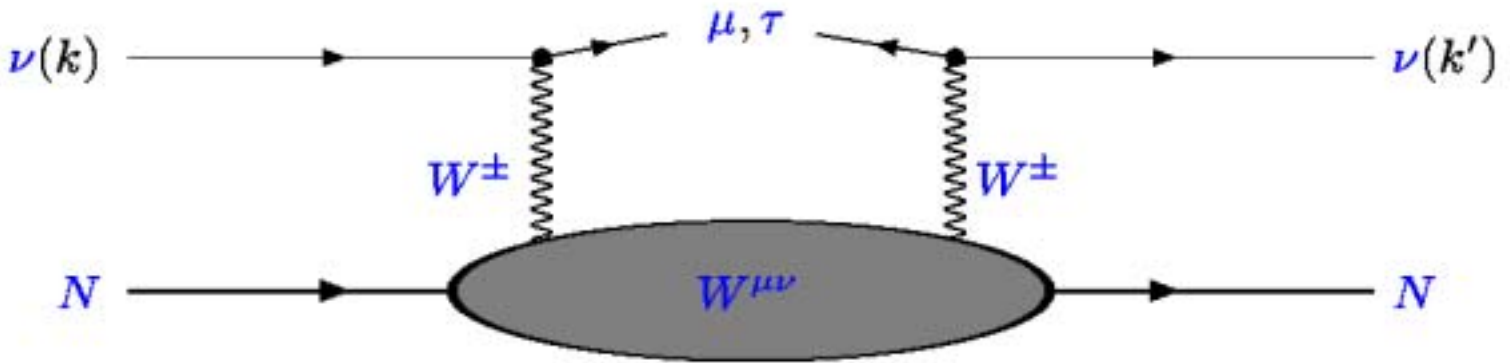


Mass Effects in Charged Current Scattering

- Collaboration with **M.H. Reno** (U. of Iowa):
Tau Neutrino Deep Inelastic Charged Current Interactions
Phys. Rev. D (2002)
- Title(s) say(s) it, and some more. Time is running, ...

Some theory to set the canvas ...

Neutrino Nucleon Scattering



$$d\sigma \propto L^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu} = \Im \left[i \int d^4x e^{iqx} \langle P | T J_\mu^+(x) J_\nu(0) | P \rangle \right]$$

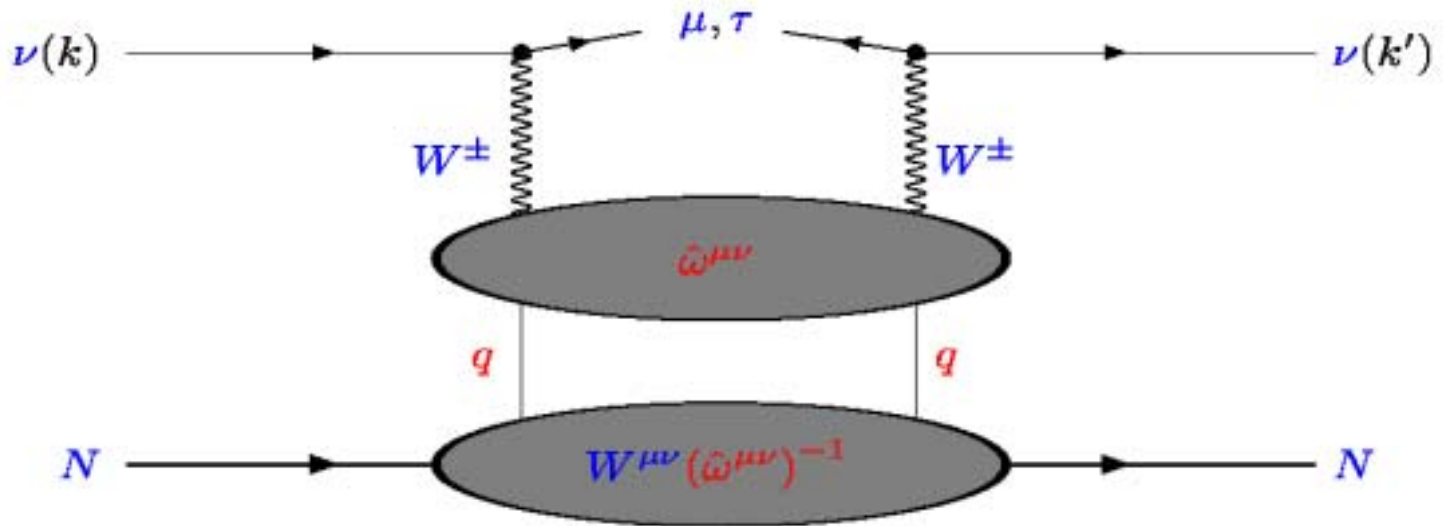
$$= \frac{1}{2} \int d^4x e^{iqx} \langle P | J_\mu^+(x) \mathbf{1} J_\nu(0) | P \rangle$$

$$\mathbf{1} = \sum_x \overbrace{|X\rangle\langle X|}^{\uparrow}$$

\sum_x : **Inclusive Sum** covers

- (Quasi-)Elastic $|X\rangle = |N\rangle$
- Resonances / low multiplicity $|X\rangle = |N\pi\rangle$
- Continuum $|X\rangle = |multi - particle\rangle$

Deep Inelastic Scattering: *Factorization...*



$$W_{\mu\nu} = q(\mu) \otimes \hat{\omega} \left(\frac{Q}{\mu}, \alpha_s(\mu) \right) + \mathcal{O} \left(\frac{1}{Q^2} \right)$$

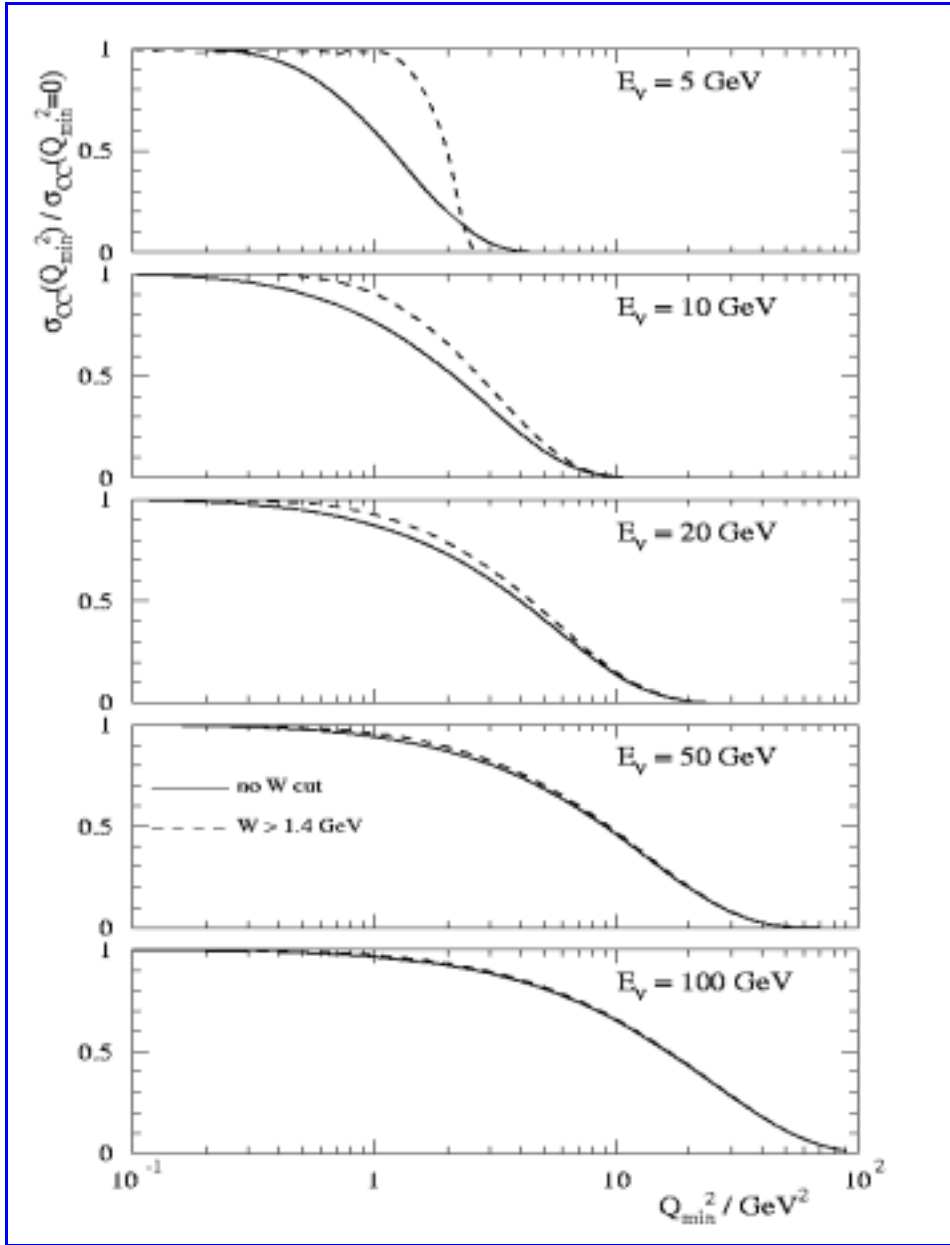
...in the limit of light-cone dominance $Q^2 \equiv p_W^2 \rightarrow \infty$:

- operator product (twist) expansion (# of participating partons) of $W_{\mu\nu}$
- perturbative expansion (# of radiation vertices) of $\hat{\omega}_{\mu\nu}$

Comments:

- $Q^2 = \mathcal{O}(1 \text{ GeV}^2)$:
 - perturbative $\alpha_s(Q^2)$
 - non-negligible/calculable $\mathcal{O}(M^2/Q^2)$ (see results ...)
- cutting on Q^2 avoids higher twist / nonperturbative QCD
- cutting on $W^2(x_{Bj}, Q) \equiv (P_N + p_W)^2$ avoids resonances / elastic (formally enhanced higher twist effects as $x_{Bj} \rightarrow 1$)
- \sum_x still includes resonant and elastic scattering:
 - overlap-problem in $\sigma_{tot} = \sigma_{DIS \& res. \& el.}$

↳ $\sigma(\nu_\tau N \rightarrow \tau X)$ is a natural starting point

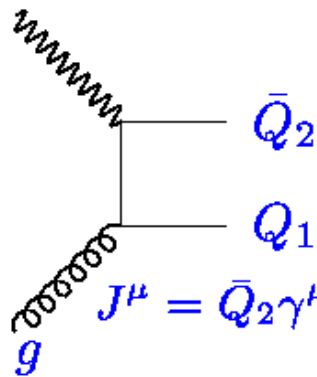


- as a theoretical playground because of a large perturbative DIS contribution from to the collinear cut-off effect of m_τ :
 - $p_w^2 \equiv Q^2 > O(m_\tau^2)$ at the $\nu_\tau \rightarrow \tau W$ emission vertex
- because of its relevance for τ appearance (even though this is a low energy workshop)

Charm Production with three partonic flavours {u,d,s}&gluons:

• Charm Production: NC \leftrightarrow CC

B^*



$$J^\mu = \bar{Q}_2 \gamma^\mu (V - A \gamma_5) Q_1$$

NC: $B = \gamma$

$$Q_{1,2} = c$$

$$V = e_c; A = 0$$

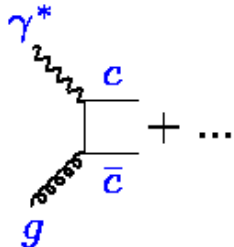
CC: $B = W$

$$Q_1 = c; Q_2 = s$$

$$V = A = 1$$

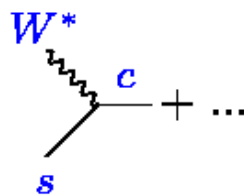
analogous, but: mass non-degeneracy

$$\begin{pmatrix} u: \text{light} \\ d: \text{light} \end{pmatrix} \begin{pmatrix} c: \text{heavy} \\ s: \text{light} \end{pmatrix} \begin{pmatrix} t: \text{heavy} \\ b: \text{heavy} \end{pmatrix} \quad \begin{matrix} \ln m_c^2: \text{perturbative} \\ \ln m_s^2: \text{non-pert.} \end{matrix}$$



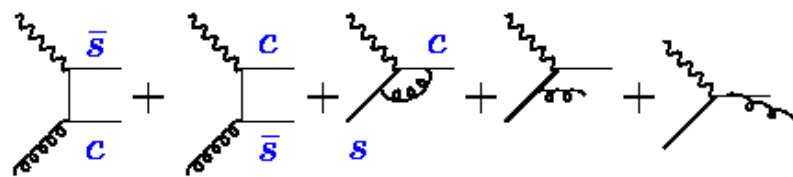
$$\mathcal{O}(\alpha_s^1) + \mathcal{O}(\alpha_s^2)$$

LO + NLO !



$$\mathcal{O}(\alpha_s^0) + \mathcal{O}(\alpha_s^1)$$

LO + NLO ?



1st $g \rightarrow c\bar{c}$
contb.: \propto
 $\ln Q^2/m_c^2$

NLO corrections to $s \rightarrow c$

\rightarrow CC DIS: $\mathcal{O}(\alpha_s^1)$ first reliable order at high Q^2 .

NLO and masses:

- $\sigma(\nu_\tau N \rightarrow [c]X): m_\tau, M_N, m_c \neq 0$

$$\begin{aligned} \frac{d^2 \sigma^{\nu(\bar{\nu})}}{dx dy} &= \frac{G_F^2 M_N E_\nu}{\pi(1 + Q^2/M_W^2)^2} \left\{ \left(y^2 x + \frac{m_\tau^2 y}{2E_\nu M_N} \right) F_1^{W^\pm} \right. \\ &+ \left[\left(1 - \frac{m_\tau^2}{4E_\nu^2} \right) - \left(1 + \frac{M_N x}{2E_\nu} \right) y \right] F_2^{W^\pm} \\ &\pm \left[xy \left(1 - \frac{y}{2} \right) - \frac{m_\tau^2 y}{4E_\nu M_N} \right] F_3^{W^\pm} \\ &\left. + \frac{m_\tau^2 (m_\tau^2 + Q^2)}{4E_\nu^2 M_N^2 x} F_4^{W^\pm} - \frac{m_\tau^2}{E_\nu M_N} F_5^{W^\pm} \right\} \end{aligned}$$

factorized pQCD for $\langle J^\mu J^\nu \rangle$:

$$W^{\mu\nu} = \int \frac{d\xi}{\xi} f(\xi, \mu^2) \hat{w}^{\mu\nu} |_{p^+ = \xi P^+}$$

$$W^{\mu\nu} = \sum_i F_i T_i^{\mu\nu}(P, q, g) \quad \hat{w}^{\mu\nu} = \sum_i \mathcal{F}_i T_i^{\mu\nu}(p, q, g)$$

$$\mathcal{F}_i(x, Q^2) = q(\eta, \mu^2) (1 - \delta_{i4}) + \mathcal{O}(\alpha_s)$$

(re-)scaling variables:

$$\eta = \frac{2x}{1 + \rho} \quad \rho^2 \equiv 1 + \left(\frac{2M_N x}{Q} \right)^2 \quad \bar{\eta} = \eta(1 + m_c^2/Q^2)$$

mixing of structure functions:

$$F_1 = \mathcal{F}_1 \quad F_2 = 2x \frac{\mathcal{F}_2}{\rho^2} \quad F_3 = 2 \frac{\mathcal{F}_3}{\rho}$$

$$\begin{aligned} F_4 &= \frac{(1-\rho)^2}{2\rho^2} \mathcal{F}_2 + \mathcal{F}_4 + \frac{1-\rho}{\rho} \mathcal{F}_5 \\ F_5 &= \frac{\mathcal{F}_5}{\rho} - \frac{(\rho-1)}{\rho^2} \mathcal{F}_2 \end{aligned}$$

Outlook:

- NLO target mass corrections for $F_{1, \dots, 5}$ (incl. Mixing in the $F_{1,2}$ sector) from the OPE (equivalent to partons with k_T)
- NLO/mass corrections to NC/Paschos-Wolfenstein-relation soon!

$F_{4,5}$ revisited

2 "new" structure functions F_4, F_5 (Albright & Jarlskog 75)

- AJ relation 1 $F_4 = 0$ is violated by $M_N, \mathcal{O}(\alpha_s)$, **not** by m_c (for $m_s = 0$)
- AJ relation 2 $F_2 - 2xF_5 = 0$ is violated by masses but holds to any $\mathcal{O}(\alpha_s^n)$ for $M_N = m_c = 0$ by "abelian" (single W exchange) gauge invariance

$$W_0 = \epsilon_0^\mu \epsilon_0^\nu W_{\mu\nu} \quad (1)$$

$$W_s = (\epsilon_0^\mu \epsilon_q^\nu + \epsilon_q^\mu \epsilon_0^\nu) W_{\mu\nu} \quad (2)$$

$$W_q = \epsilon_q^\mu \epsilon_q^\nu W_{\mu\nu} \quad (3)$$

$$W_\pm = \epsilon_\pm^\mu \epsilon_\pm^{\nu*} W_{\mu\nu} \quad (4)$$

$$\epsilon_\pm^\mu = \frac{1}{\sqrt{2}} (0, \mp 1, -i, 0) \quad (\text{transverse}) \quad (5)$$

$$\epsilon_q^\mu = \frac{q^\mu}{\sqrt{-q^2}} \quad (\text{scalar}) \quad (6)$$

$$\epsilon_0^\mu = \frac{(-q^2)k^\mu + (k \cdot q)q^\mu}{\sqrt{(-q^2)[(k \cdot q)^2]}} \quad (\text{longitudinal}) \quad (7)$$

$$\mathcal{F}_1 = \frac{1}{2}(W_+ + W_-) \quad (8)$$

$$\mathcal{F}_2 = \frac{\lambda}{2}(W_+ + W_- + 2W_0) \quad (9)$$

$$\mathcal{F}_3 = \mp \frac{1}{2}(W_+ - W_-) \quad (10)$$

$$\mathcal{F}_4 = \frac{1}{2}(W_0 + W_q - W_s) \quad (11)$$

$$\mathcal{F}_5 = \frac{1}{2}(W_+ + W_- + 2W_0 - W_s) . \quad (12)$$

$$F_2 - 2xF_5 = xW_s \quad (13)$$

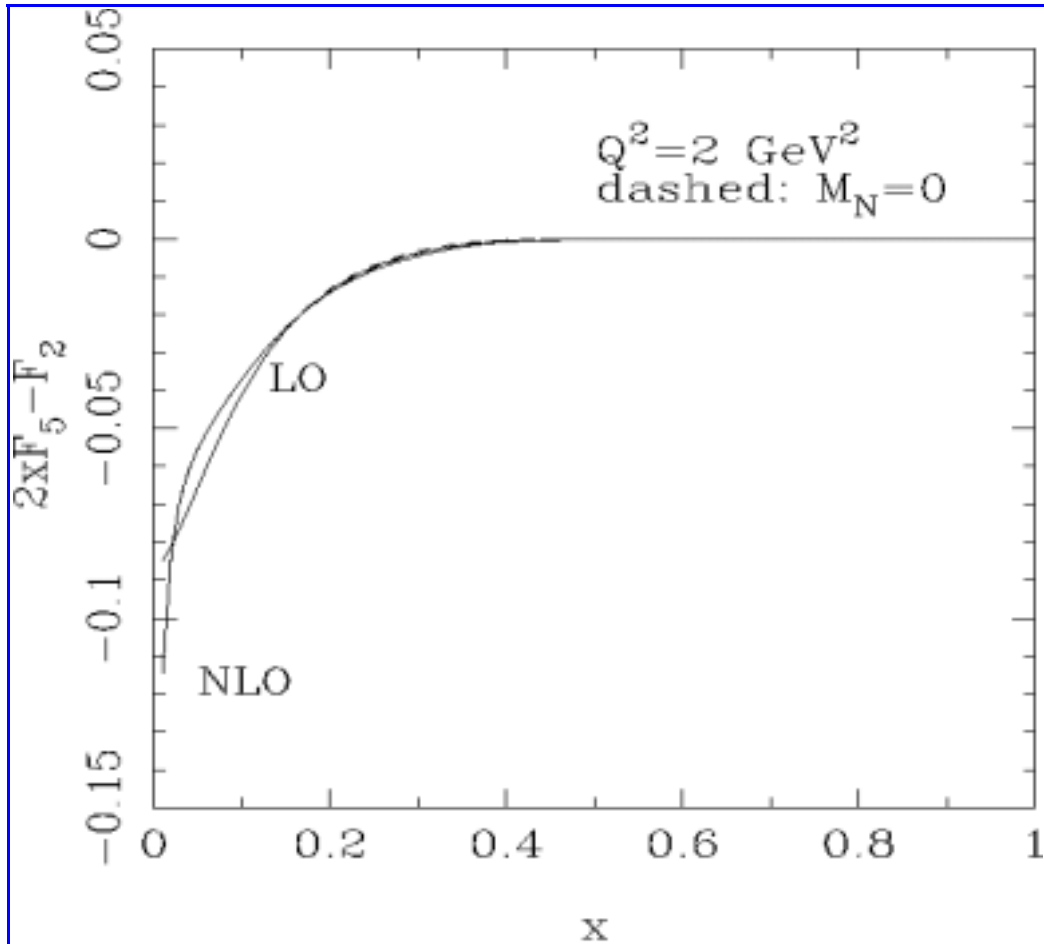
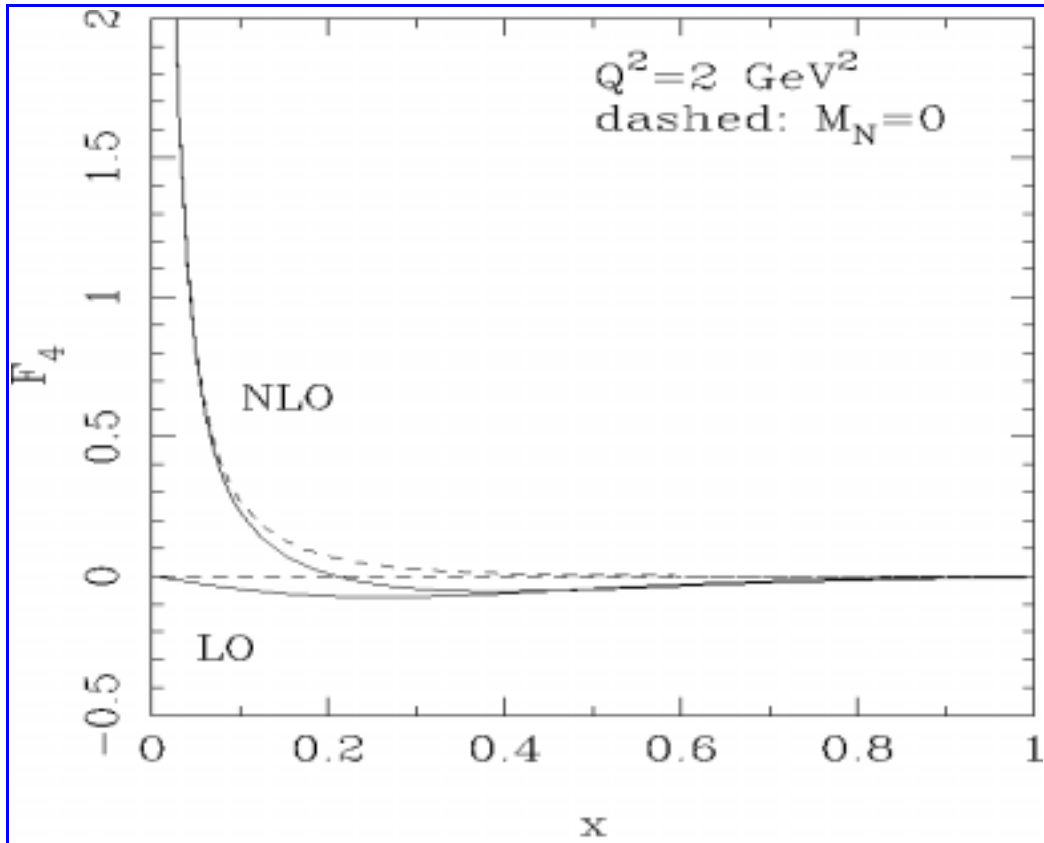
$$\mathcal{F}_4^c = \frac{k^\mu k^\nu}{Q^2} W_{\mu\nu} \quad (14)$$

Comment:

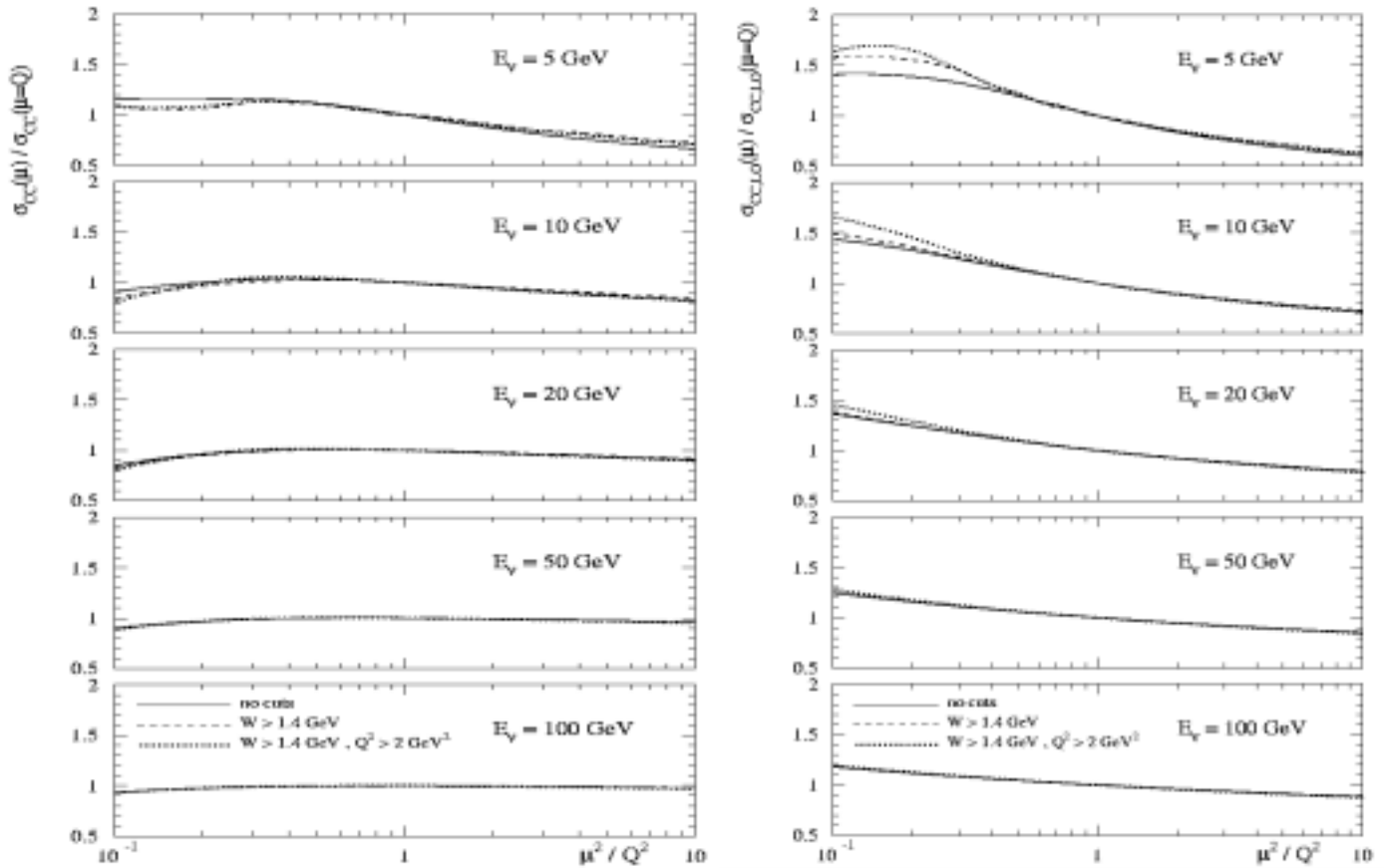
OPE is not tied to perturbation theory, so (13)=0 should hold for the (equivalent) elastic form factors as well.

Results ...

Violation of the Albright&Jarlskog Relations

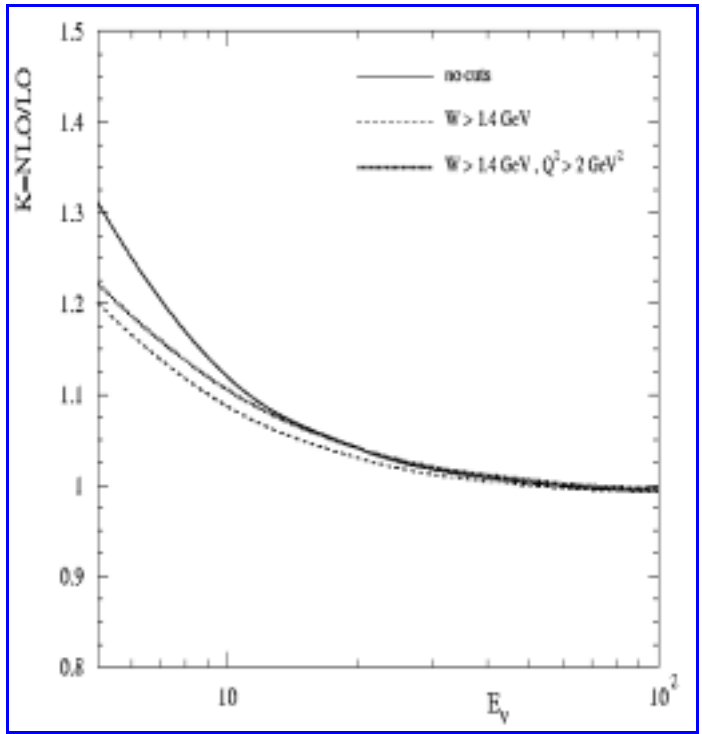
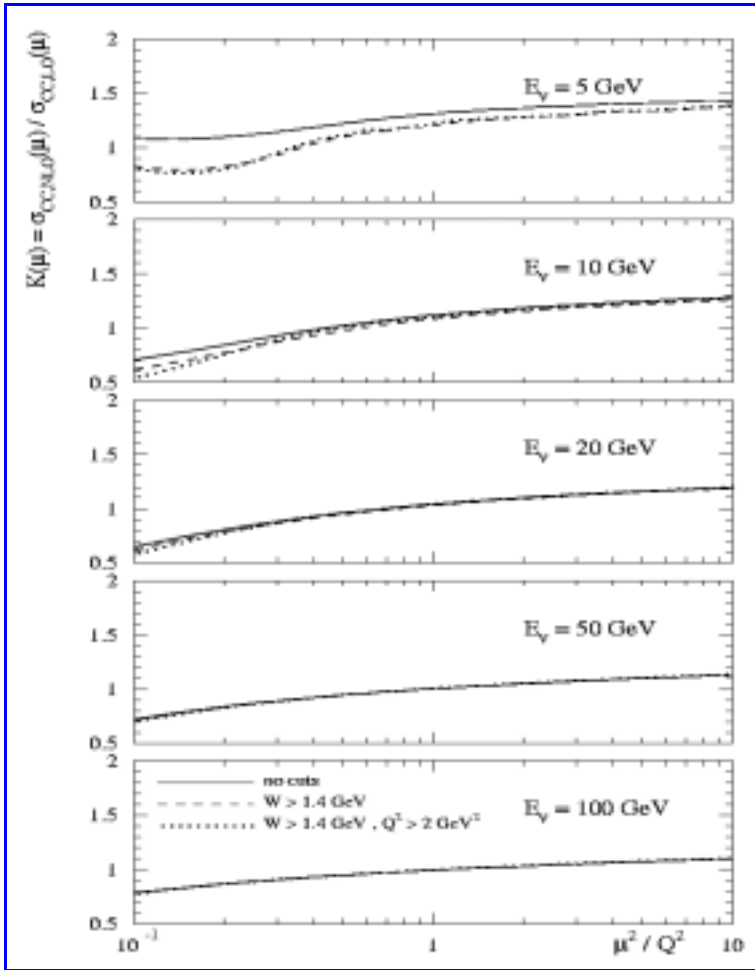


Scale dependence: $\sigma(\mu)/\sigma(\mu=Q)$



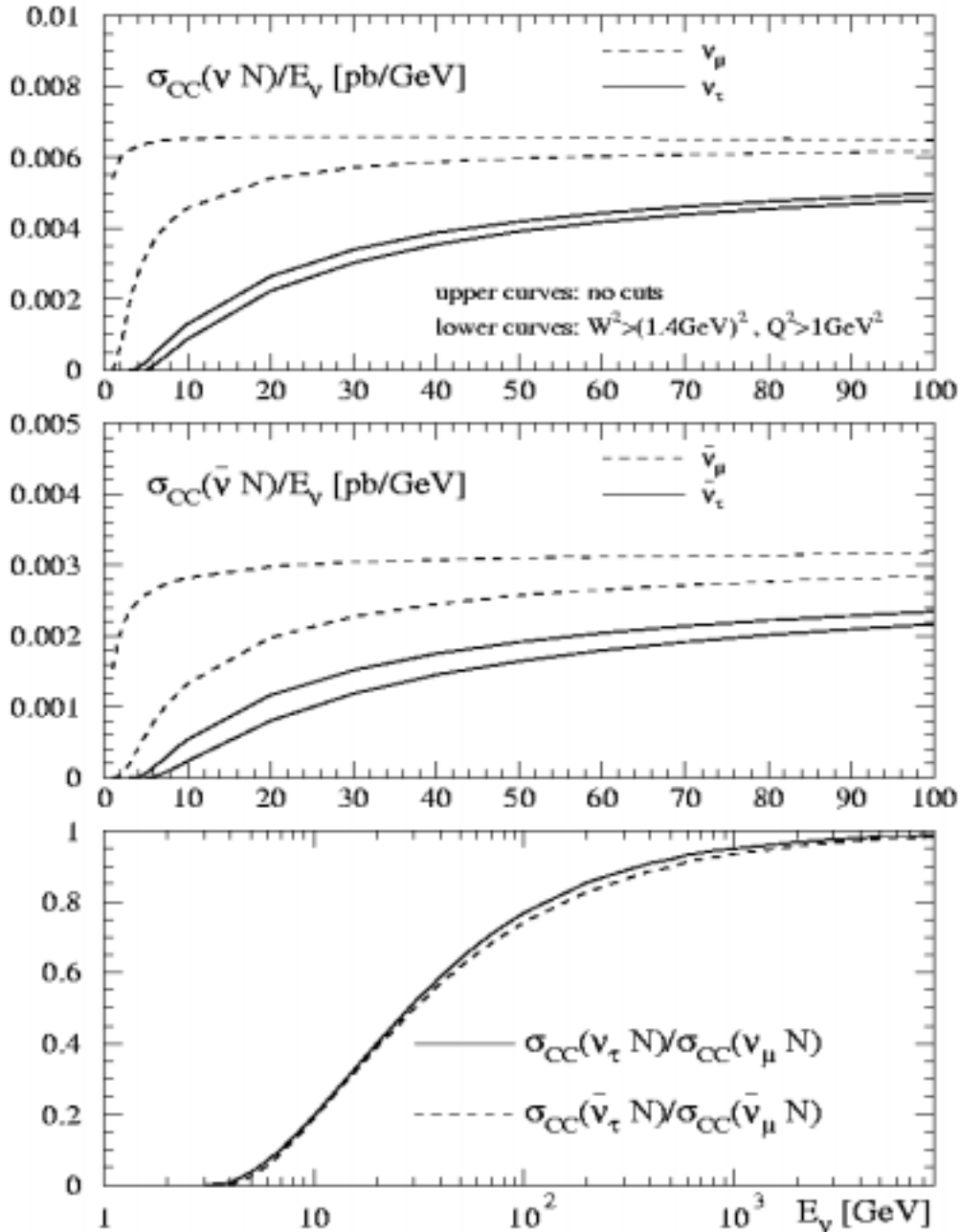
substantial improvement from NLO corrections

K-factor: NLO / LO



reasonably stable (but energy dependent); confirming canonical DIS choice $\mu = Q$

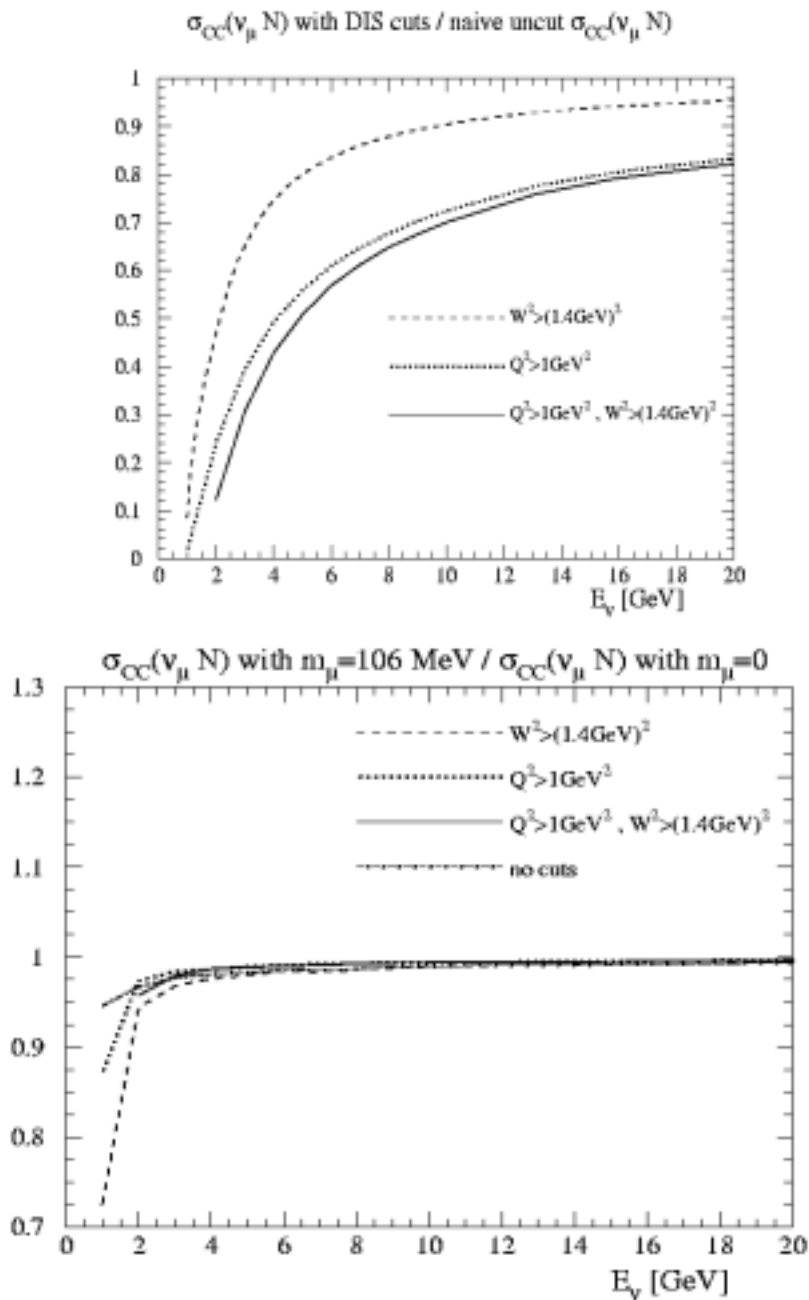
DIS cuts and/or lepton threshold in $\nu_{\tau,\mu}$ CC scattering



(Prior to solving the overlap problem)

What can we say about the scattering of muon neutrinos ... ?

- Reduction of *bona fide* DIS component
- Since $m_\tau \neq 0$ effects extend (at the 5% level) up to 1TeV(!), do $m_\mu \neq 0$ effects vanish only at (the equivalent) energy 3.5 GeV? (actually the scaling is not that simple ...)



- As to be expected, the *bona fide* DIS component becomes very small at low neutrino energies
- Muon mass corrections range within [5%;30%] at 1GeV but depend sensitively on DIS cuts
- A reliable quantitative statement will have to await the combination of deep inelastic scattering with resonant and quasi-elastic scattering (or an extended duality approach)

Conclusions / Outlook:

- **Tau neutrino** (differential and integrated) **CC DIS cross section** at NLO including all **mass effects** (target mass effects in the collinear approximation) as:
 - a signal for tau appearance
 - a playground for the DIS component in neutrino scattering (of any neutrino flavour)
- **NLO features:**
 - **stable** under scale variations around $\mu = Q$
 - energy dependent **K-factors** reflecting **20%-30%** corrections (depending on DIS cuts) at $E_\nu = 5 \text{ GeV}$ to $\sim 0\%$ around $E_\nu = 50 \text{ GeV}$
 - The uncertainties from **scale variations** and **PDF-errors** are very **moderate**.
- **NC cross-sections** and **NLO target mass effects** from the OPE approach ($O(M^2/Q^2)$ mixing in the $F_{1,2}$ sector) **nearly finished** (for e.g. **Paschos-Wolfenstein** type analyses at NLO)
- For low energies: **Combination** of **DIS** with **resonant** and **quasi-elastic** scattering (cuts, duality, ...?) and/or the inclusion of higher twist still to do.
- As long as this is unsolved we have no reliable quantitative statement on **muon mass effects**; but it seems they *can be* as large as **30%** at low energy.