Neutrino-Nucleus Cross Section: Fermi Gas Model vs Spectral function

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Introduction

- Recent development of neutrino experiments requires more accurate cross section of neutrino-nucleus scattering.
- Neutrino-nucleus cross section using realistic spectral function of nucleon in the target nucleus is compared with that of Fermi gas model.
- Quasi-elastic scattering is investigated mainly. Δ resonance is also calculated.

Models

1. Realistic spectral function.
   - Initial State → Realistic Spectral Function of ^{16}O
   - Final State Interactions → PWIA(NO FSI)
   - Vertex → On-shell $\nu + N$ Form Factors
2. Fermi Gas Model
   - Initial State → Fermi Gas
   - Final State Interactions → Pauli Blocking
3. Fermi Gas Model without Pauli Blocking
   - Initial State → Fermi Gas
   - Final State Interactions → PWIA(NO FSI)
4. Fermi Gas with Nucleon Potential

   Nucleon Energy: $E_p = \sqrt{p^2 + m^2} + V(p)$. ← NEUT

Differential Cross Section of $\nu + A \to \ell + N' + \cdots$

$$\frac{d\sigma}{dE_\ell d\Omega_\ell} = \frac{k_\ell}{8(2\pi)^4 M_A E_\nu} \int d^3\boldsymbol{p} F(\boldsymbol{p},\boldsymbol{q},\omega) \sum_{\text{spin}} |\mathcal{M}_{\nu N}|^2$$

$\boldsymbol{p}$: initial nucleon momentum, $\boldsymbol{q}$: momentum transfer
Imaginary part of $1h1p$ Green's function (involving all nuclear effects)

$$F(\boldsymbol{p},\boldsymbol{q},\omega) = \langle A | a^\dagger_{\boldsymbol{p}} a_{\boldsymbol{p}+\boldsymbol{q}} \delta(H - M_A - \omega) a^\dagger_{\boldsymbol{p}+\boldsymbol{q}} a_{\boldsymbol{p}} | A \rangle$$

Approximation : $1p1h \to$ convolution of $1p$ and $1h$

$$F(\boldsymbol{p},\boldsymbol{q},\omega) = \frac{1}{2M_A} \int d\omega' P_h(\boldsymbol{p},\omega') P_p(\boldsymbol{p}+\boldsymbol{q},\omega-\omega')$$

$P_h = \langle A | a^\dagger_{\boldsymbol{p}} \delta(H - M_A - \omega) a_{\boldsymbol{p}} | A \rangle$: Initial Spec Func (1h)
$P_p = \langle A | a_{\boldsymbol{p}+\boldsymbol{q}} \delta(H - M_A - \omega) a^\dagger_{\boldsymbol{p}+\boldsymbol{q}} | A \rangle$: Final Spec Func (1p)

Quasielastic Scattering

Invariant Amplitude of $\nu + N \to \ell + N'$

$$|\mathcal{M}_{\nu N}|^2 = \frac{G^2 \cos^2\theta_c}{2}\left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2 \eta_{\mu\nu} T^{\mu\nu}$$

$\eta_{\mu\nu}$: leptonic tensor, $T_{\mu\nu}$: hadronic tensor;

$$\begin{aligned}\eta_{\mu\nu} &= \textstyle\sum_{\text{spin}} \langle \nu|\gamma_\nu(1-\gamma_5)|\ell^-\rangle\langle \ell^-|\gamma_\nu(1-\gamma_5)|\nu\rangle \\ T_{\mu\nu} &= \textstyle\sum_{\text{spin},X} \langle N|\hat{J}_\nu^-|X\rangle\langle X|\hat{J}_\mu^+|N\rangle\end{aligned}$$

Nucleon current—in the case of $\nu + n \to \mu^- + p$

$$\begin{aligned}\langle p|\hat{J}_\mu^+|n\rangle \quad &= \overline{u_p}(\boldsymbol{p}')\Big[\gamma_\mu F_1(Q^2) + \tfrac{i\sigma_{\mu\nu}q^\nu}{2M} F_2(Q^2) \\ &\quad -\gamma_\mu\gamma_5 F_A(Q^2) - \gamma_5 q_\mu F_p(Q^2)\Big] u_n(\boldsymbol{p})\end{aligned}$$

Form Factors

For quasi-elastic scattering,

$$F_1(Q^2) = \left(1 + Q^2/(4M^2)\right)^{-1} \left[G_E(Q^2) + Q^2/(4M^2) G_M(Q^2)\right]$$
$$F_2(Q^2) = \left(1 + Q^2/(4M^2)\right)^{-1} \left[-G_E(Q^2) + G_M(Q^2)\right]$$
$$F_A(Q^2) = -1.26 \times (1 + Q^2/(1.03\mathrm{GeV})^2)^{-2}$$

where

$$\begin{cases} G_E(Q^2) = (1 + Q^2/0.71\mathrm{GeV}^2)^{-2} \\ G_M(Q^2) = 4.71 \times G_E(Q^2) \end{cases}$$

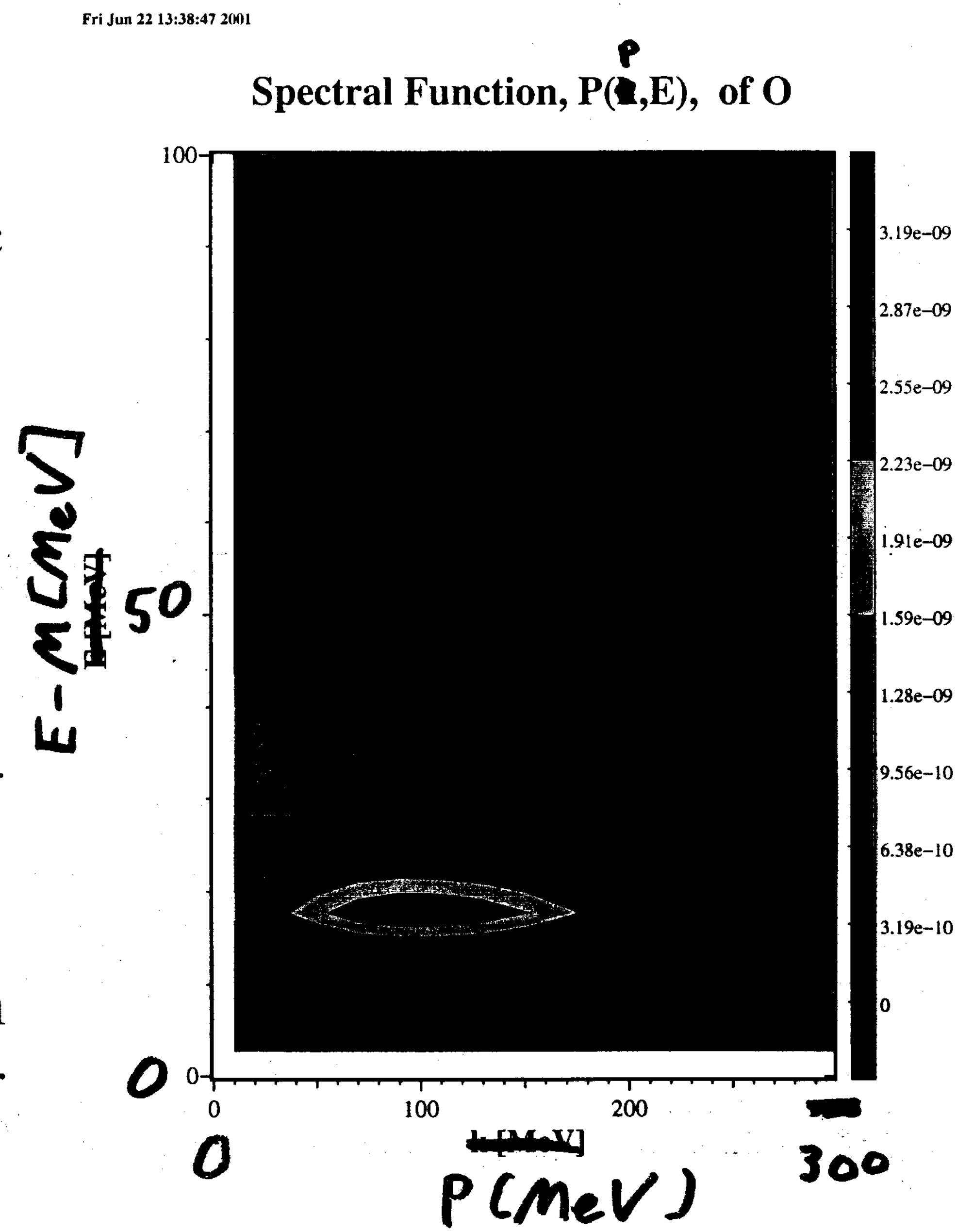
$$Q^2 = -(k_\nu - k_\ell)^2$$

Spectral Function of ^{16}O

Model 1
Initial State $\leftarrow$ Realistic Spectral Function
Final State $\leftarrow$ Free

$$P_h(\boldsymbol{p},\omega) = \frac{1}{E_p} P(\boldsymbol{p},\omega)$$
$$P_p(\boldsymbol{p}',\omega) = \frac{1}{E'_p}\delta(E'_p - \omega)$$

$P(\boldsymbol{p}, E) \leftarrow$ single particle + correlation part
with local density approx.
(SF of ^{16}O is estimated from that of ^{12}C made by Benhar et al)



Fermi Gas Model

- Model 2: Simple Fermi Gas Model (Moniz et al)

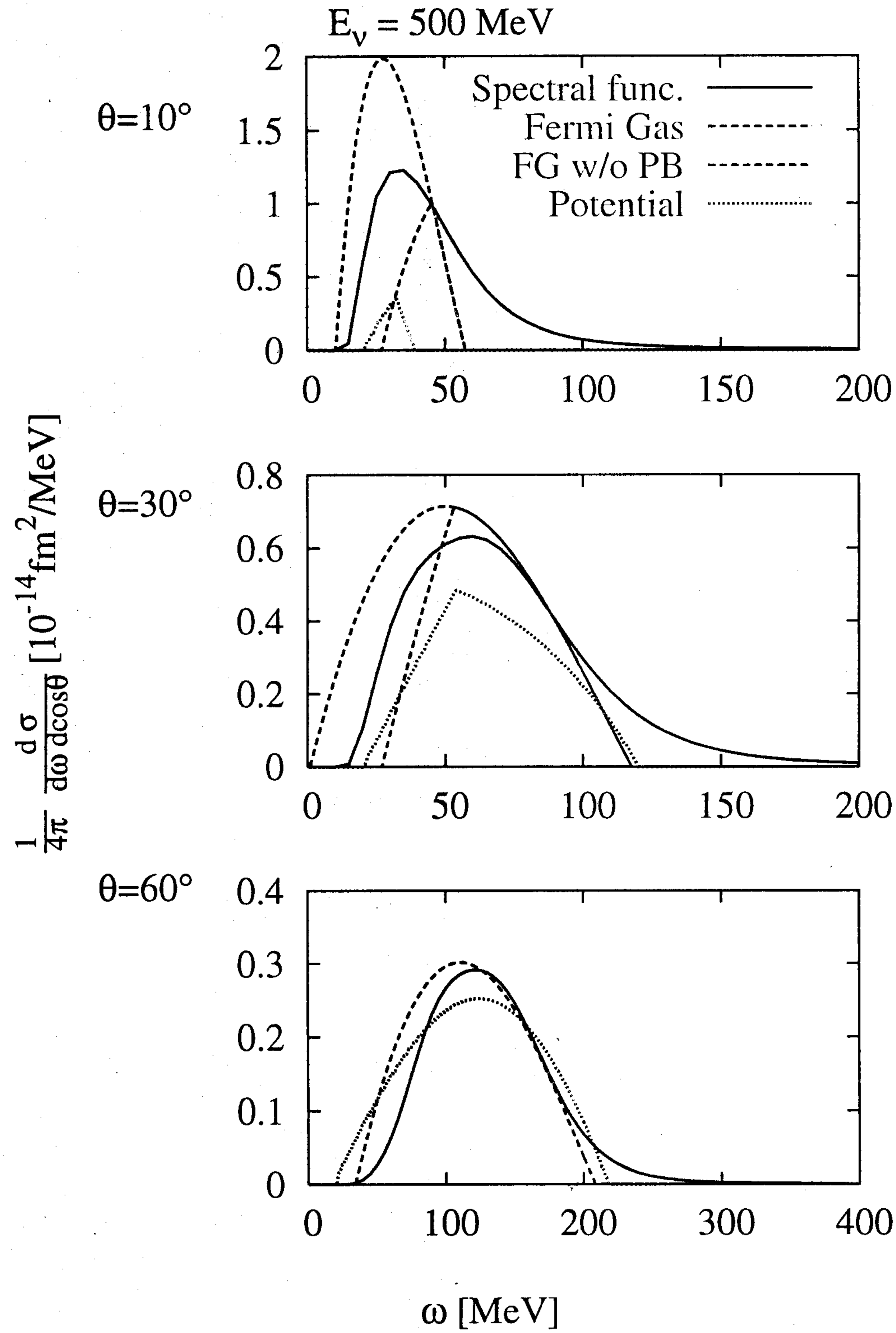
$$P_h(\boldsymbol{p},\omega) = \tfrac{1}{E_p}\theta(P_F - |\boldsymbol{p}|)\delta(E_p + \omega)$$
$$P_p(\boldsymbol{p}',\omega) = \tfrac{1}{E'_p}\theta(|p'| - P_F)\delta(E'_p - \omega)$$
$$E_p = \sqrt{p^2 + M^2} - E_B, \qquad E'_p = \sqrt{p'^2 + M^2}$$

P_F : Fermi momentum (225 MeV),
E_B : 'Binding' Energy (27 MeV)

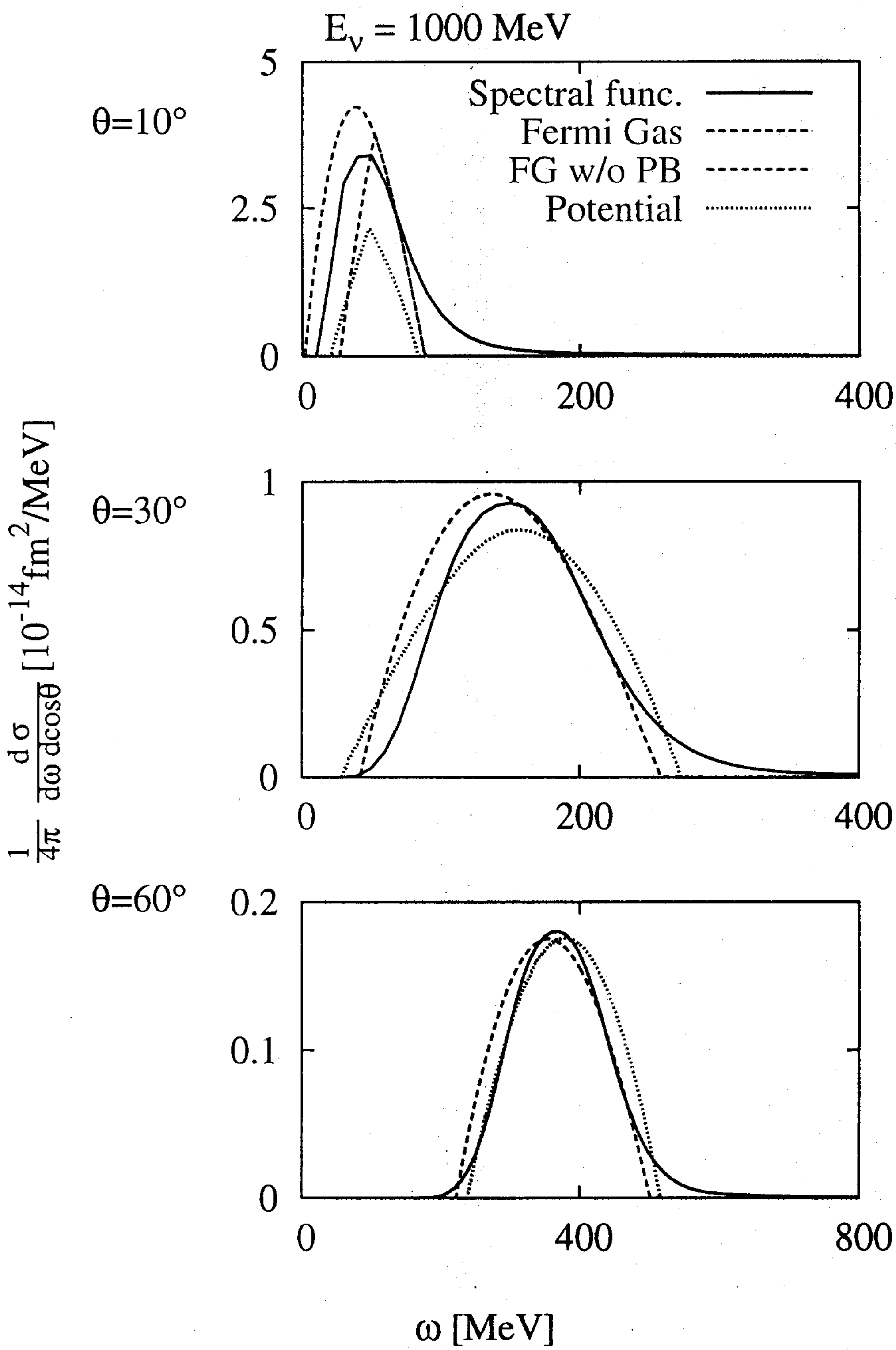
- Model 3: Fermi Gas without Pauli Blocking
$P_h(\boldsymbol{p},\omega)$ is the same as above.
$P_p(\boldsymbol{p}',\omega) = \frac{1}{E'_p}\delta(E'_p - \omega)$

- Model 4: Fermi Gas Model with Nucleon Potential (Brieva, Dellafiore)
$E_p = \sqrt{p^2 + M^2} + V(p)$,
$V(p)$: Nucleon Potential

$$\frac{d\sigma}{d\omega d\cos\theta_\mu} \text{ (QE } \nu_\mu + {}^{16}\text{O)}$$

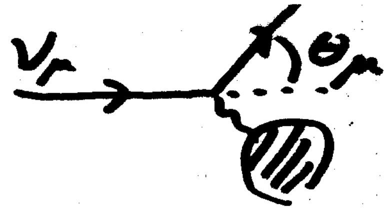


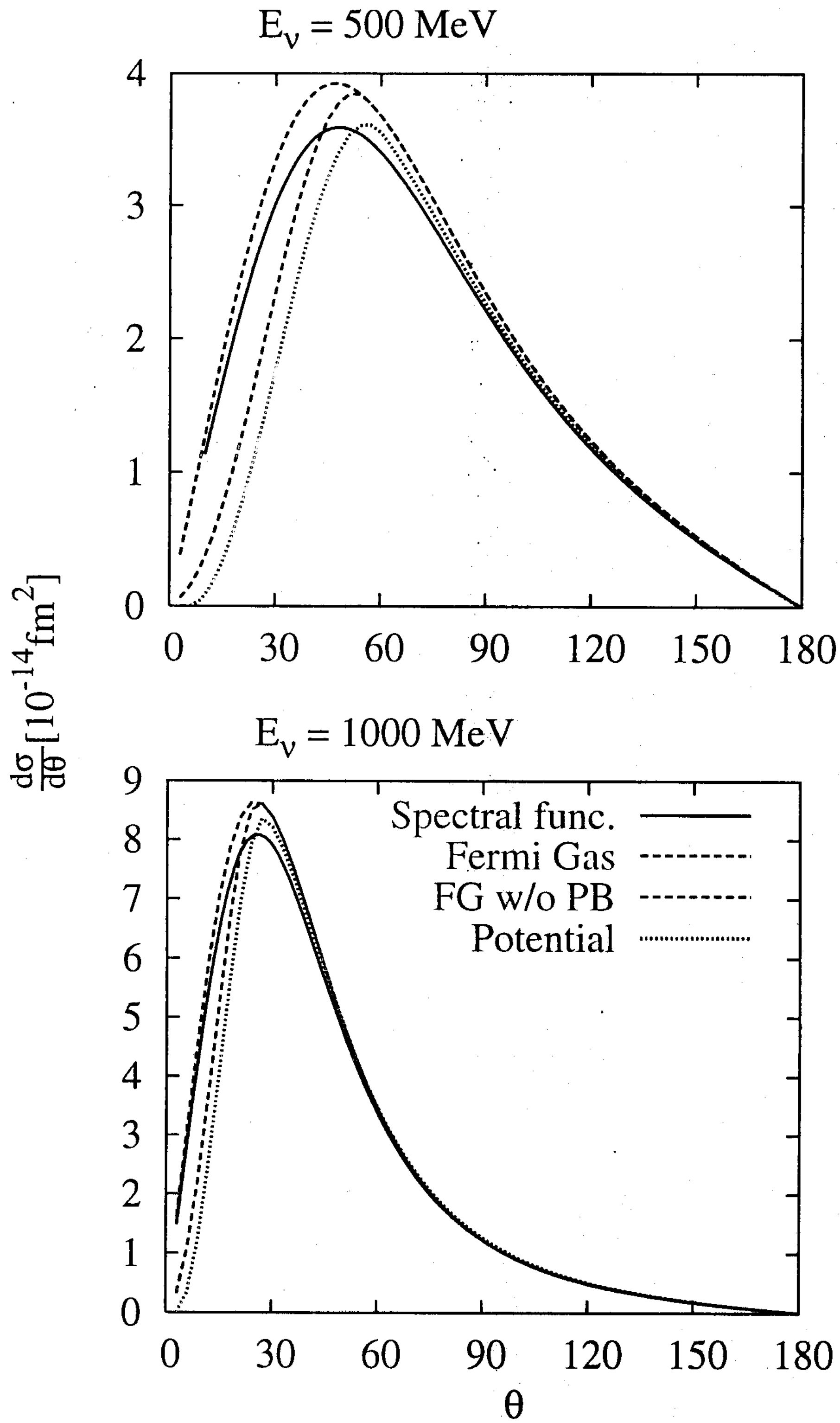
$$\omega = E_\nu - E_\ell$$



E_ν = 1000 MeV
θ=10°
Spectral func.
Fermi Gas
FG w/o PB
Potential
5
2.5
0
0
200
400
θ=30°
1
0.5
0
0
200
400
θ=60°
0.2
0.1
0
0
400
800
$\frac{1}{4\pi}\frac{d\sigma}{d\omega\, d\cos\theta}$ [10^{-14}fm^2/MeV]
ω [MeV]

$\frac{d\sigma}{d\theta_\mu}$ QE : $\nu_\mu + {}^{16}O$





Δ Resonance

$$T_{\mu\nu} = \sum_{\text{spin}} J^-_\mu J^+_\nu \times (\text{Breit–Wigner})$$

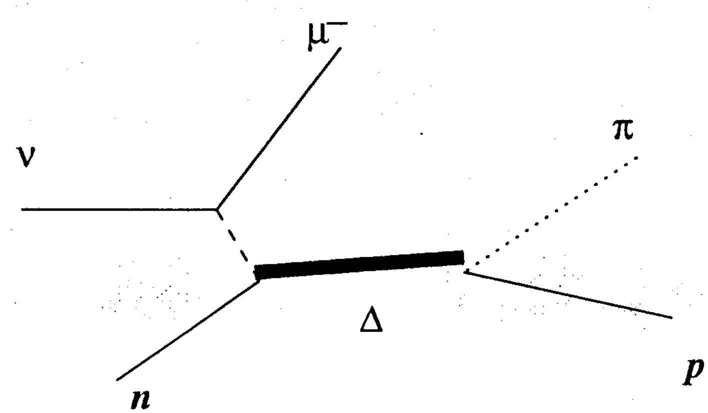
$$J^+_\mu = \overline{U}^\rho(p')\left[\frac{C_3^V(Q^2)}{M}(\not{q}g_{\rho\mu} - q_\rho\gamma_\mu) + \frac{C_4^V(Q^2)}{M^2}(g_{\rho\mu}p'\cdot q - q_\rho p'_\mu) + \frac{C_5^V(Q^2)}{M^2}(g_{\rho\mu}p\cdot q - q_\rho p_\mu) - g_{\rho\mu}C_5^A(Q^2)\gamma_5\right]\gamma_5 u(p)$$

U_ρ : spin 3/2 spinor (Rarita–Schwinger spinor)
Form Factors,

$t = -Q^2$

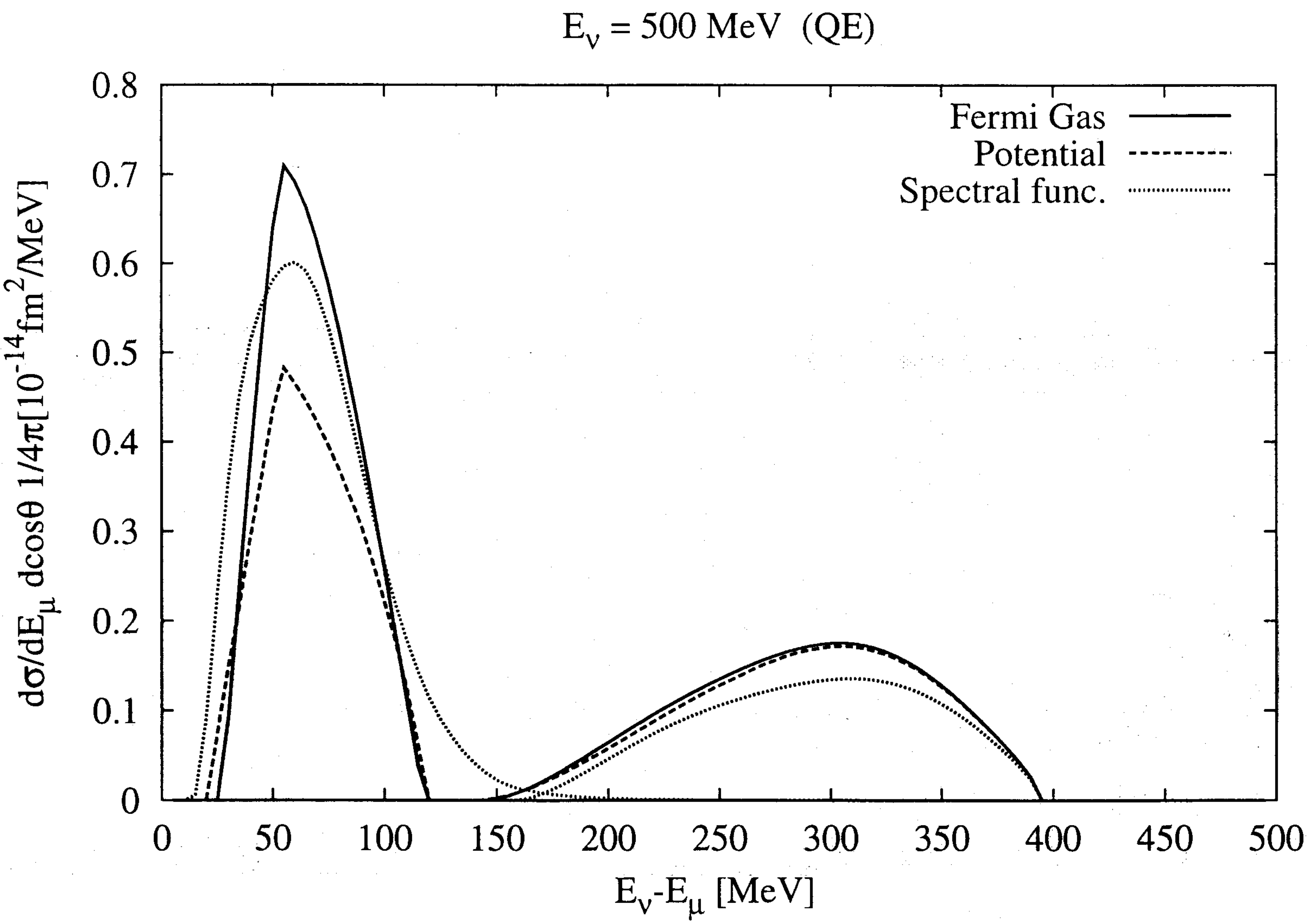
$$C_3^V = \frac{M_\Delta}{M}C_4^V = 2.07\left\{e^{-6.3\sqrt{-t}}(1+9\sqrt{-t})\right\}^{\frac{1}{2}} \qquad (t \text{ in GeV})$$

$$C_5^A = 1.18 \times (1 - t/(0.65\text{GeV})^2)^{-2}$$

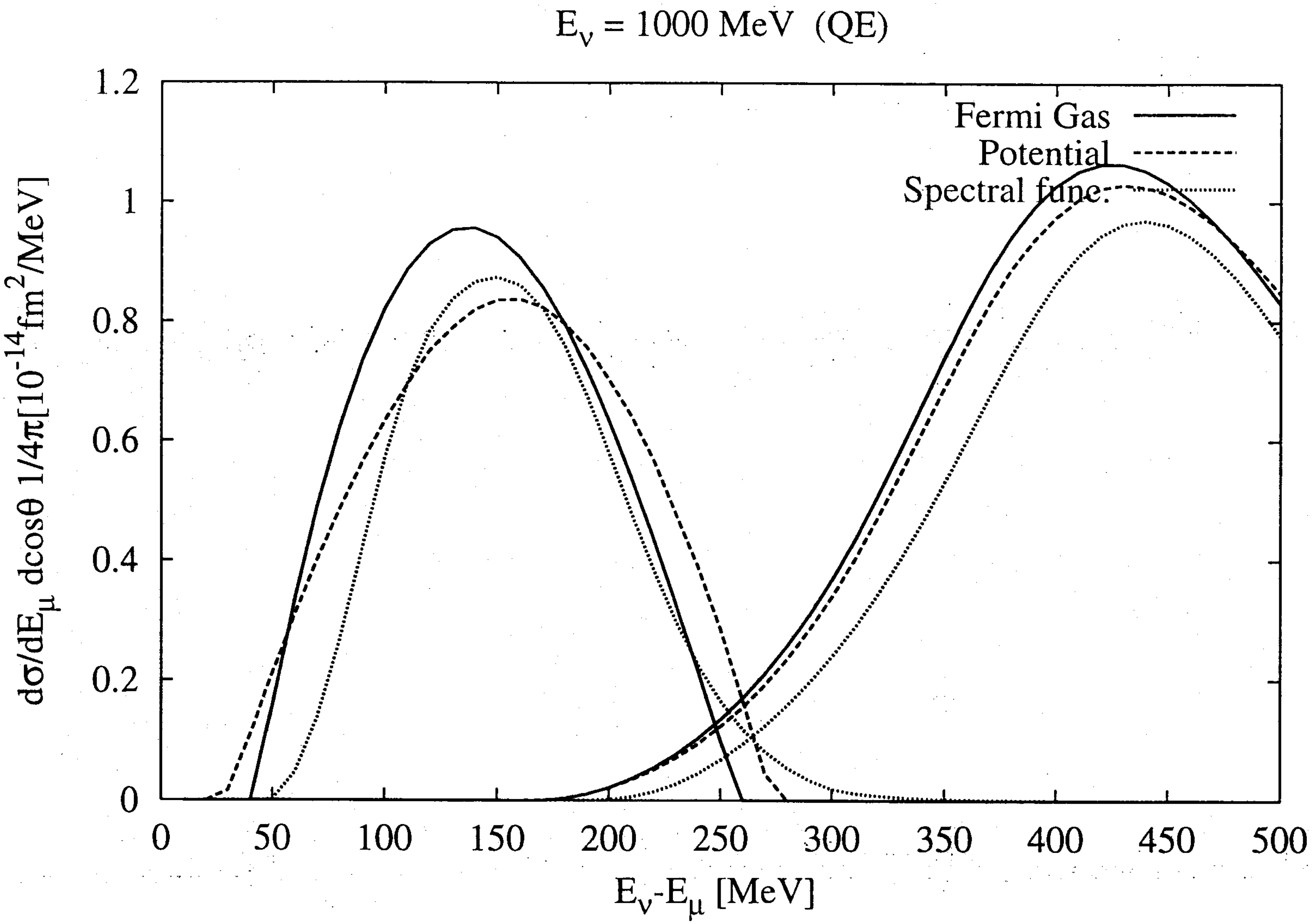


Results

$\theta_\mu = 30^\circ$



$\theta_\mu = 30^\circ$



Summary

- neutrino-nucleus cross sections for quasi-elastic scattering and Δ resonance are investigated using Fermi Gas model and ^{16}O Spectral function.
- Cross sections of Spectral Function have long tails at larger ω that doesn't appear in the case of Fermi Gas.
- Generally, cross sections of Spectral function are smaller than those of Fermi gas model (except for FG with Potential).

Future Problems

- Final State Interaction and Vertex Correction.
  Especially, final state interaction is important at small scattering angle.