Modeling Quark-Hadron Duality

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- What is duality?
- Applications - why should we think about duality?
- Modeling duality - what we do about duality

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What is Quark-Hadron Duality?

Duality means: In a certain kinematic regime, properly averaged hadronic observables can be described by a partonic picture (all or some of the physics described by perturbative QCD.)

We observe duality in many different reactions; some are

- $e^+ e^- \longrightarrow$ hadrons
- semileptonic decays of heavy quarks
- inclusive inelastic electron scattering

Duality has interesting and useful applications! Several duality experiments have been completed at JLab, and there will be a large duality program at CEBAF @ 12 GeV.

Kinematics & Observables

$$e\, p \longrightarrow e\, X$$

Inelastic electron scattering from a nucleon:

$$\frac{d\sigma}{d\Omega dE_f} = \sigma_{\text{Mott}} \left(W_2 + 2\, W_1 \tan^2 \frac{\vartheta_e}{2} \right)$$

The structure functions W_1, W_2 depend on the transferred energy ν and the negative transferred four-momentum, $Q^2 = \vec{q}^{\,2} - \nu^2$.

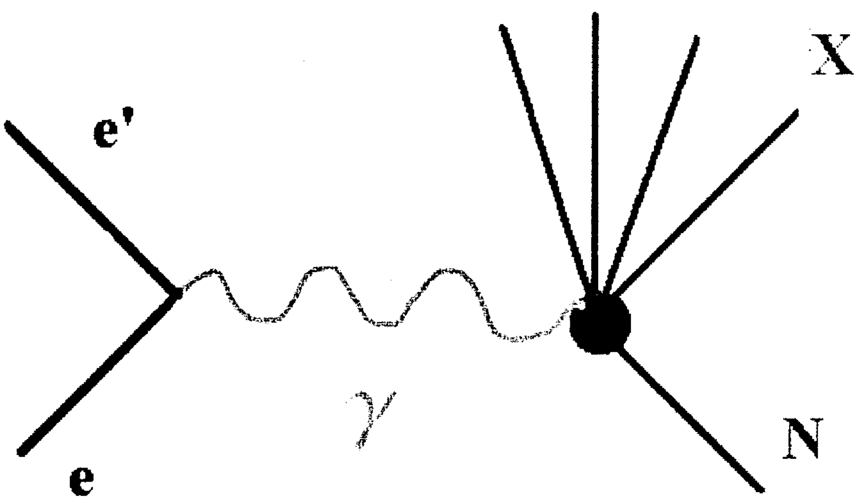
In the Bjorken limit, $Q^2 \to \infty$, $\frac{\nu}{Q^2} = \text{const.}$, we have scaling:

$\nu W_2(\nu, Q^2) \to F_2(x)$ [**actually,** $F_2(x, \log Q^2)$]

where $x_{Bj} = \frac{Q^2}{2M_N \nu}$.

Where does Duality hold?

Consider $eN \rightarrow e'X$



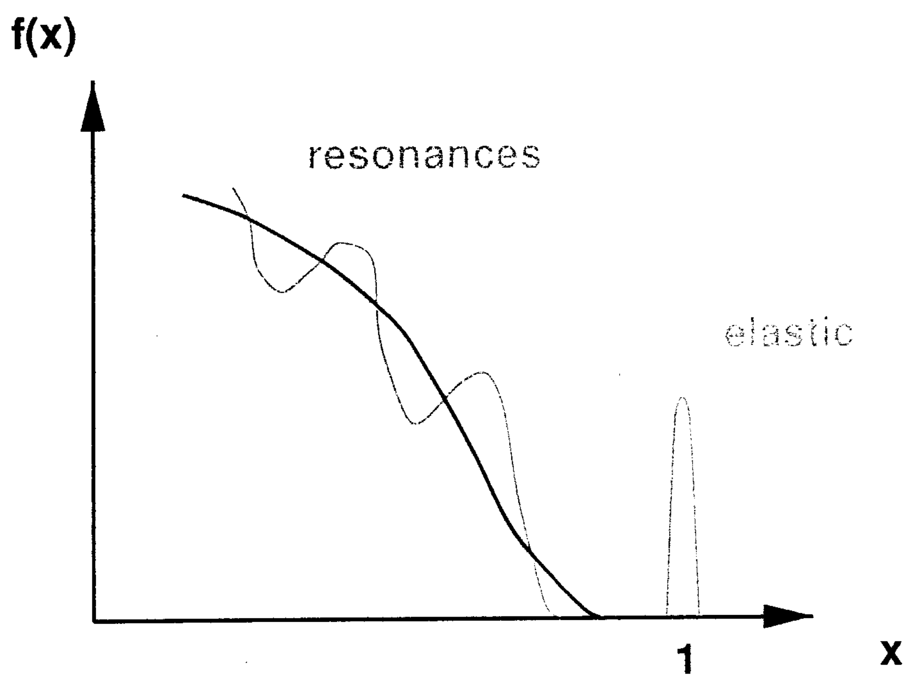
It must hold in the scaling region, as perturbative QCD = full QCD in this case.

It must break down at very low Q^2, $Q^2 \rightarrow 0$, where QCD is strong.

Interesting question: How does the transition from high Q^2 to low Q^2 take place?

Where does Duality hold in inelastic inclusive electron scattering?

Answer from experiment: Duality works well down to low $Q^2 \approx 0.5$ GeV^2!

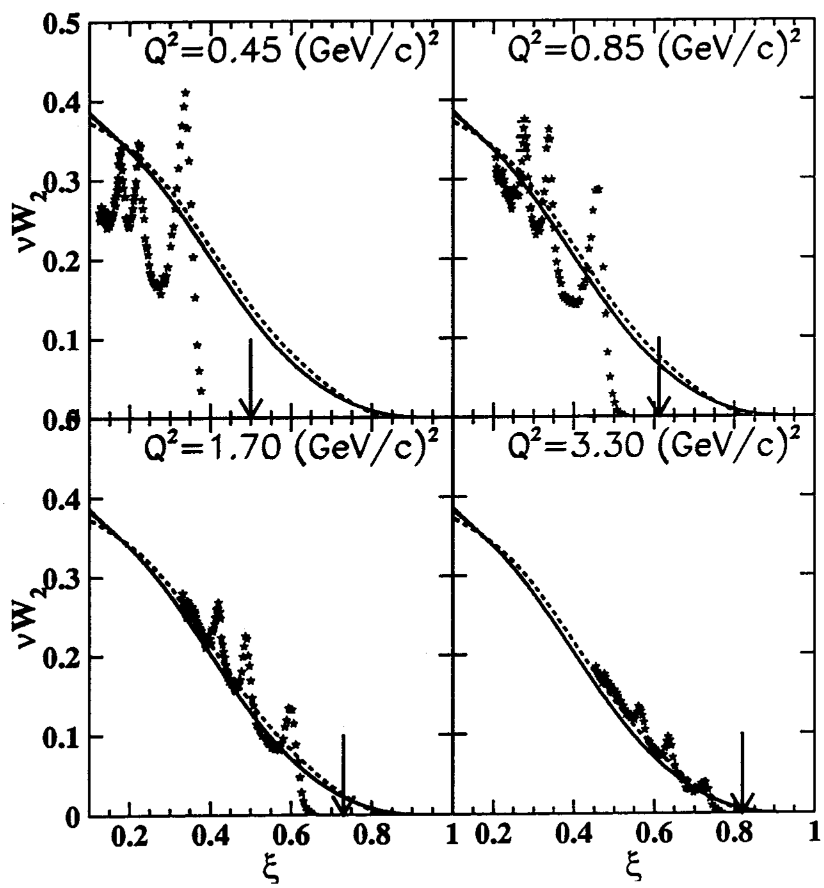


Data from I. Niculescu et al., PRL 85, 1186 (2000)

νW_2 at various Q^2

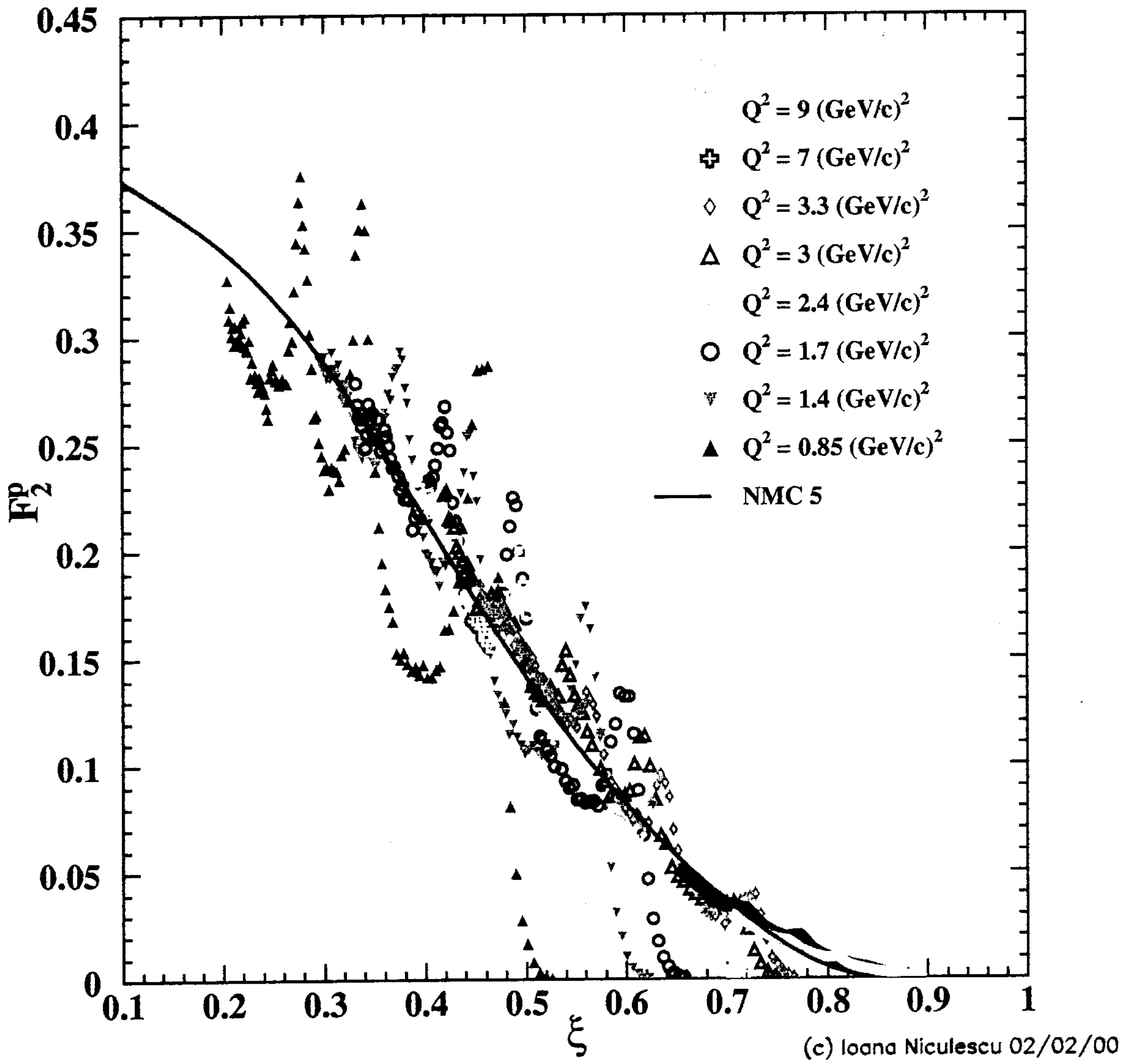
solid line: NMC scaling curve for $Q^2 = 10 GeV^2$

dashed line: NMC scaling curve for $Q^2 = 5 GeV^2$



New Data from Jefferson Lab

Data from I. Niculescu et al., PRL 85 (2000)



scaling variable:

$$\xi = \frac{2x}{1 + \sqrt{1 + \frac{4M^2x^2}{Q^2}}} = \frac{1}{M}(\sqrt{\nu^2 + Q^2} - \nu)$$

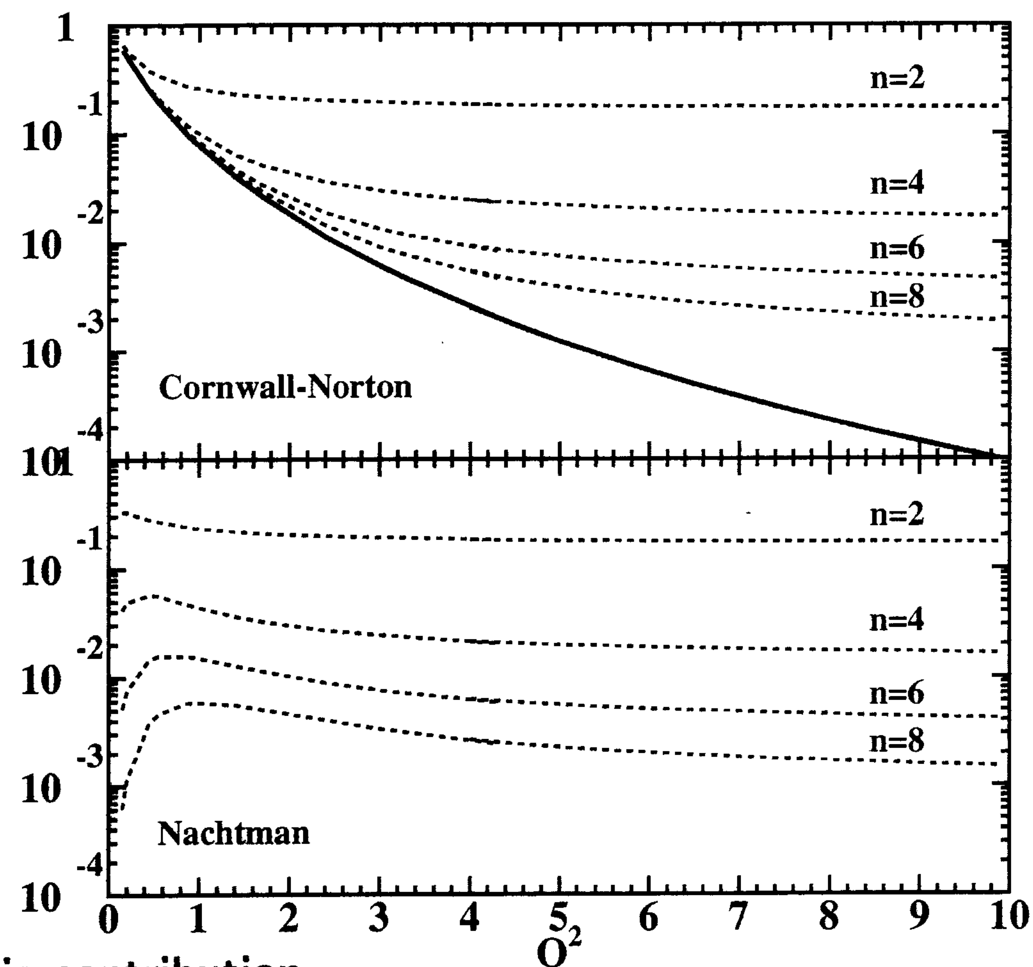
New Data from JLab: Moments

Data from C. S. Armstrong et al., PRD 63, 094008 (2001)

$$M_n(Q^2) = \int dx\, x^{n-2}\, F(x, Q^2)$$

Cornwall–Norton **Moments:** $x = x_{Bj}$

Nachtmann **Moments:** $x = \xi$

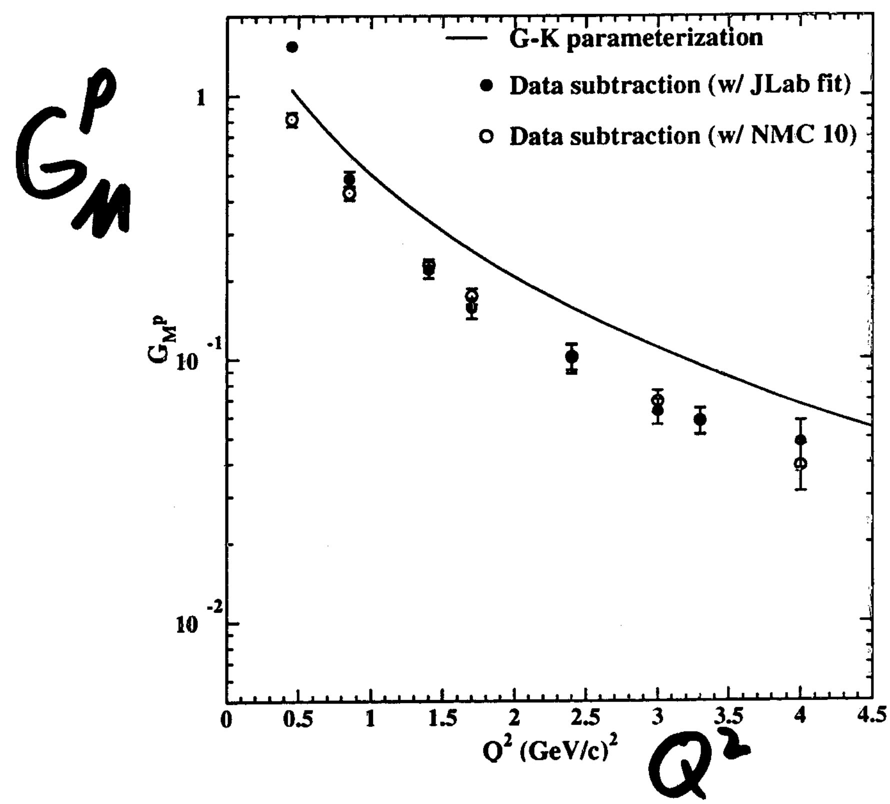


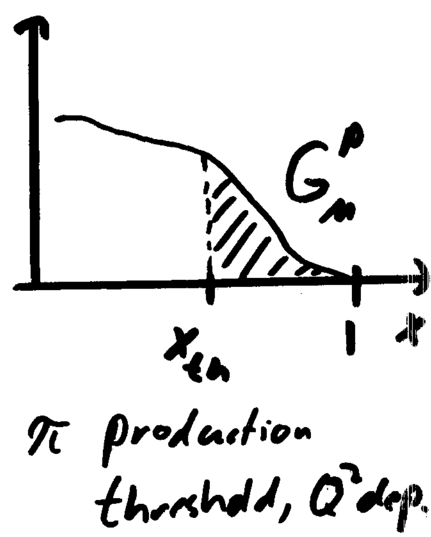
elastic contribution

Applications of Duality

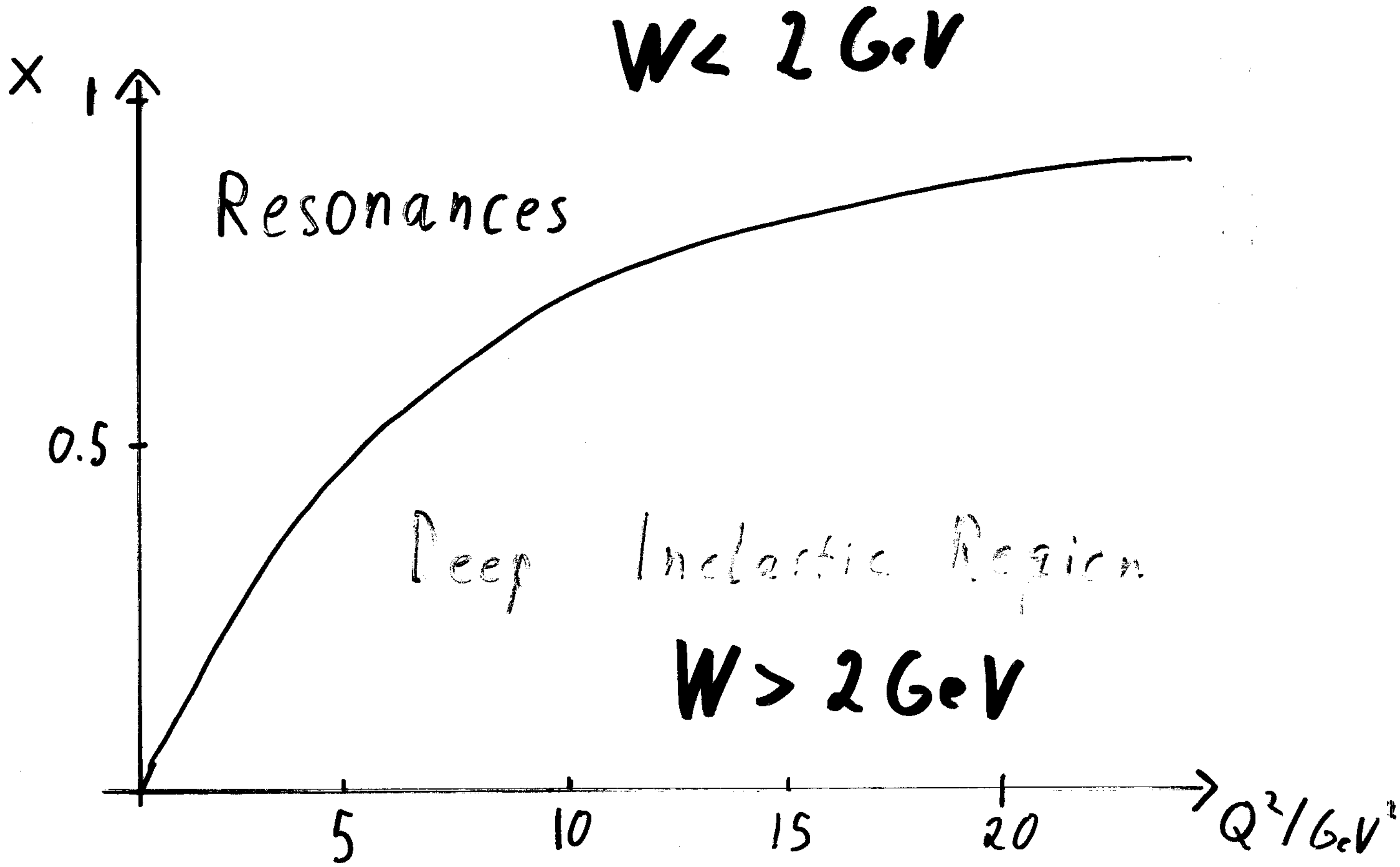
Duality establishes a relation between the resonance region and the deep inelastic region.

Nice examples in X. Ji & P. Unrau, PRD 52, 72 (1995), A. DeRujula, H. Georgi, and H. D. Politzer, Ann. Phys. 103, 315 (1977)





Duality may allow us to learn about the deep inelastic region at high x by measuring the resonances at high x with CEBAF @ 12 GeV.



$$\sigma_{Mott} \propto \frac{1}{Q^4}$$

$$W^2 = M^2 + 2M\nu - Q^2$$
$$= M^2 + Q^2 \frac{1-x}{x}$$

$$x = \frac{Q^2}{2M\nu}$$

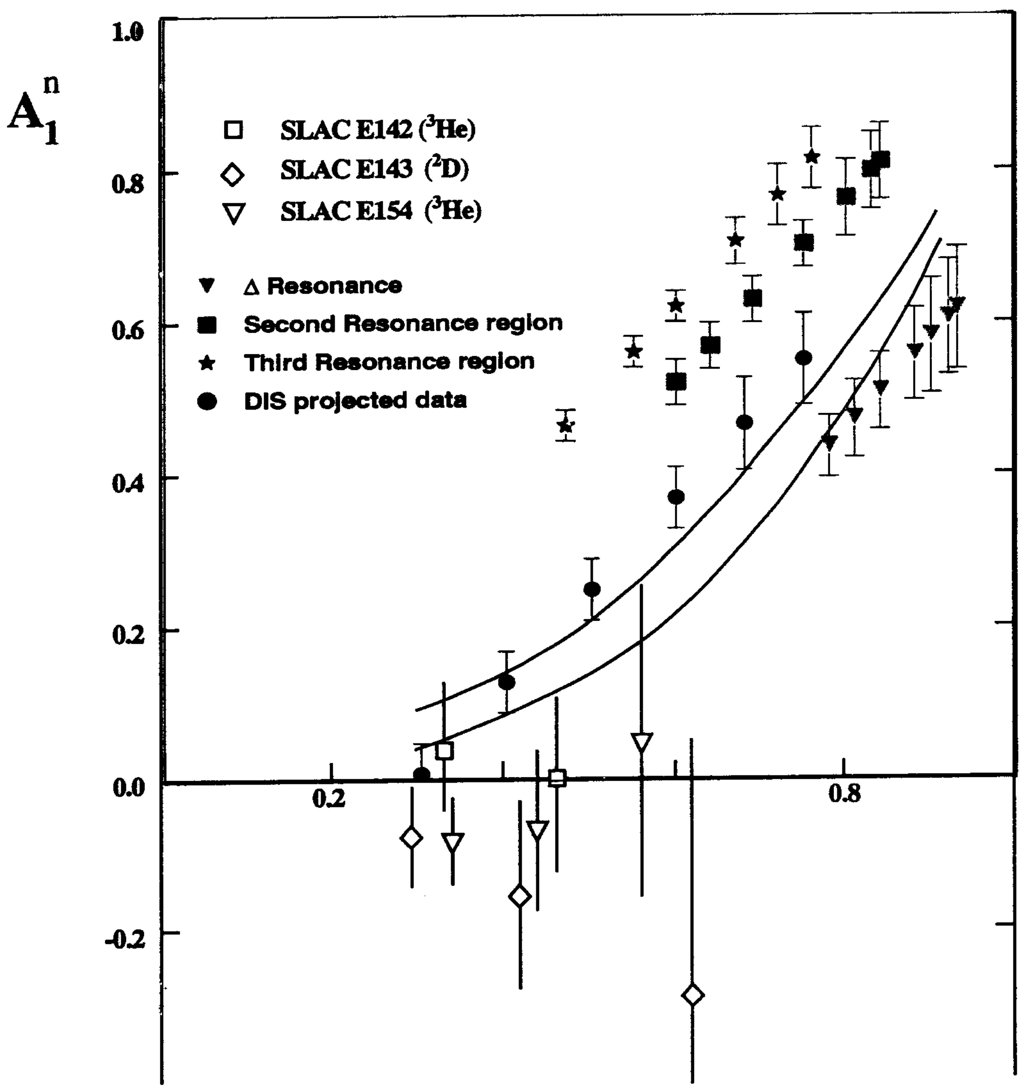
Applications of Duality: A_1^n

Polarization asymmetry **of the neutron,**
$A_1^n(x)$ **at high x:**

- **many, widely differing theoretical predictions, ranging from 0 to 1**
- **available data have large error bars, are taken at lower x**
- **Physics we can learn:** valence quark spin distribution functions

use known elastic form factor data to extract A_1^n,
Melnitchouk, PRL 86, 35 (2001)

Projections for 11 GeV beam



Nilanga Liyanage et al.,

Jefferson Lab Proposal 01-012

Towards Understanding Duality

Goal: develop a qualitative understanding of duality

How to tackle this complicated problem?

Try to make it as simple as possible!

Take a fully solvable model for hadrons, and compare the results to the free quark results.

Disclaimer: at first, no intention to quantitatively describe any data

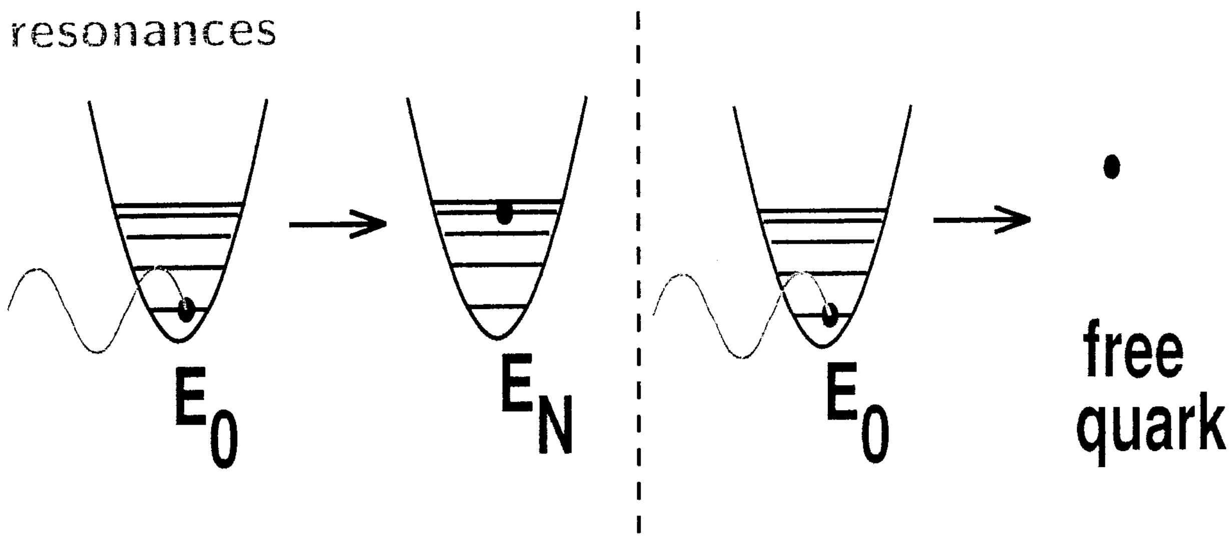
Isgur, SJ, Melnitchouk, Van Orden, PRD64, 054005 (2001)
SJ & Van Orden, PRD 65, 094038 (2002)

Paris & Pandharipande, PLB514, 361 (2001); PRC65, 035203 (2002)

F. E. Close and Q. Zhao, Phys. Rev. D 66, 054001 (2002)

Duality: Requirements for Models

Current models describe the excitation of resonances



no decays yet: $\nu = E_N - E_0 \Rightarrow \delta(\nu - E_N - E_0)$

A model must reproduce:

- **scaling: scaling curve for the bound-bound transition must be the same as for the bound-free transition**
- **moments flatten out with large enough** Q^2
- **resonance region results oscillate around the scaling curve**

A Model for Duality

basic ingredients:

- **relativity**
- **valence quarks only, treat them as scalars**
- **confinement: linear potential** ("relativistic harmonic oscillator")

simplify: reduce from the nucleon target to a one-body problem: light quark bound to an infinitely heavy diquark or antiquark

solve KG equation, obtain relativistic energy spectrum $E_N \propto \sqrt{N}$; **wave functions are identical to the NR HO case**

parameters: constituent quark mass $m = 0.33$ GeV, string tension $= 0.16$ GeV^2

all scalar case Isgur, SJ, Melnitchouk, Van Orden, PRD64, 054005 (2001)

electromagnetic current & scalar quarks **SJ & Van Orden, hep-ph/0202113**

Structure Functions

1) All Scalar Case Isgur, SJ, Melnitchouk, Van Orden, PRD 64, 054005 (2001)

$$\frac{d\sigma}{d\Omega dE_f} = \frac{g^2}{(2\pi^2)^2} \frac{E_f}{4E_i} \frac{1}{Q^4} W_{scalar}$$

structure function $W_{scalar}(\nu, Q^2) =$

$$\sum_N \frac{1}{4E_0 E_N} |F_{0N}(\sqrt{Q^2+\nu^2})|^2 \delta(\nu - E_N - E_0)$$

2) Electromagnetic Current SJ & Van Orden, PRD 65, 094038 (2002)

$$W_2(\nu, Q^2) = \sum_N \frac{1}{4E_0 E_N} |F_{0N}(\sqrt{Q^2+\nu^2})|^2$$

$$\left(\frac{Q^4}{Q^2+\nu^2} (E_0 + E_N)^2 + 4N\beta^4 \frac{Q^2}{Q^2+\nu^2} \right)$$
$$\delta(\nu - E_N - E_0)$$

Scaling Variables & Scaling Functions

$$x_{Bj} \text{ and } F_2(x_{Bj}) = \nu W_2(\nu, Q^2)$$

appropriate for the Bjorken limit where $Q^2 \gg M_T^2, m_q^2$

Duality is observed at much lower $Q^2 \approx M_T^2, Q^2 > m_q^2$, scaling variable and scaling function need to reflect that:

ad hoc variable of Bloom & Gilman: $x' = \frac{Q^2}{W^2+Q^2}$

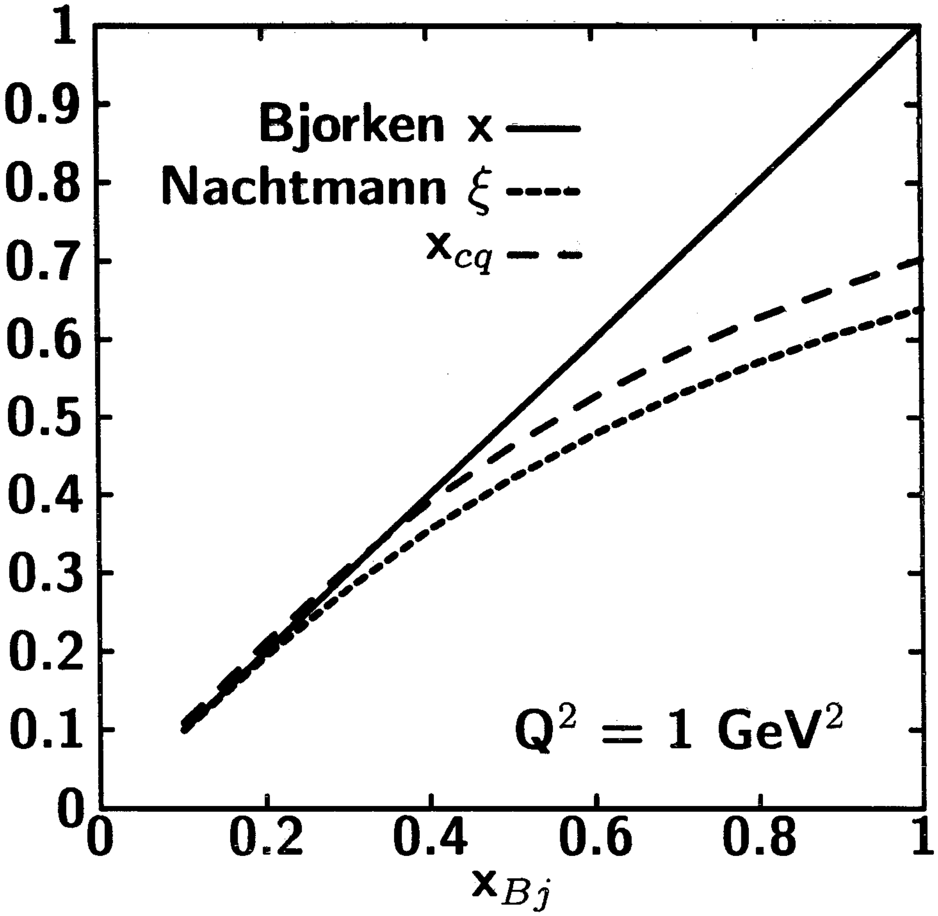
derive Barbieri et al, PLB 64, 171 (1976)

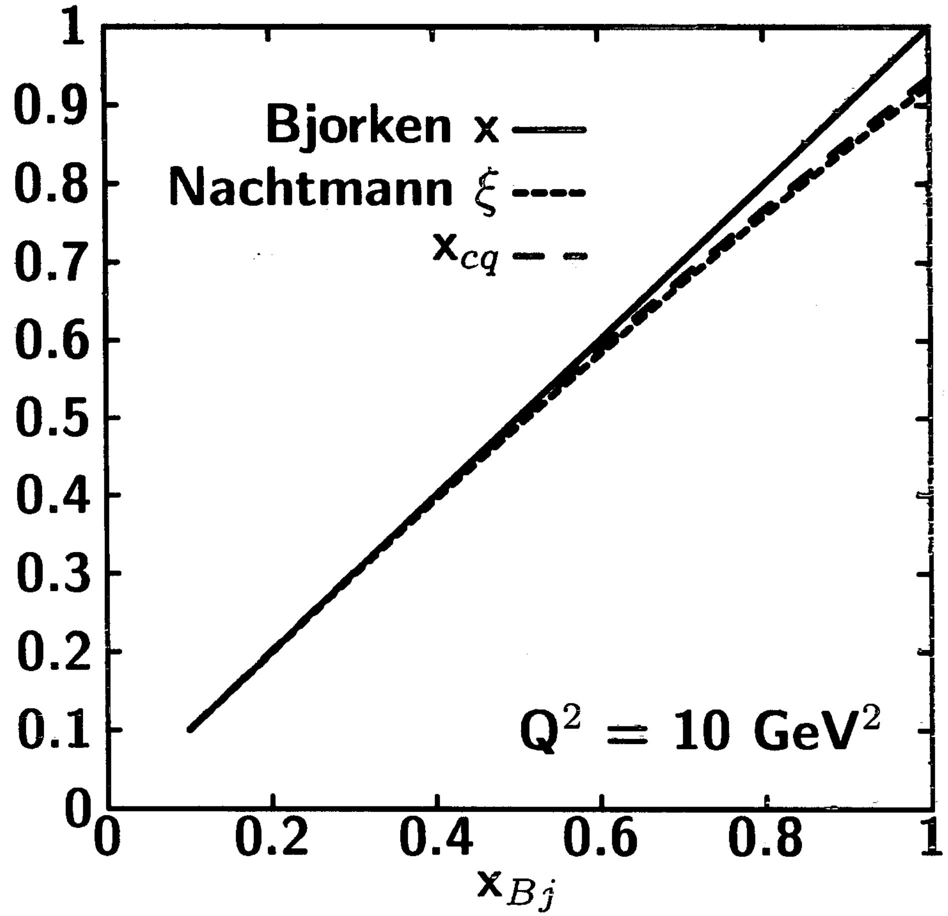
$$x_{cq} \equiv \frac{1}{2M_T}\left(\sqrt{\nu^2+Q^2}-\nu\right)\left(1+\sqrt{1+\frac{4m^2}{Q^2}}\right)$$

$$\mathcal{S}_{2cq} \equiv |\vec{q}|W_2 = \sqrt{\nu^2+Q^2}W_2$$

In the Bjorken limit, all versions of scaling variable and scaling function must converge to x_{Bj} and F_2.

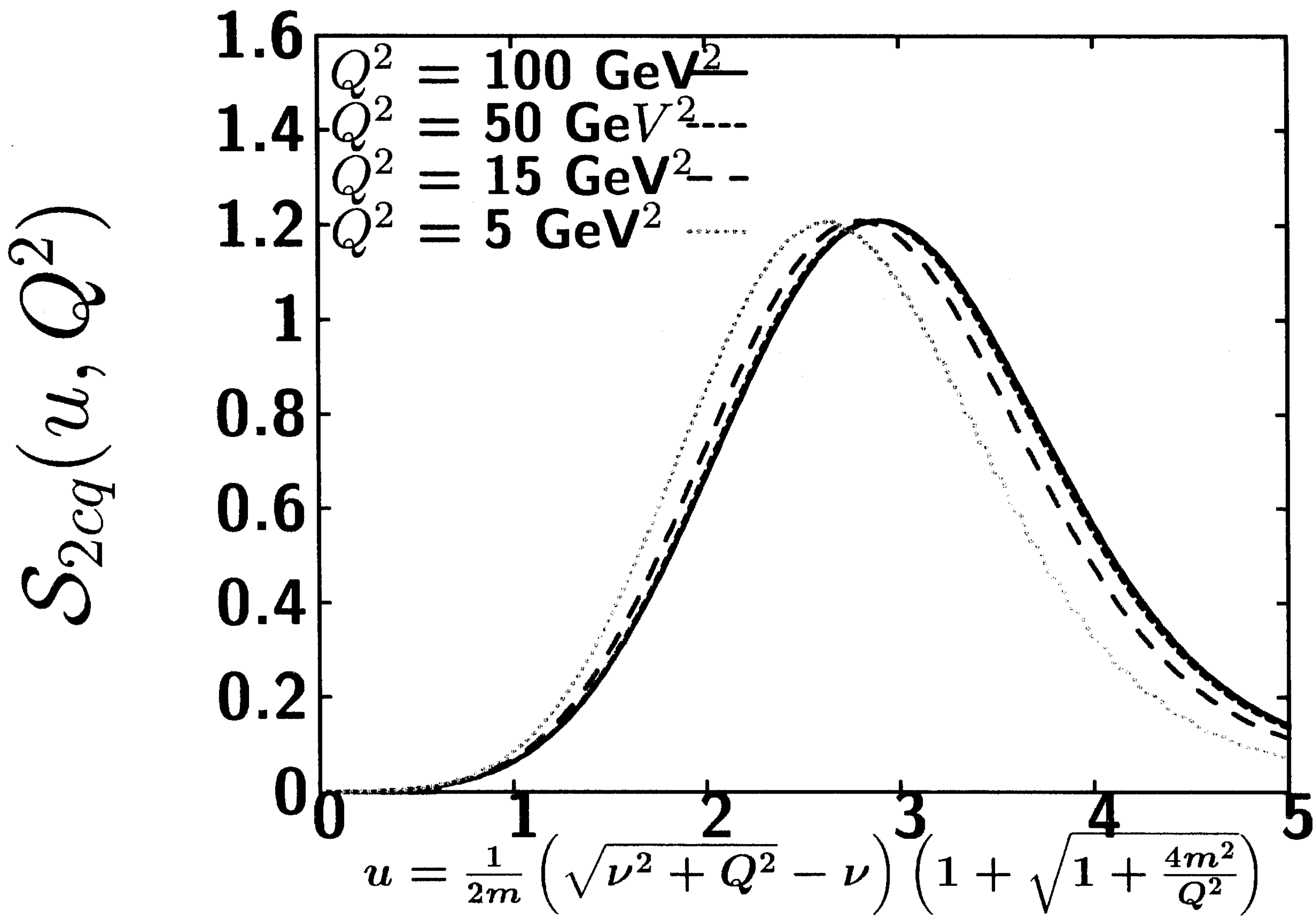
Comparison of Scaling Variables



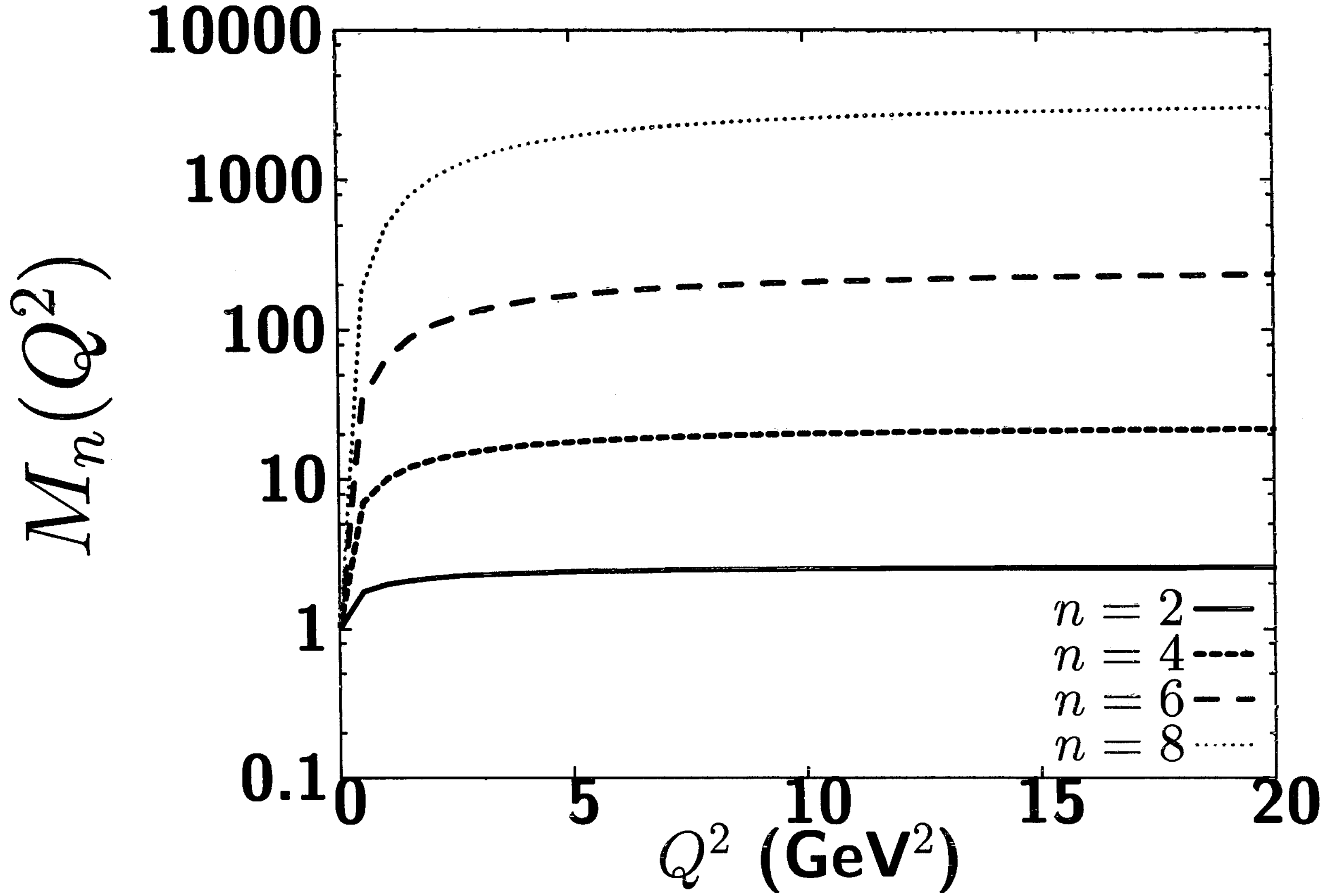


Scaling: Bound-Bound Transition

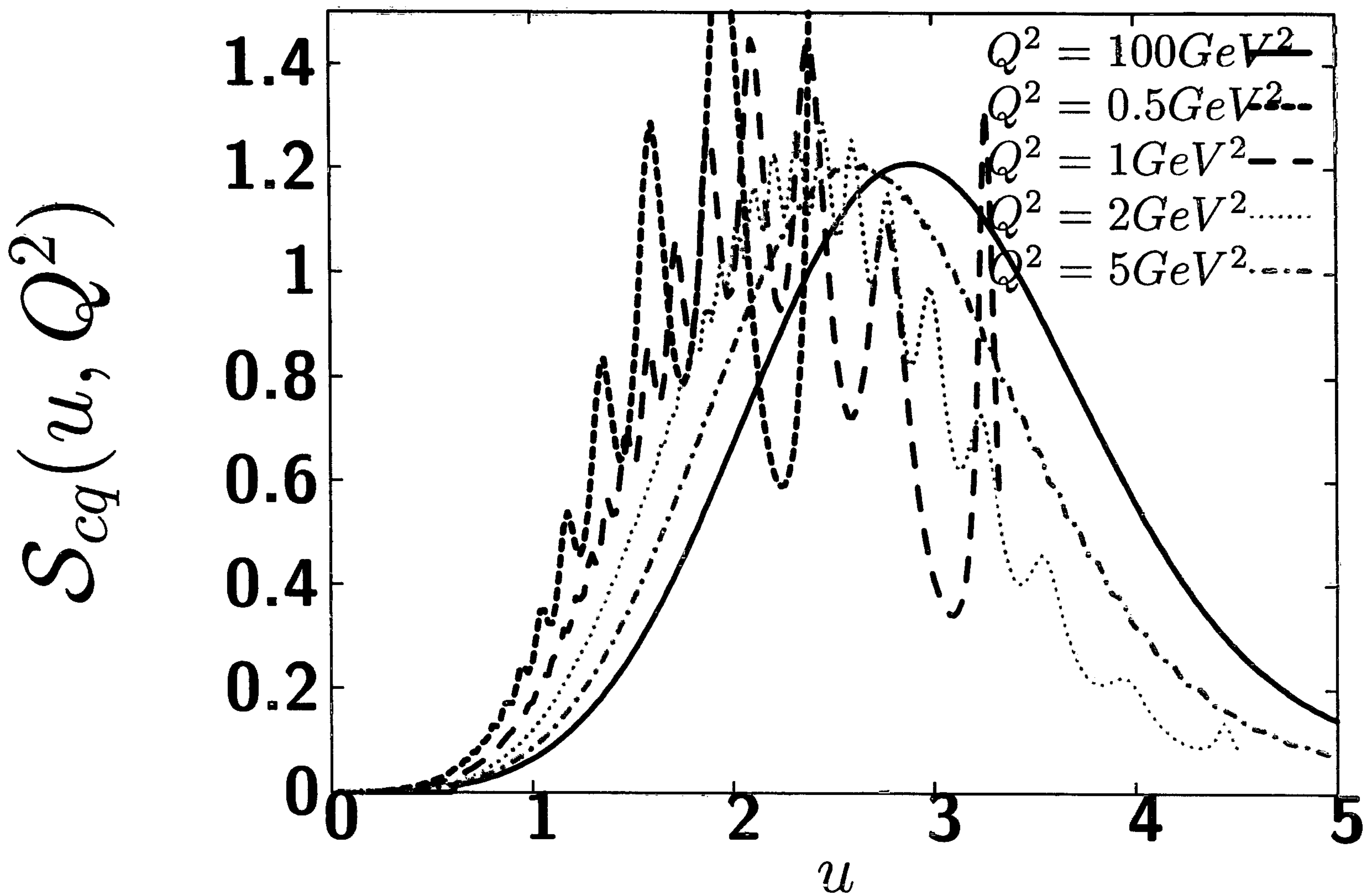
electromagnetic current; $\mathcal{S}_2 cq = |\vec{q}|W_2 = \sqrt{\nu^2 + Q^2}W_2$



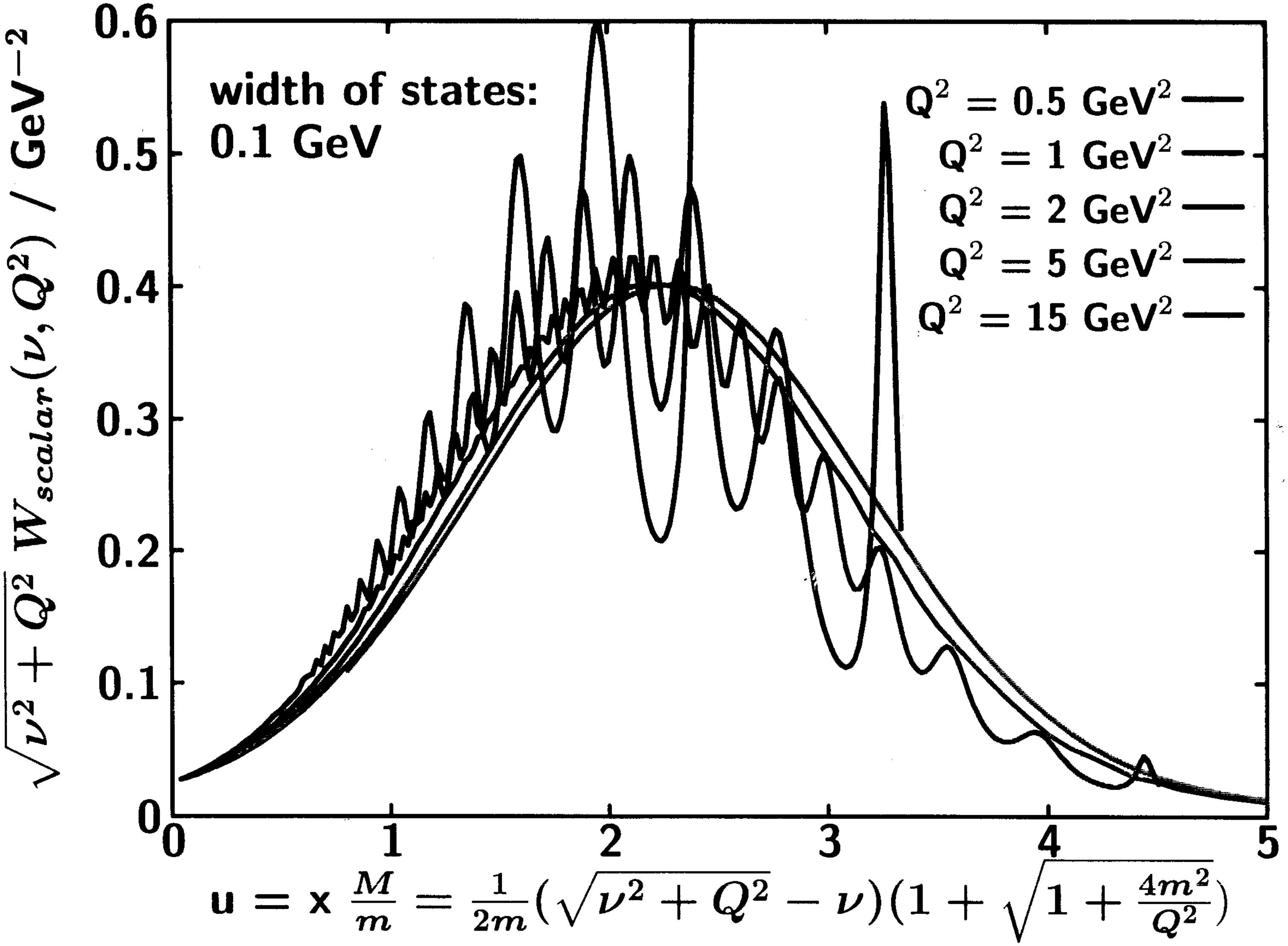
Moments



Local Duality



Local Duality for the All Scalar Case



Summary & Outlook

Duality

- appears in many processes
- is well established experimentally
- has useful applications
- in a simplified version, it can be modelled successfully

The future holds

- more new data on F_1 and F_L, F_2 at low Q^2
- development of more realistic models
  - use spin 1/2 quarks (almost ready!)
  - produce mesons