Nuclear modification of structure functions in lepton scattering

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Purposes

Determination of parton distributions

- unpolarized distributions in the nucleon

  3 major groups (CTEQ, GRV, MRST)

- polarized distributions in the nucleon

  several groups (GS, ...)

- distributions in nuclei

  only a few papers (Eskola, Honkanen, Kolhinen, Ruuskanen, Salgado; Hirai, SK, Miyama)

→ for understanding nuclear mechanisms in the high-enegy region

→ for heavy-ion physics

→ for long baseline neutrino physics

→ re-examination of $q_V(x)$ in the nucleon

Situation of our nuclear PDF studies

Refs. (1) M. Hirai, SK, M. Miyama,

Phys. Rev. D64 (2001) 034003.

1st version: DY data are not included

(2) research in progress

2nd version: with DY and $F_2^A/F_2^{A'}$

We expect to write soon.

- Nuclear parton distributions (per nucleon)

  if there *were* no modification

$$A\,u^A = Z\,u^p + N\,u^n, \quad A\,d^A = Z\,d^p + N\,d^n$$

Isospin symmetry: $u^n = d^p \equiv d\,,\ d^n = u^p \equiv u$

$$\rightarrow u^A = \frac{Z\,u + N\,d}{A}, \quad d^A = \frac{Z\,d + N\,u}{A}$$

- Take into accont the nuclear modification

  by the factors $w_i(x,A)$

$$u_v^A(x) = w_{u_v}(x,A)\,\frac{Z\,u_v(x) + N\,d_v(x)}{A}$$

$$d_v^A(x) = w_{d_v}(x,A)\,\frac{Z\,d_v(x) + N\,u_v(x)}{A}$$

$$\bar{q}^A(x) = w_{\bar{q}}(x,A)\,\bar{q}(x)$$

$$g^A(x) = w_g(x,A)\,g(x)$$

Functional form of $w_i(x,A)$

$$f_i^A(x) = w_i(x,A)\, f_i(x), \quad i = u_v, d_v, \bar{q}, g$$

first, assume the A dependence as $1/A^{1/3}$

$$w_i(x,A) = 1 + (1-1/A^{1/3}) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}$$

a_i, b_i, c_i, d_i, β_i: parameters to be determined by χ^2 analysis

Fermi motion: $\dfrac{1}{(1-x)^{\beta_i}} \to \infty$ as $x\to 1$ if $\beta_i > 0$

Shadowing: $w_i(x\to 0, A) = 1 + (1-1/A^{1/3})\, a_i < 1$

Fine tuning: b_i, c_i, d_i

Constraints

- **Nuclear charge**

$$Z = A \int dx \left[\frac{2}{3}(u^A - \bar{u}^A) - \frac{1}{3}(d^A - \bar{d}^A) - \frac{1}{3}(s^A - \bar{s}^A) \right]$$

$$= A \int dx \left(\frac{2}{3} u_v^A - \frac{1}{3} d_v^A \right)$$

- **Baryon number:** $A = A \int dx\, \frac{1}{3}(u_v^A + d_v^A)$

- **Momentum:** $A = A \int dx\, x\,(u_v^A + d_v^A + 6\,\bar{q}^A + g^A)$

Three parameters can be determined by these conditions.

Experimental data

(1) F_2^A / F_2^D

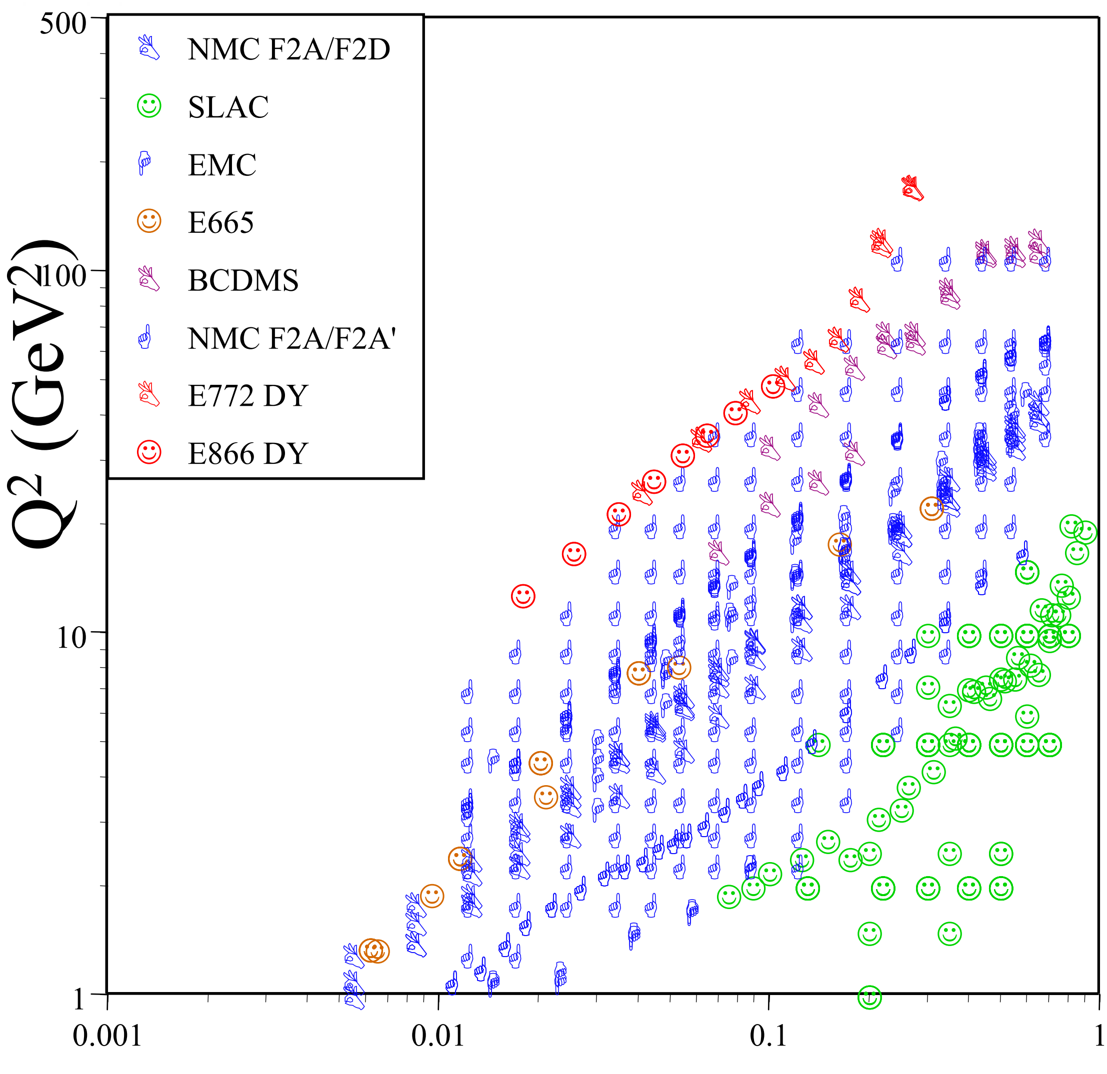
NMC: He, Li, C, Ca
SLAC: He, Be, C, Al, Ca, Fe, Ag, Au
EMC: C, Ca, Cu, Sn
E665: C, Ca, Xe, Pb
BCDMS: N, Fe

(2) $F_2^A / F_2^{A'}$

NMC: Be / C, Al / C, Ca / C, Fe / C, Sn / C, Pb / C, C / Li, Ca / Li

(3) $\sigma_{DY}^A / \sigma_{\square\square}^{A'}$

E772: C / D, Ca / D, Fe / D, W / D
E866: Fe / Be, W / Be



Analysis conditions

- parton distributions in the nucleon

  MRST98 - LO (Λ_{QCD}=174 MeV)

- Q^2 point at which the parametrized distributions are defined: $Q^2 = 1\ GeV^2$

- used experimental data: $Q^2 \geq 1\ GeV^2$

- total number of data: 669

  309 (F_2^A/F_2^D) + 308 ($F_2^A/F_2^{A'}$) + 52 (Drell-Yan)

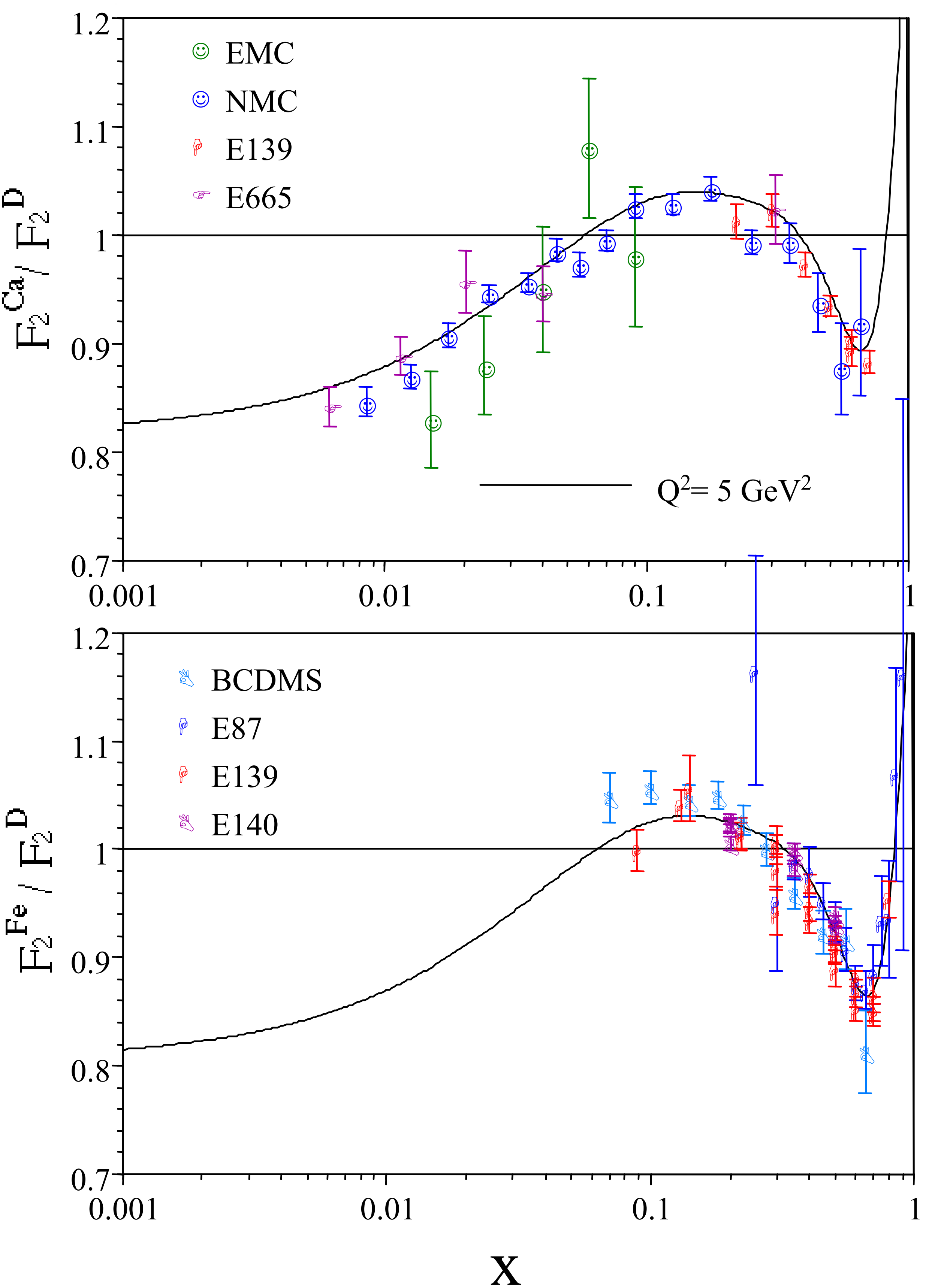
- subroutine for the χ^2 analysis: CERN - Minuit

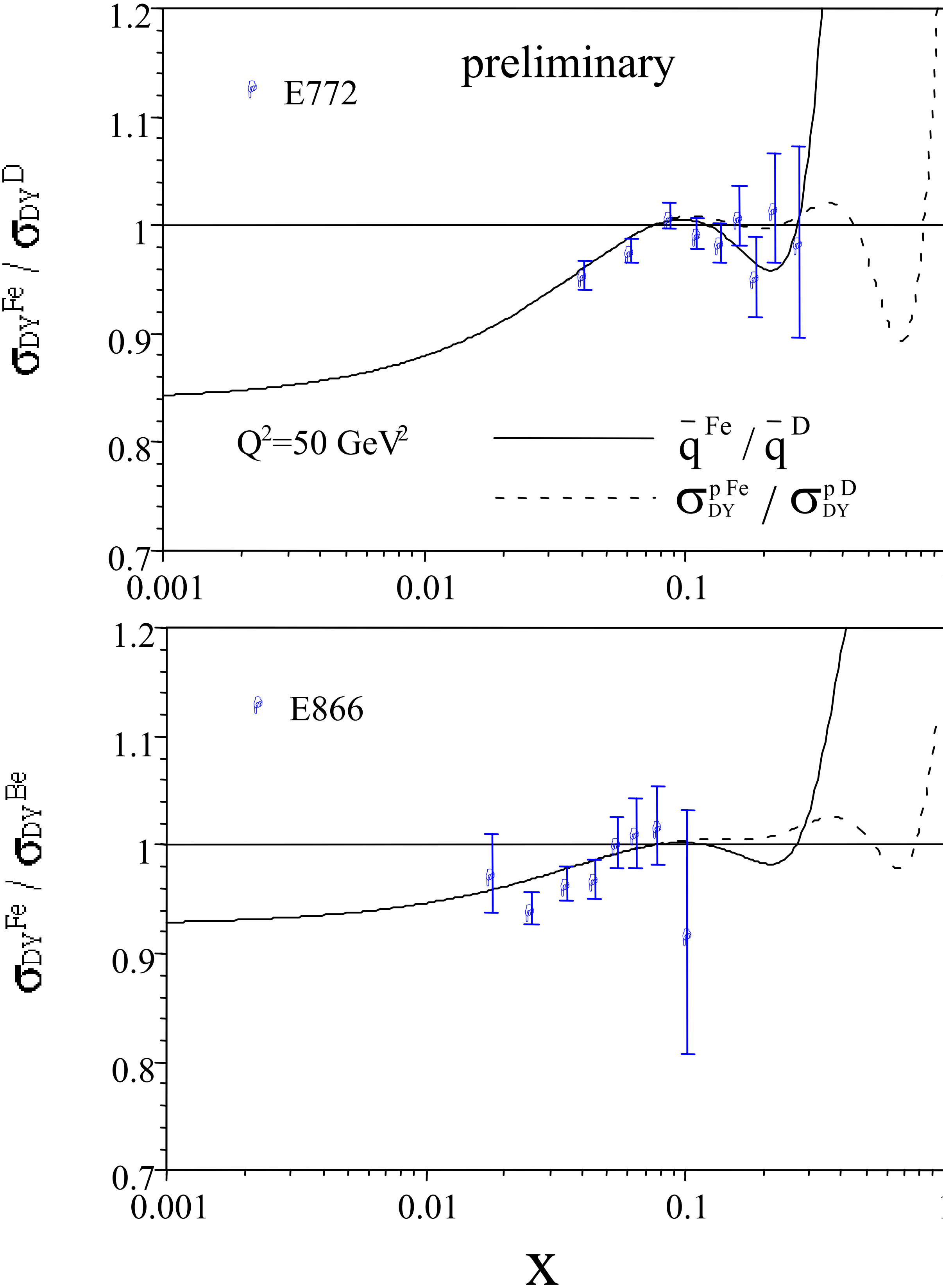
$$\chi^2 = \sum_i \frac{(R_i^{data} - R_i^{calc})^2}{(\sigma_i^{data})^2}$$

$$R = \frac{F_2^A}{F_2^D}, \quad \frac{F_2^A}{F_2^{A'}}, \quad \frac{\sigma_2^{pA}}{\sigma_2^{pA'}}, \qquad \sigma_i^{data} = \sqrt{(\sigma_i^{sys})^2 + (\sigma_i^{stat})^2}$$

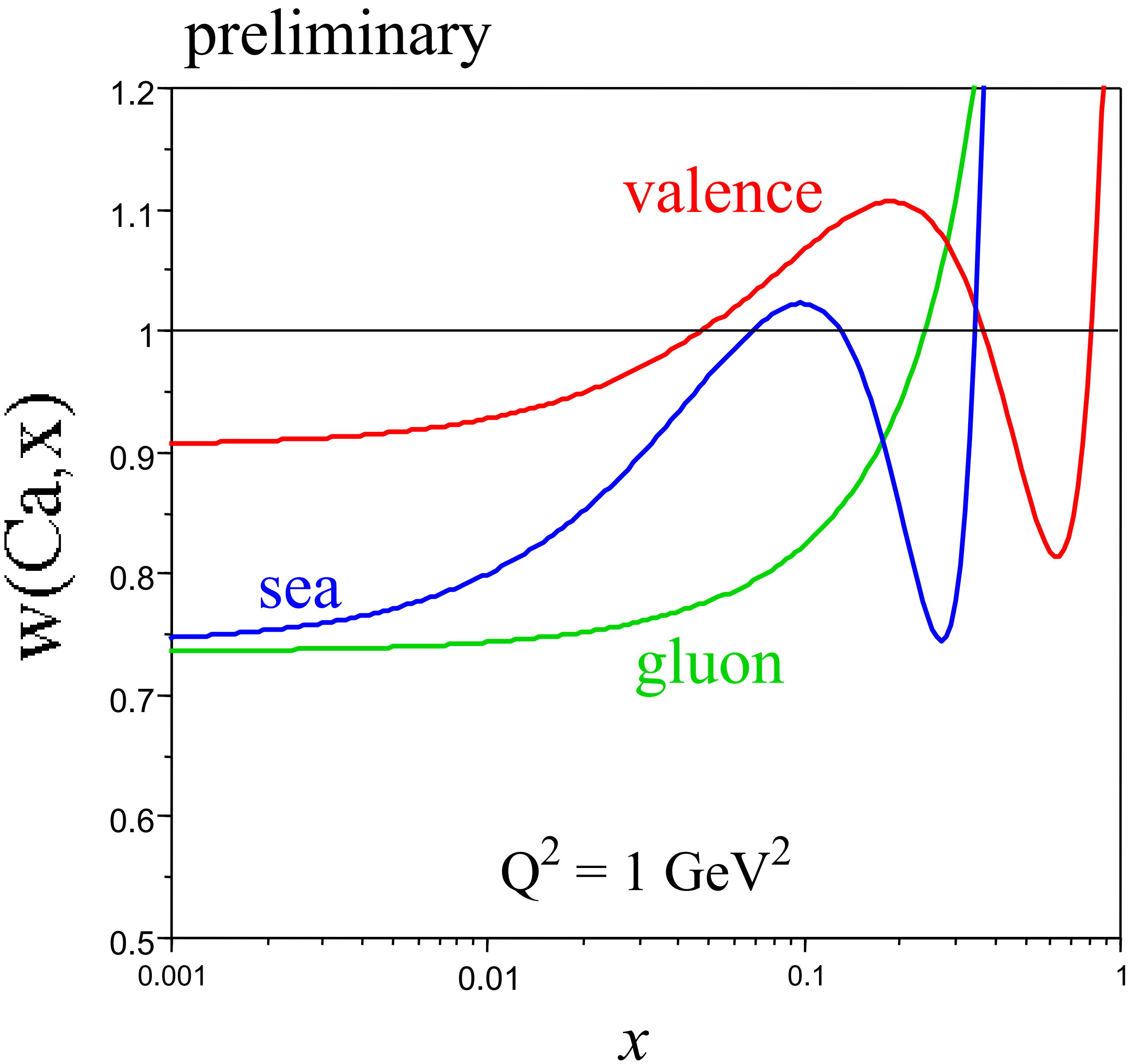
→ obtained χ^2_{min}/d.o.f.=1337.6 / 658 (preliminary)

Analysis results (preliminary)




preliminary
E772
$\sigma_{DY}{}^{Fe} / \sigma_{DY}{}^{D}$
Q^2=50 GeV^2
$\bar{q}^{Fe} / \bar{q}^{D}$
$\sigma_{DY}^{p\,Fe} / \sigma_{DY}^{p\,D}$
E866
$\sigma_{DY}{}^{Fe} / \sigma_{DY}{}^{Be}$
X

Nuclear corrections for Ca



Summary on nuclear PDFs

- χ^2 analysis

  for the nuclear parton distributions

  Computer codes could be obtained from

  http://hs.phys.saga-u.ac.jp/nuclp.html.

- nuclear PDFs are still premature

  $\rightarrow$ need analysis refinements

- reasonably good fit with χ^2_{min} / d.o.f. = 2.03

- $g^A(x)$?
- $\bar{q}^A(x)$ at medium x $\leftrightarrow$ Drell - Yan
- $q_V(x)$ at small & medium x $\rightarrow$ ν factory

- applications to

  high-eneygy nuclear reactions ?

- future experiments ?

Selected topics

(1) weak mixing angle $\sin^2\theta_W$

(2) “HERMES effect”

NuTeV $\sin^2\theta_W$ anomaly

NuTeV, Phys. Rev. Lett. 88 (2002) 091802

Others: $\sin^2\theta_W = 1 - m_W^2/m_Z^2 = 0.2227 \pm 0.0004$

NuTeV: $\sin^2\theta_W = 0.2277 \pm 0.0013$ (stat) ± 0.0009 (syst)

(talk by G. P. Zeller / K. S. McFarland)

- Studies on nuclear effects in iron

McFarland et. al., in http://neutrino.kek.jp (NuInt01)
nuclear modification of structure functions,
deviation from isoscalar nucleus

Miller and Thomas, hep-ex/0204007
shadowing effects, vector meson dominance (VMD)

Zeller et. al., hep-ex/0207052
VMD issues: Paschos-Wolfenstein, Q^2 dependence,
each NC/CC ratio

Kovalenko, Schmidt, and Yang, Phys. Lett. B546 (2002) 68
modifications of nuclear PDFs in the iron

SK, hep-ph/0209200 v2 (Phys. Rev. D in press)
difference between nuclear modifications of u_v and d_v
This paper is explained in the following.

Paschos-Wolfenstein (PW) relation

$$R^{-} = \frac{\sigma_{NC}^{\nu N} - \sigma_{NC}^{\bar{\nu} N}}{\sigma_{CC}^{\nu N} - \sigma_{CC}^{\bar{\nu} N}} = \frac{1}{2} - \sin^2\theta_W$$

N = isoscalar nucleon

NuTeV target: ^{56}Fe (Z = 26, N = 30), not isoscalar nucleus

⟶ nuclear effects should be carefully taken into account

Charged current (CC) cross sections for νA and $\bar{\nu} A$:

$$\frac{d\sigma_{CC}^{\nu A}}{dx\,dy} = \sigma_0\, x\, [\, d^A(x) + s^A(x) + \{\bar{u}^A(x) + \bar{c}^A(x)\}\,(1-y)^2]$$

$$\frac{d\sigma_{CC}^{\bar{\nu} A}}{dx\,dy} = \sigma_0\, x\, [\, \bar{d}^A(x) + \bar{s}^A(x) + \{u^A(x) + c^A(x)\}\,(1-y)^2]$$

where $\sigma_0 = G_F^2\, s / \pi$

Neutral current (NC):

$$\frac{d\sigma_{NC}^{\nu A}}{dx\,dy} = \sigma_0\, x\, [\, \{u_L^2 + u_R^2\,(1-y)^2\}\{u^A(x) + c^A(x)\}$$
$$+ \{u_R^2 + u_L^2\,(1-y)^2\}\{\bar{u}^A(x) + \bar{c}^A(x)\}$$
$$+ \{d_L^2 + d_R^2\,(1-y)^2\}\{d^A(x) + s^A(x)\}$$
$$+ \{d_R^2 + d_L^2\,(1-y)^2\}\{\bar{d}^A(x) + \bar{s}^A(x)\}\,]$$

$$\frac{d\sigma_{NC}^{\bar{\nu} A}}{dx\,dy} = \frac{d\sigma_{NC}^{\nu A}}{dx\,dy}\,(L \leftrightarrow R)$$

where $u_L = +\frac{1}{2} - \frac{2}{3}\sin\theta_W^2$, $u_R = -\frac{2}{3}\sin\theta_W^2$,

$d_L = -\frac{1}{2} + \frac{1}{3}\sin\theta_W^2$, $u_R = +\frac{1}{3}\sin\theta_W^2$

Modified PW relation for nuclei

$$R_A^- = \frac{\sigma_{NC}^{\nu A} - \sigma_{NC}^{\bar{\nu} A}}{\sigma_{CC}^{\nu A} - \sigma_{CC}^{\bar{\nu} A}}$$

$$= \frac{\{1-(1-y)^2\}\ [\,(u_L^2 - u_R^2)\{u_v^A(x) + c_v^A(x)\} + (d_L^2 - d_R^2)\{d_v^A(x) + s_v^A(x)\}]}{d_v^A(x) + s_v^A(x) - (1-y)^2\{u_v^A(x) + c_v^A(x)\}}$$

where $q_v^A(x) \equiv q^A(x) - \bar{q}^A(x)$

Nuclear effects are in the weight functions: w_{u_v} and w_{d_v}

$$u_v^A(x) = w_{u_v}(x, A, Z)\ \frac{Z\,u_v(x) + N\,d_v(x)}{A}$$

$$d_v^A(x) = w_{d_v}(x, A, Z)\ \frac{Z\,d_v(x) + N\,u_v(x)}{A}$$

Neutron excess and a related function

$$\hat{\varepsilon}_n = \frac{N-Z}{A}\ , \quad \varepsilon_n(x) = \hat{\varepsilon}_n\ \frac{u_v(x) - d_v(x)}{u_v(x) + d_v(x)}$$

Difference between nuclear modifications of u_v and d_v

$$\varepsilon_v(x) = \frac{w_{d_v}(x, A, Z) - w_{u_v}(x, A, Z)}{w_{d_v}(x, A, Z) + w_{u_v}(x, A, Z)}$$

$$R_A^- = \frac{(\frac{1}{2} - \sin^2\theta_W)\ \{1 + \varepsilon_v(x)\,\varepsilon_n(x)\} + \frac{1}{3}\sin^2\theta_W\ \{\varepsilon_v(x) + \varepsilon_n(x)\} + (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\ \varepsilon_s(x) + (\frac{1}{2} - \frac{4}{3}\sin^2\theta_W)\ \varepsilon_c(x)}{1 + \varepsilon_v(x)\,\varepsilon_n(x) + \frac{1+(1-y)^2}{1-(1-y)^2}\ \{\varepsilon_v(x) + \varepsilon_n(x)\} + \frac{2\{\varepsilon_s(x) - (1-y)^2\,\varepsilon_c(x)\}}{1-(1-y)^2}}$$

Expand in ε_v, ε_n, ε_s, ε_c << 1

$$R_A^- = \frac{1}{2} - \sin^2\theta_W$$

$$- \varepsilon_v(x) \left\{ \left(\frac{1}{2} - \sin^2\theta_W\right) \frac{1+(1-y)^2}{1-(1-y)^2} - \frac{1}{3}\sin^2\theta_W \right\} + O(\varepsilon_v^2)$$

$$+ O(\varepsilon_n) + O(\varepsilon_s) + O(\varepsilon_c)$$

small effect which increases the deviation, Zeller et al., PRD 65 (2002) 111103

taken into account in the NuTeV analyis

my studies in hep-ph/0209200

note: $\varepsilon_v(x)$ is not known !

constraints of baryon number and charge

$$Z = \int dx\, A \sum_q e_q (q^A - \bar{q}^A) = \int dx\, \frac{A}{3} (2 u_V^A - d_V^A)$$

$$A = \int dx\, A \sum_q \frac{1}{3} (q^A - \bar{q}^A) = \int dx\, \frac{A}{3} (u_V^A + d_V^A)$$

$$\int dx\, (u_v + d_v) [\, \Delta w_v + w_v\, \varepsilon_v(x)\, \varepsilon_n(x) \,] = 0$$

$$\int dx\, (u_v + d_v) [\, \Delta w_v \{1 - 3\,\varepsilon_n(x)\} - w_v\, \varepsilon_v(x) \{3 - \varepsilon_n(x)\} \,] = 0$$

where $w_v = \dfrac{w_{u_v} + w_{d_v}}{2}$, $\Delta w_v = w_v - 1$

There is no unique solution. The $\varepsilon_v(x)$ should be determined experimentally. We studied some possibilities theoretically.

Possible prescriptions

note: merely estimates of the magnitude

(A) $\int dx\,(u_v + d_v)\,[\Delta w_v + w_v\,\varepsilon_v(x)\,\varepsilon_n(x)] = 0$

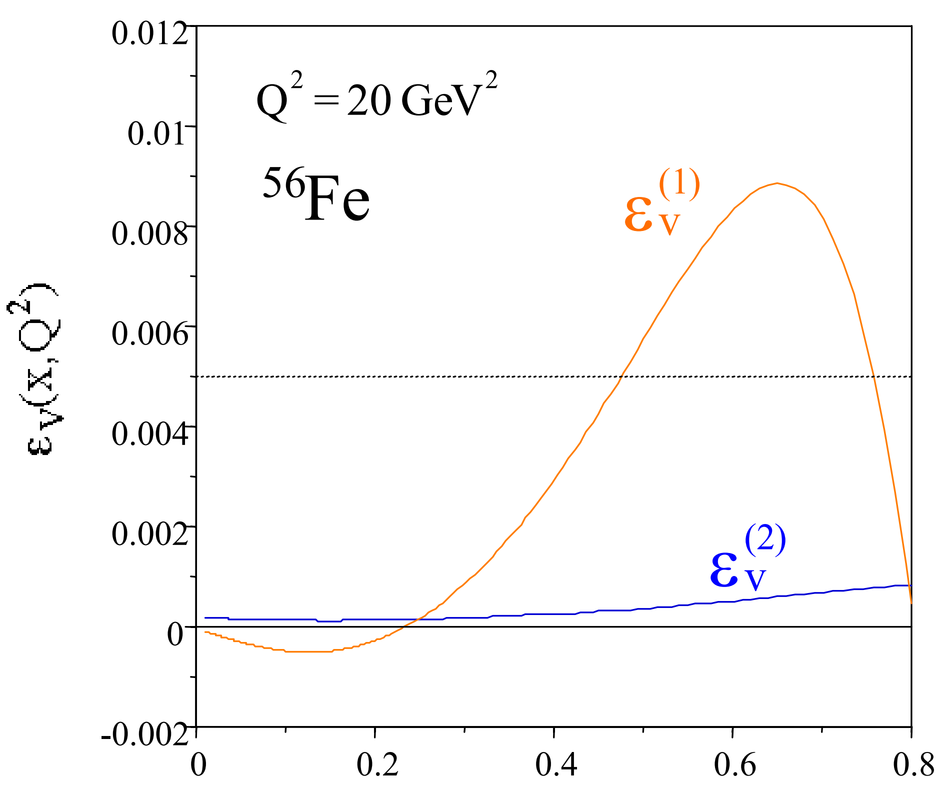
(B) $\int dx\,(u_v + d_v)\,[\Delta w_v\{1 - 3\,\varepsilon_n(x)\} - w_v\varepsilon_v(x)\{3 - \varepsilon_n(x)\}] = 0$

Prescription 1. Neglect $O(\varepsilon^2)$, then integrand (B) = 0

$$\varepsilon_v^{(1)}(x) = -\,\hat{\varepsilon}_n\,\frac{u_v(x) - d_v(x)}{u_v(x) + d_v(x)}\,\frac{\Delta w_v(x)}{w_v(x)}$$

Prescription 2. χ^2 analysis of NPDFs

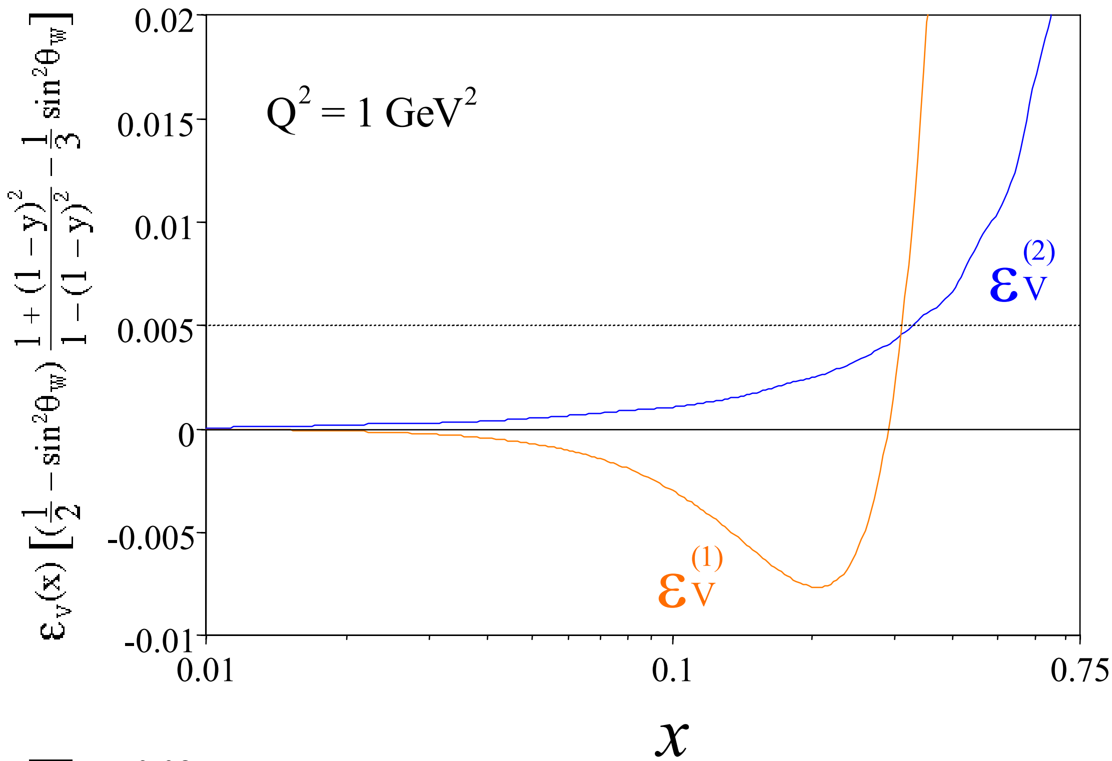
$$\varepsilon_v^{(2)}(x) = \frac{w_{d_v}(x) - w_{u_v}(x)}{w_{d_v}(x) + w_{u_v}(x)}$$

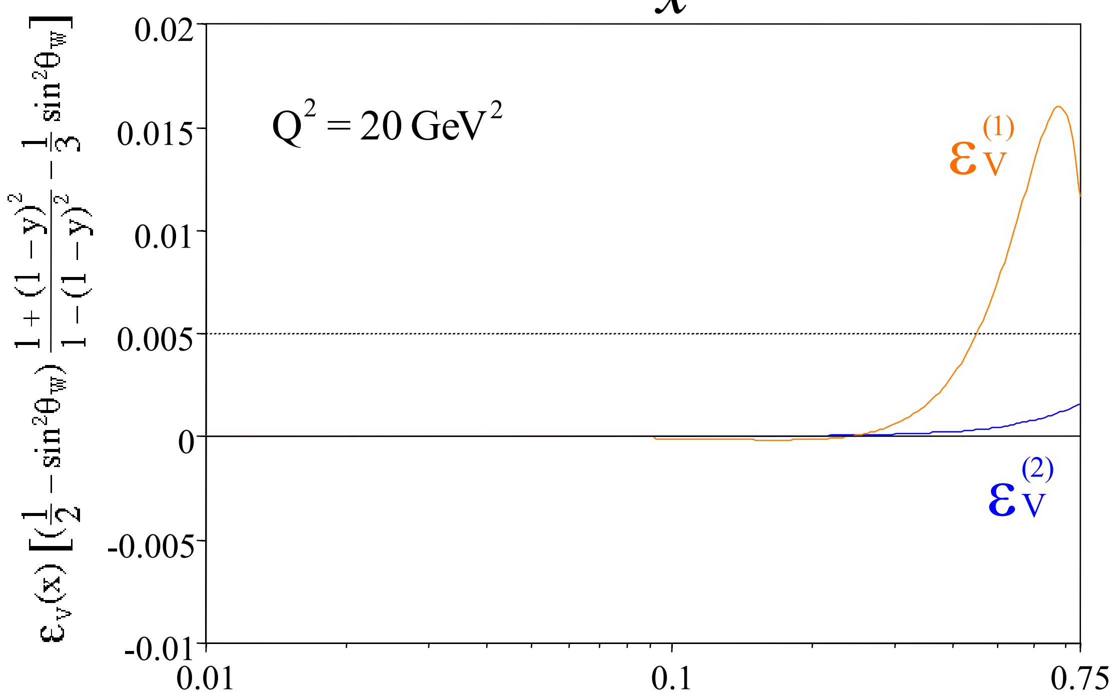


Effects on $\sin^2\theta_W$ determination

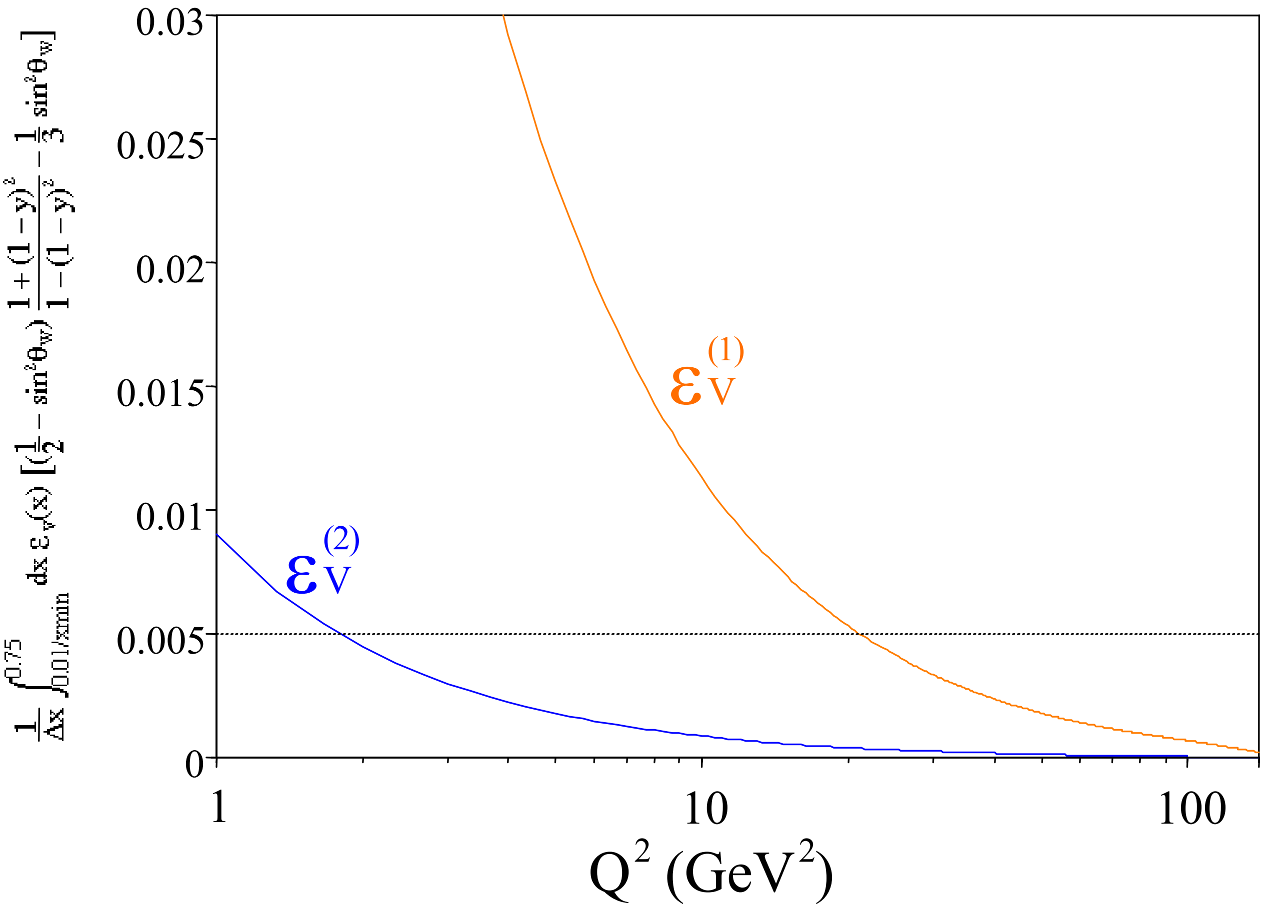
$$R_A^- = \frac{1}{2} - \sin^2\theta_W$$

$$- \varepsilon_v(x) \ \{(\frac{1}{2} - \sin^2\theta_W) \frac{1 + (1-y)^2}{1 - (1-y)^2} - \frac{1}{3} \sin^2\theta_W\}$$





Effects on $\sin^2\theta_W$ by simple x average



If the simple x average is taken, the contribution is of the order of the NuTeV deviation (0.0050). However, it seems to be an overestimation due to lack of large x data in the NuTeV experiment.

↓

take into account the NuTeV kinematics (next page)

Take NuTeV kinematics into account

thank K. S. McFarland, G. P. Zeller

our PDFs ↔ NuTeV PDFs (*)

$$xu_V^A = w_{u_v} \frac{Z\, xu_v + N\, xd_v}{A} = \frac{Z\, u^*_{vp} + N\, u^*_{vn}}{A}$$

$$xd_V^A = w_{d_v} \frac{Z\, xd_v + N\, xu_v}{A} = \frac{Z\, d^*_{vp} + N\, d^*_{vn}}{A}$$

$$\rightarrow\ u^*_{vp} = w_{u_v} xu_v\ ,\ \ u^*_{vn} = w_{u_v} xd_v\ ,\ \ d^*_{vp} = w_{d_v} xd_v\ ,\ \ d^*_{vn} = w_{d_v} xu_v$$

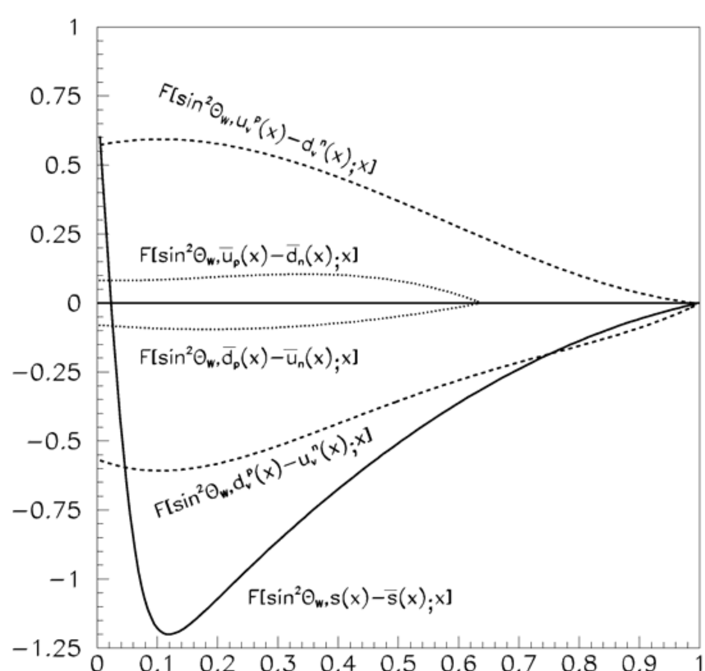
$$\rightarrow\ \delta u^*_v = u^*_{vp} - d^*_{vn} = -\,\varepsilon_v\,(w_{u_v} + w_{d_v})\, xu_v$$

$$\delta d^*_v = d^*_{vp} - u^*_{vn} = +\,\varepsilon_v\,(w_{u_v} + w_{d_v})\, xd_v$$

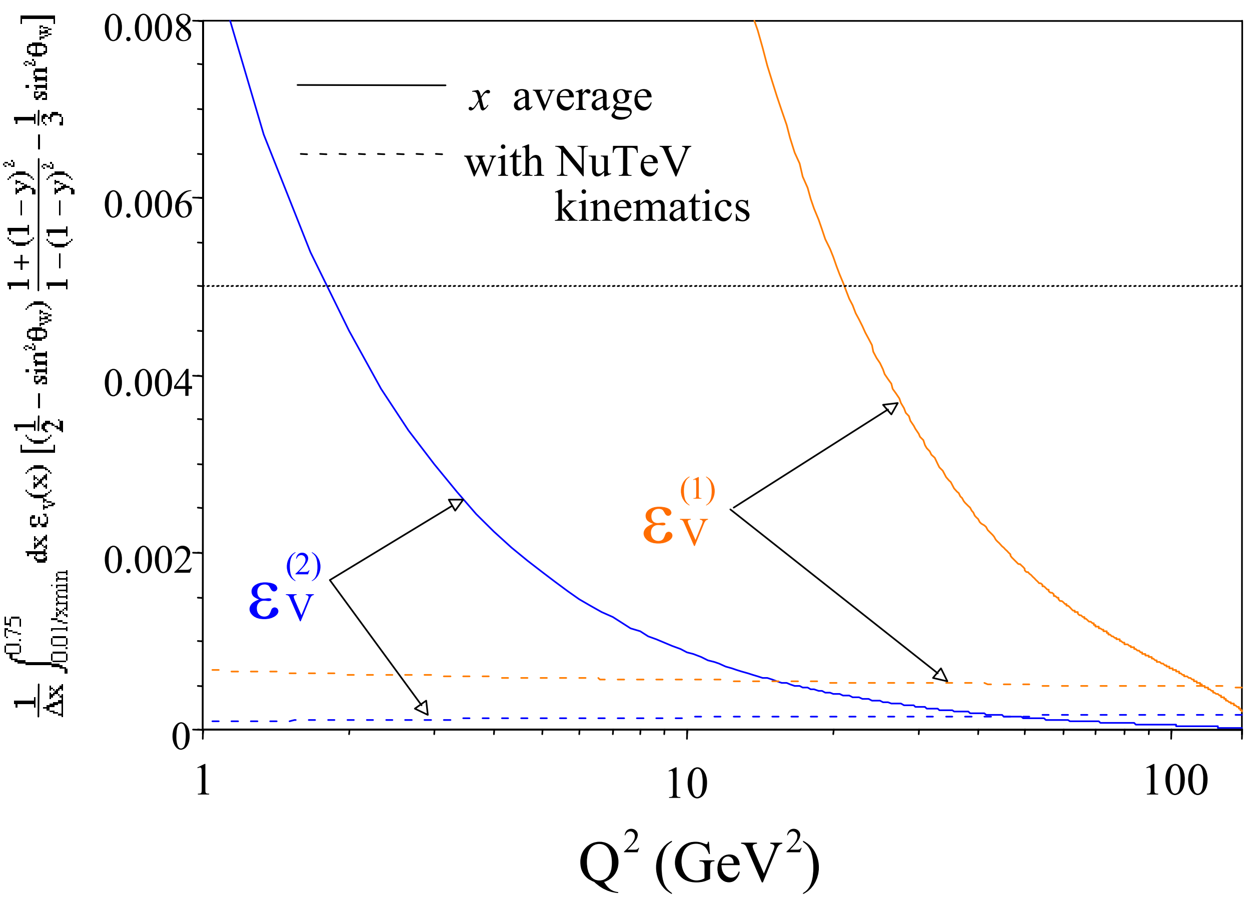
$$\Delta \sin^2\theta_W = -\int dx\, \{ F[\delta u^*_v, x]\, \delta u^*_v + F[\delta d^*_v, x]\, \delta d^*_v \}$$

The functionals F[…] are taken from

G. P. Zeller et al. Phys. Rev. D65 (2002) 111103.



Effects on $\sin^2\theta_W$ with NuTeV kinematics



Summary on $\sin^2\theta_W$

(1) As far as the considered descriptions are concerned, the effects are too small to explain the whole NuTeV deviation (0.0050).

(2) However, it is too early to exclude the mechanism because the difference between u_v and d_v modifications is not measured.

(3) It is experimentally interesting to investigate $\varepsilon_v(x)$ at NuMI and ν factory.

“HERMES effect” (nuclear effect on R=L/T)

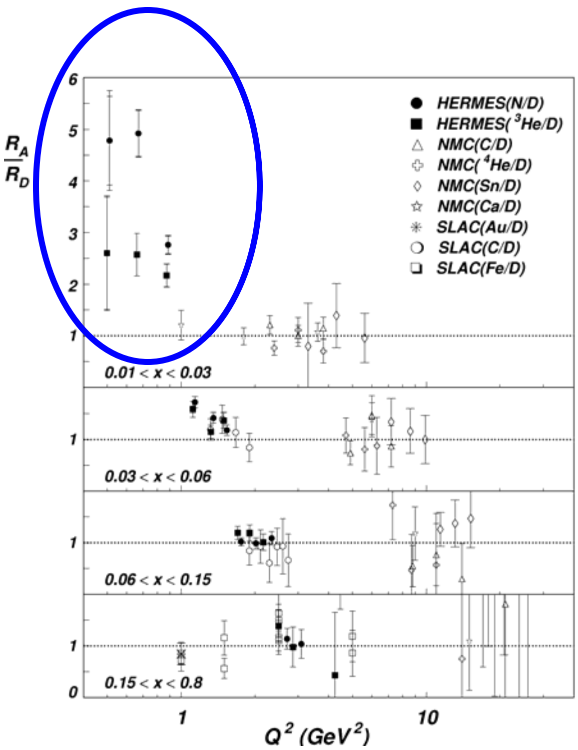
HERMES, Ackerstall el al., Phys. Rev. Lett. B 475 (2000) 386;

Erratum, hep-ex/0210067; hep-ex/0210068.

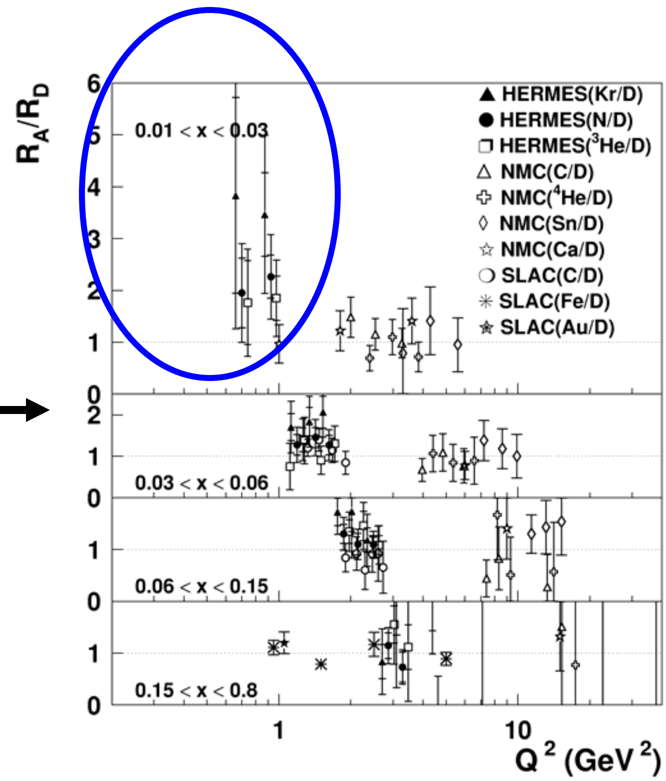
CCFR/NuTeV, U.K.Yang et al., Phys.Rev.Lett. 87 (2001) 251802.

Theoretical studies e.g. by Miller, Brodsky, and Karliner,

in Phys. Lett. B 481, 245 (2000).



(2000)



(2002)-preprint

M. Ericson and SK, hep-ph/0212001

- Nuclear modification of transverse-longitudinal ratio does exist in medium and large x regions.
- Mechanisms

  (1) transverse nucleon motion

  → T-L admixture of nucleon structure functions

  (2) binding and Fermi-motion effects

  in the spectral function

Formalism (hep-ph/0212001)

eA cross section: $$\frac{d\sigma}{dE_e' \, d\Omega_e'} = \frac{|\vec{p}_e'|}{|\vec{p}_e|} \frac{\alpha^2}{(q^2)^2} L^{\mu\nu}(p_e,q) \, W_{\mu\nu}^{A}(p_A,q)$$

$$W_{\mu\nu}^{A,N}(p_{A,N},q) = -W_1^{A,N}(p_{A,N},q)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + W_2^{A,N}(p_{A,N},q)\frac{\tilde{p}_{A,N\,\mu}\tilde{p}_{A,N\,\nu}}{p_{A,N}^2}$$

where $\tilde{p}_\mu = p_\mu - \frac{p \cdot q}{q^2} q_\mu$

Projection operators of W_1^A and W_2^A

$$\hat{P}_1^{\mu\nu} = -\frac{1}{2}\left(g_{\mu\nu} - \frac{\tilde{p}_{A\mu}\tilde{p}_{A\nu}}{\tilde{p}_A^2}\right), \quad \hat{P}_2^{\mu\nu} = -\frac{p_A^2}{2\,\tilde{p}_A^2}\left(g_{\mu\nu} - \frac{3\,\tilde{p}_{A\mu}\tilde{p}_{A\nu}}{\tilde{p}_A^2}\right)$$

so as to have $\hat{P}_1^{\mu\nu} W_{\mu\nu}^A = W_1^A$, $\hat{P}_2^{\mu\nu} W_{\mu\nu}^A = W_2^A$

Convolution: $W_{\mu\nu}^A(p_A,q) = \int d^4p_N \, S(p_N) \, W_{\mu\nu}^N(p_N,q)$

$S(p_N)$ = spectral function

$$W_{1,2}^A(p_A,q) = \int d^4p_N \, S(p_N) \, \hat{P}_{1,2}^{\mu\nu} \, W_{\mu\nu}^N(p_N,q)$$

Longitudinal and transverse components

$W_\lambda^{A,N} = \varepsilon_\lambda^{\mu\,*} \, \varepsilon_\lambda^\nu \, W_{\mu,\nu}^{A,N}$, photon polarization vector ε_λ^μ

$$W_T^{A,N} = \frac{1}{2}\left(W_{\lambda=+1}^{A,N} + W_{\lambda=-1}^{A,N}\right) = W_1^{A,N}$$

$$W_L^{A,N} = W_{\lambda=0}^{A,N} = \left(1 + \frac{\nu_{A,N}^2}{Q^2}\right) W_2^{A,N} - W_1^{A,N}$$

where $\nu_A^2 = \nu^2$, $\nu_N^2 = \frac{(p_N \cdot q)^2}{p_N^2}$

Formalism (continued)

Scaling variables

$$x_A = \frac{Q^2}{2\,p_A\cdot q} = \frac{M_N}{M_A}\,x,\quad x_N = \frac{Q^2}{2\,p_N\cdot q} = \frac{x}{z},\quad x = \frac{Q^2}{2\,M_N\,\nu},\quad z = \frac{p_N\cdot q}{M_N\,\nu}$$

Structure functions F_1 and F_2

$$F_1^{A,N}(x_{A,N},q) = \sqrt{p_{A,N}^2}\,W_1^{A,N},\quad F_2^{A,N}(x_{A,N},q) = \frac{p_{A,N,}\cdot q}{\sqrt{p_{A,N}^2}}\,W_2^{A,N}$$

Longitudinal structure function F_L

$$F_L^{A,N} = \left(1 + \frac{Q^2}{\nu_{A,N}^2}\right) F_2^{A,N} - 2\,x_{A,N}\,F_1^{A,N}$$

Transverse-longitudinal ratio: $R_{A,N} = \frac{F_L^{A,N}}{2\,x_{A,N}\,F_1^{A,N}}$

Calculating $W_{1,2}^A = \int d^4p_N\,S(p_N)\,\hat{P}_{1,2}^{\mu\nu}\,W_{\mu\nu}^N$, we have

$$2\,x_A F_1^A = \int d^4p_N\,S(p_N)\,z\,\frac{M_N}{\sqrt{p_N^2}}\left[\left(1 + \frac{\vec{p}_{N\perp}^{\,2}}{2\,\tilde{p}_N^2}\right) 2\,x_N F_1^N(x_N, Q^2) + \frac{\vec{p}_{N\perp}^{\,2}}{2\,\tilde{p}_N^2}\,F_L^N(x_N, Q^2)\right]$$

$$F_L^A = \int d^4p_N\,S(p_N)\,z\,\frac{M_N}{\sqrt{p_N^2}}\left[\left(1 + \frac{\vec{p}_{N\perp}^{\,2}}{\tilde{p}_N^2}\right) F_L^N(x_N, Q^2) + \frac{\vec{p}_{N\perp}^{\,2}}{\tilde{p}_N^2}\,2\,x_N F_1^N(x_N, Q^2)\right]$$

Transverse-longitudinal admixture

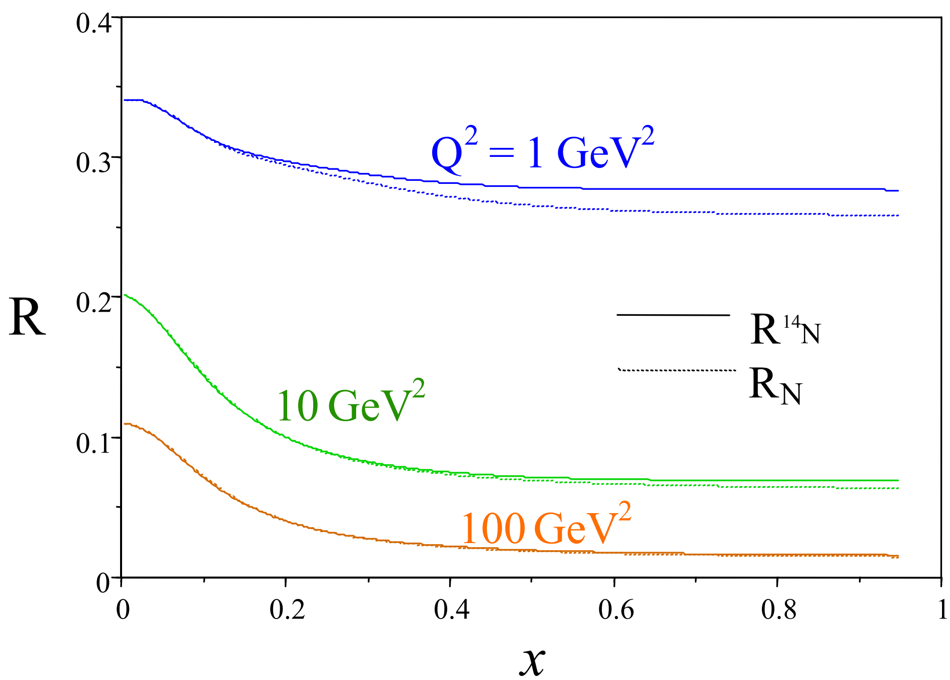
$$\frac{\vec{p}_{N\perp}^{\,2}}{\tilde{p}_N^2} = \frac{4\,x_N^2\,\vec{p}_{N\perp}^{\,2}}{Q^2\,(1 + 4\,x_N^2\,p_N^2/Q^2)} \approx \frac{4\,x_N^2\,\vec{p}_{N\perp}^{\,2}}{Q^2}$$

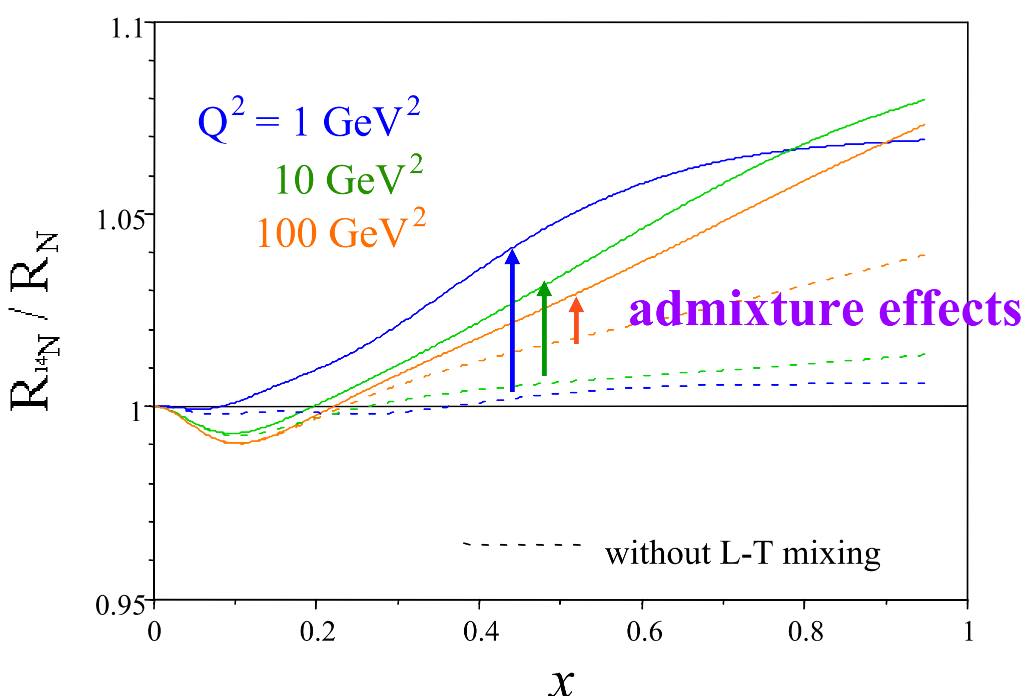
Results

- Spectral function $(M_{A-i} = M_A - M_N - \varepsilon_i)$

$$S(p_N) = \sum_i |\phi(\vec{p}_N)|^2 \, \delta\left(p_N^0 - M_A + \sqrt{M_{A-i}^2 + \vec{p}_N^{\,2}}\right) \quad \text{for } {}^{14}N$$

- Transverse-longitudinal ratio: R_{1990}
- F_2^N (PDFs): MRST98-LO





Summary on "the HERMES effect"

(1) After the HERMES (CCFR/NuTeV) re-analysis, people tend to lose interest in the nuclear effect on R.
However, we claim that nuclear modification should exist in medium and large x regions.

(2) Physical origin

- transverse-longitudinal admixture due to the transverse Fermi motion
- binding and Fermi motion effects in the spectral function

(3) Need future experimental investigations
JLab, EIC, NuMI, ν factory, …