ GeV}$$

2. Strategy for extracting coherent components

1. Plot the t-dependence

$$-t = -(q - p_\pi)^2 = \left(\sum_{\mu,\pi} p_i\right)^2 + \left[\sum_{\mu,\pi}(E_i - P_i^{11})\right]^2$$

Identify forward peak

2. Q^2 dependence $\left(\dfrac{m_A^2}{Q^2 + m_A^2}\right)^2$

3. Azimuthal Depencence

$$d\sigma \sim A + B\cos\phi + C\cos 2\phi$$

In the model of Rein + Sehgal or Belkov + Kopeliovich ones uses the formula
[see Willocq et al., Phys. Rev. D47 (1993) 2661]

$$\frac{d\sigma}{dx\,dy\,dt} = \frac{G^2 ME}{2\pi^2}(1-y)(1-x)\frac{g_{A_1}^2}{m_A^2}\left(\frac{m_A^2}{Q^2 + m_A^2}\right)^2$$

$$\otimes \frac{1}{16\pi}\left(\sigma_{tot}(\pi N)\right)^2\left(1 + r^2\right)e^{-bt} \qquad \Leftarrow \left(\frac{d\sigma}{dt}\right)_{\pi N} \approx e^{-bt}$$

$r = {}^{\mathrm{Re}\,f}\!/_{\mathrm{Im}\,f}$ with f the scattering amplitude

- There are several assumptions going into this formula, especially for Pion-Nucleus Scattering

- From $\tau \rightarrow A_1 + \nu; \quad g_{A_1}^2 = 0.024\ \mathrm{GeV}^4$

  upper bound from -decay, entire decay is resonant

  Axial Meson Dominance (AMD)

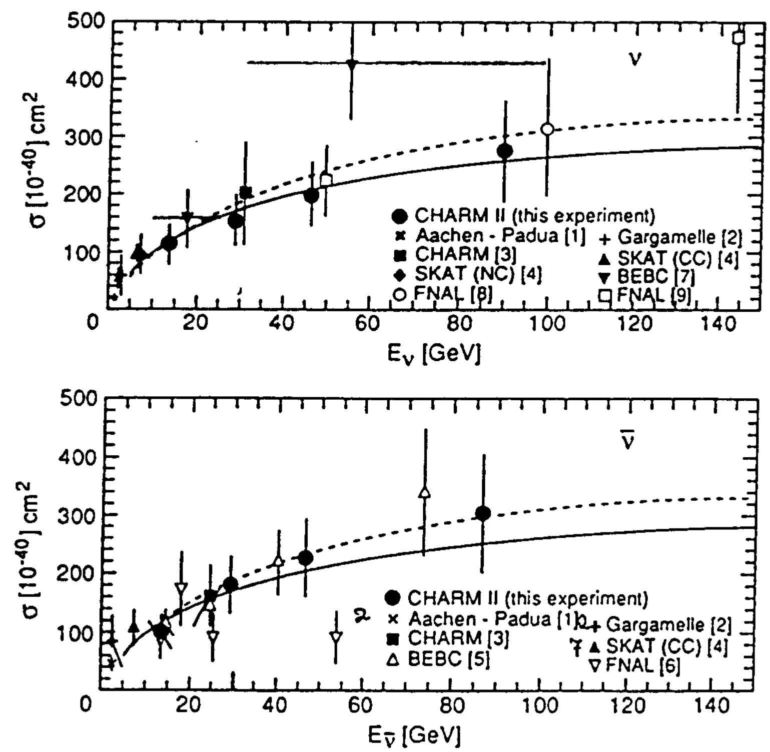
- $g_{A_1}^2 \approx g_\rho^2$ from Weinberg's sum rule

- The coefficient in the exponential *b* has still a large uncertainty.
  There is a large range for b-reported

$\mathrm{GeV}^2 \cdot b \sim$	20 ...	33 ...	80
	SKAT	BEBC E632	BEBC

2b. Experimental Evidence for Coherent pion production

(There are eight experiments that reported results)
See references



Charm 25
30
45
85

Fig. 6. Compilation of experiments on coherent single pion production. Shown are the results from both neutral current [1–4] and charged current [4–9] data, for neutrino and antineutrino induced interactions. The FNAL [8,9] values from combined neutrino and antineutrino data have been included in the upper diagram. For this experiment the results for the visible cross section were corrected for the selection of $E_\pi \geqslant 5$ GeV according to the Bel'kov–Kopeliovich approach. Data from other experiments have been scaled, where necessary, to allow comparison. The predictions of the Rein–Sehgal model (full line) and of the Bel'kov–Kopeliovich model (dashed line) are indicated.

Charm Collaboration (ref. 7)

5.

DATA

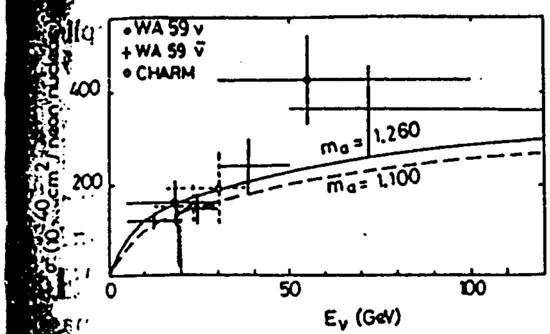


Fig. 3. Cross section for coherent π^+ production by ν interactions on neon, as a function of the ν energy E_ν; the curves are the predictions of the model for two choices of the axial-vector mass m_a. Data for π^0 production in the CHARM experiment have been scaled to correspond to charged current interactions on neon

tions of $|t|$ and $t'=|t|-t_{min}$, where $t_{min} \simeq \left(\frac{Q^2+m_\pi^2}{2\nu}\right)^2$ is the square of the minimum 4-momentum transfered to the nucleus when a pion of energy ν is produced at a given Q^2. The solid curves superimposed on the distributions are the predictions of the model given in the appendix with $m_a = 1.260$ GeV; if m_a is taken to be 1.100 GeV, the curves are nearly indistinguishable. Table 2 gives the mean values for the distributions of Fig. 4. One observes that, given the small statistics, the agreement is good between the data and the model predictions for all kinematical variables.

In conclusion, the coherent π^+ production in neutrino interactions on neon nuclei has been studied in the energy range between 5 and 150 GeV. The ab-

Marage et al. (ref. 6)
Z. Phys. C 43 (1989) 523
BEBC - Collaboration
Neon
⟨E⟩ ≈ 10 - 30 GeV
30 - 50 "

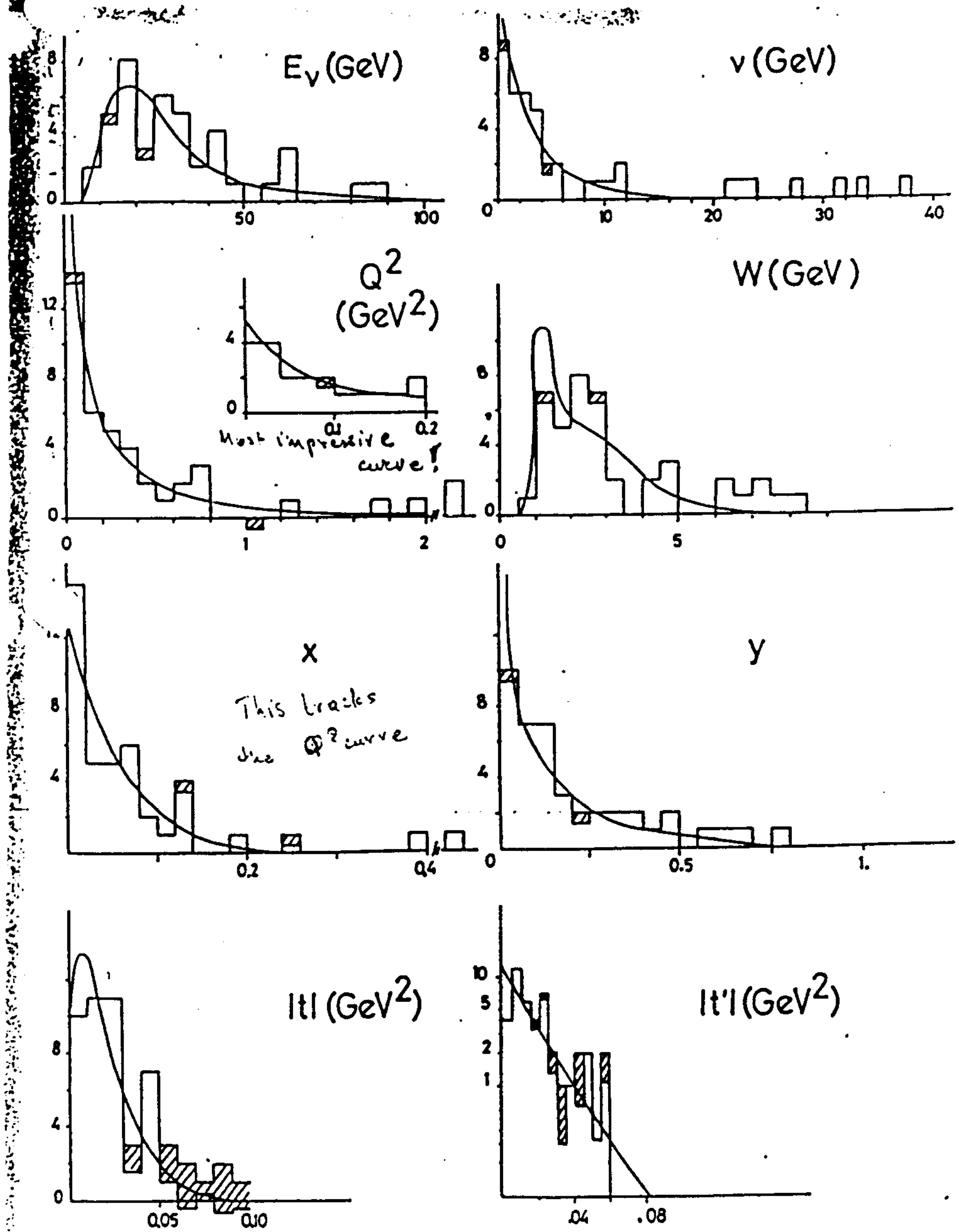
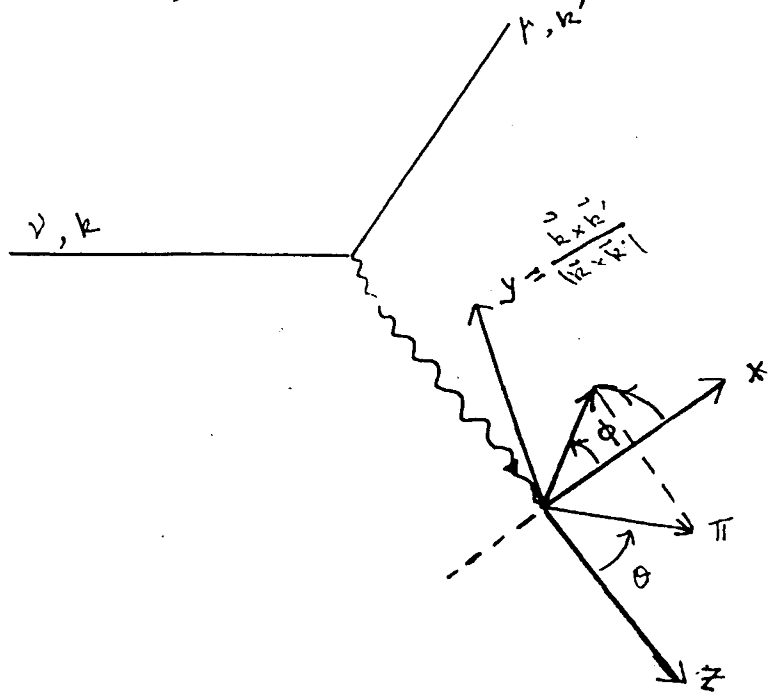
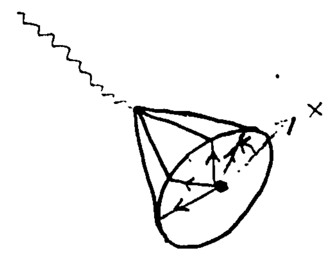


Fig. 4. Distributions of kinematical variables for the $\mu^-\pi^+$ coherent events with $|t|<0.05$ GeV^2; the incoherent background, estimated from the events with stubs, is shown hatched. The curves, normalised to the coherent signal, are the predictions of the model for $m_a = 1.260$ GeV

$t' = |t| - t_{min}$

2 C.1 AZIMUTHAL DEPENDENCE





H.-H. Grabosch et al.: Coherent Pion Production (ref. 5)

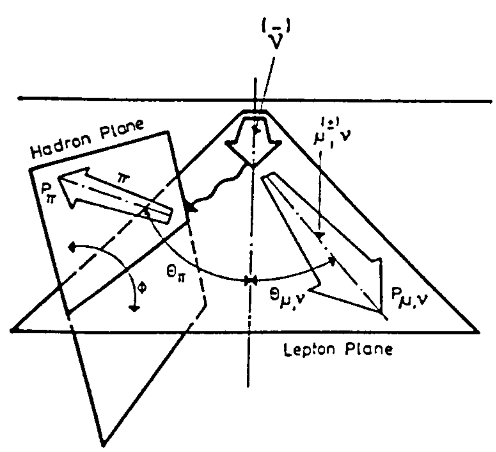


Fig. 2. Definition of the pion direction angles Θ_π, Φ

$$d\sigma \propto |A|^2 = A + B\cos\phi + C\cos 2\phi$$

$$A = c_1 Y_0^1(\theta,\phi) + c_1 Y_1^1(\theta,\phi) + c_2 Y_{-1}^1(\theta,\phi)$$

$$Y_1^1 = \sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi}$$

$$Y_{-1}^1 = \sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\varphi}$$

$$Y_0^1 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

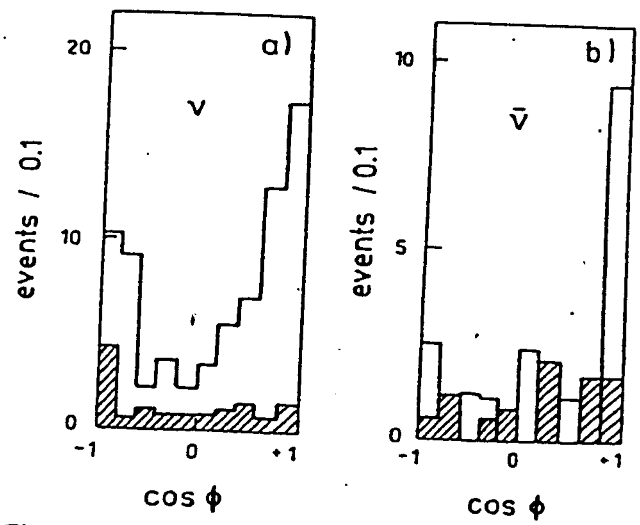


Fig. 8 a, b. Distribution of cos ϕ (ϕ azimuthal angle) for with $|t| \leqq 0.15\ \mathrm{GeV}^2$. (Hatched histograms: background butions); **a** neutrino data, **b** antineutrino data

Azimuthal for coherent

Grabosch Z. Phys. ref 5

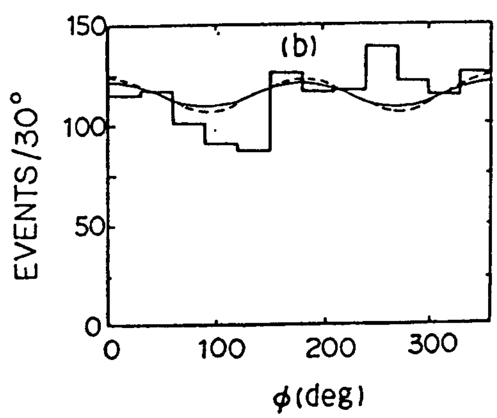


FIG. 12. Decay angular distributions of (a) cosθ and (b) azimuthal angle φ for the final state $\mu^- p\pi^+$ with $M(p\pi^+) < 1.4$ GeV and $0.5 < E_\nu < 6.0$ GeV. The solid curves are obtained from the measured density matrix elements. The dashed curves are the predictions of the Adler model with $M_A = 1.28$ GeV.

Azimuthal for Δ- production (Very different)

Kitagaki et al. PR D

3. THEORY

3a. FACTORIZATION THEOREM

Bjorken and EAP Phys. Rev. D1 (70) 3151

$$\frac{d\sigma}{dx\,dy\,d\Gamma} = \frac{G^2}{4\pi^2}(2ME')\,Q^2\left\{2\frac{d\sigma_S}{d\Gamma}+\left(\frac{E'}{E}\right)\frac{d\sigma_R}{d\Gamma}+\left(\frac{E}{E'}\right)\frac{d\sigma_L}{d\Gamma}\right\}$$

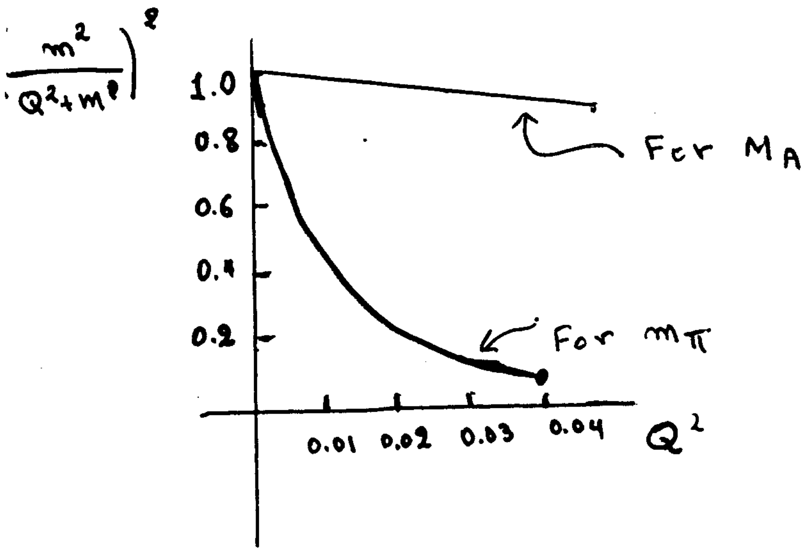
with

$$d\sigma_i = (1-x)\frac{1}{\nu m_T}\left|\langle n|\,\varepsilon^i\cdot J\,|p\rangle\right|^2(2\pi)^4\delta(p_f-q-p_i) \qquad d\,\{\text{phase space}\}$$

For $\pi N \to \pi N$

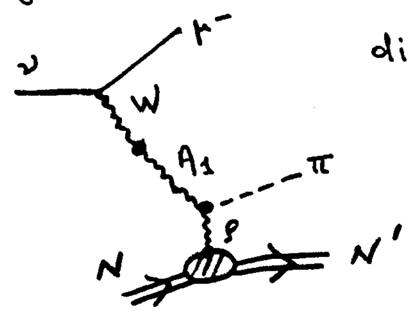
$$\frac{d\sigma_S}{dt} = (1-x)\left(\frac{m_\pi^2}{Q^2+m_\pi^2}\right)^2\frac{f_\pi^2}{Q^2}\frac{d\sigma_{\pi N\to\pi N}}{dt}F^2(t)$$

$f_\pi = 0.8\,m_\pi$



New Calculation of a diagram

Deriving a formula for the diagram:



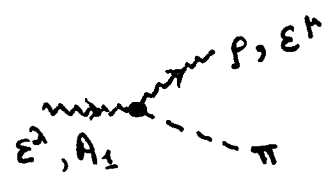
$$\frac{d\sigma}{dx\,dy\,dt} = \frac{G^2}{2\pi^3}\, g_A^2\, g_{A\rho\pi}^2\, \frac{Q^2+m_\pi^2}{m_A^2} \left(\frac{1-y}{y}\right)\left(\frac{m_A^2}{Q^2+m_A^2}\right)^2 \left(\frac{1}{|t|+m_\rho^2}\right)^2 F^2(t)$$

with $x = \frac{Q^2}{2M\nu}$, $y = \frac{\nu}{E_\nu}$ and $t = (q - p_\pi)^2$

We shall select

1) $\boxed{g_A^2 = g_\rho^2 = 0.024 \text{ GeV}^4}$ from Eq. (5) of Lalakulich + EAP : hep-ph 020627.

2) $g_{A\rho\pi}$ we obtain from $A_1 \to \rho\pi$ decay



: $\mathcal{M} = -i\, g_{A\rho\pi}\, m_A\, \epsilon^\lambda(A)\, \epsilon^\mu(\rho)$

then $\Gamma(A_1 \to \rho\pi) = \frac{g_{A\rho\pi}^2}{24\pi}\left(2 + \frac{(m_A^2 + m_\rho^2 - m_\pi^2)^2}{4 m_\rho^2 m_A^2}\right)|\vec{k}_\pi|$

The PDG gives $\Gamma_A = 250 - 600$ MeV (full width)

$$\boxed{\frac{g_{A\rho\pi}^2}{4\pi} = 1.78 \text{ to } 4.30 \text{ (dimensionless)}}$$

3) Pamettrizing $F(t) = e^{-bt}$ for Oxygen $b \simeq 20.2 \text{ GeV}^{-2}$

for a radius $r_0 = 1.8$ fm

The integrations are done as follows.

I integrate first over t, ignoring t-dependence everywhere except in the Form Factor:

$$\frac{d\sigma_0}{dx\,dy} = (1.18 \text{ to } 2.95)\cdot 10^{-12}\left(\frac{2ME}{m_A^2}\right)\left\{x(1-y)\left(\frac{m_A^2}{2MExy+m_A^2}\right)^2\right\}\frac{1}{2b}\,\frac{1}{\mathrm{GeV}^4}$$

$$\sigma_0(E_\nu) = (\underline{1.18 \text{ to } 2.95})\,\frac{1}{2b}\,\frac{1}{\mathrm{GeV}^4}\left\{1 - \frac{m_A^2}{2ME_\nu}\ln\left(\frac{2ME_\nu+m_A^2}{m_A^2}\right)\right\}$$

I have carried out the integrations over x and y analytically, integrating first over y and then over x.

$$\sigma_{coh}(E_\nu{=}1.5\,\mathrm{GeV}) = A^2\,\sigma_0(E_\nu{=}1.5\,\mathrm{GeV}) = \frac{1.18 \text{ to } 2.95}{40.5}\times 10^{-12}\times 4\times 10^{-28}\,\mathrm{cm}^2\left\{1 - 0.53\times 1.06\right\}(16)^2$$

$$= (13.1 \text{ to } 32.7)\cdot 10^{-40}\,\mathrm{cm}^2$$

$$= 3.3\times 10^{-39}\,\mathrm{cm}^2$$

Form Factor Calculation

$$F_1(q) = \frac{1}{Ze}\int \rho(r)\, e^{i\vec{q}\cdot\vec{r}} d^3$$

$$\text{for } \rho(r) = \left(\frac{a^2}{2\pi}\right)^{3/2} e^{-a^2 r^2/2}$$

the Form Factor is: $F_1(q) = e^{-q^2/2a^2}$ $\quad |\vec{q}| = |\vec{k} - \vec{k}'|$

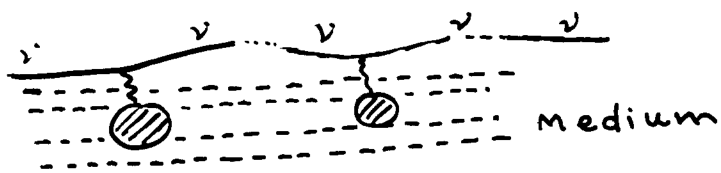
Thus $a^2/2 = \frac{1}{r_0^2}$

For Oxygen $r_0 = 1.8\,\text{fm} = \frac{1}{0.11} GeV^{-1}$

$$F_1(t) = e^{-bt} = e^{-20.3 \frac{t}{GeV^2}}$$

$$\boxed{b = \frac{20.3}{GeV^2}}$$

Ultimate coherence occurs in the Wolfenstein, Mikeyev, Smirnov Effect where the momentum transfer between two neutrinos - interacting with the medium - is very small, so that it involves the interaction with the density of the medium.



$$\mathcal{H}_{int.} = \frac{G}{\sqrt{2}} \bar{\psi}_\nu \gamma^\mu (1-\gamma_5) \psi_\nu \bar{\psi}_m \gamma_\mu (g_V - g_A \gamma_5) \psi_m$$

$$= \frac{G}{\sqrt{2}} \bar{\psi}_\nu \gamma_0 (1-\gamma_5) \psi_\nu \, g_V \psi_m^+ \psi_m$$

$$= \frac{G}{\sqrt{2}} \psi_\nu^+ \psi_\nu \cdot g_V \, n(x)$$ ⇐ This reduces to a mass term

The interaction with the medium acts as a polarizer changing the values of the masses.

There are attemts to observe similar coherent effects in the Laboratory without success so far.

SUMMARY

1. Coherent pion production has been observed at several experiments

2. It has characteristic t, Q^2 and azimuthal dependences.

3. The production rate is still investigated especially at low energies

4. It will be a challenge to observe and determine properties and parameters of the effect - in the new experiments.

Experimental References

1. H. Faissner et al. (Aachen–Padova) Phys. Lett. B 125 (83) 230

2. P. Marage et al (BEBC) Phys. Lett B 140 (84) 137

3. E. Isiksal et al. (Gargamelle) Phys. Rev. Lett. 52 (84) 1096

4. P. Marage et al (BEBC) Zeitsch. Phys. C 31 (86) 191

5. H. Grabosch et. al. (SKAT-coll.) Z. Phys. C 31 (86) 203

6. P. Marage et. al. (BEBC) Z. Phys. C 43 (89) 523

7. P. Villain et. al. (CHARM) Phys. Lett B 313 (93) 267

8. S. Willocq et. al (E 632) Phys. Rev D 47 (93) 2661

Theory References

9. D. Rein and L. Sehgal Nucl. Phys. B229 (83) 29

10. A. Belkov and B. Kopeliovich Sov. J. Nucl. Phys 46 (87) 499