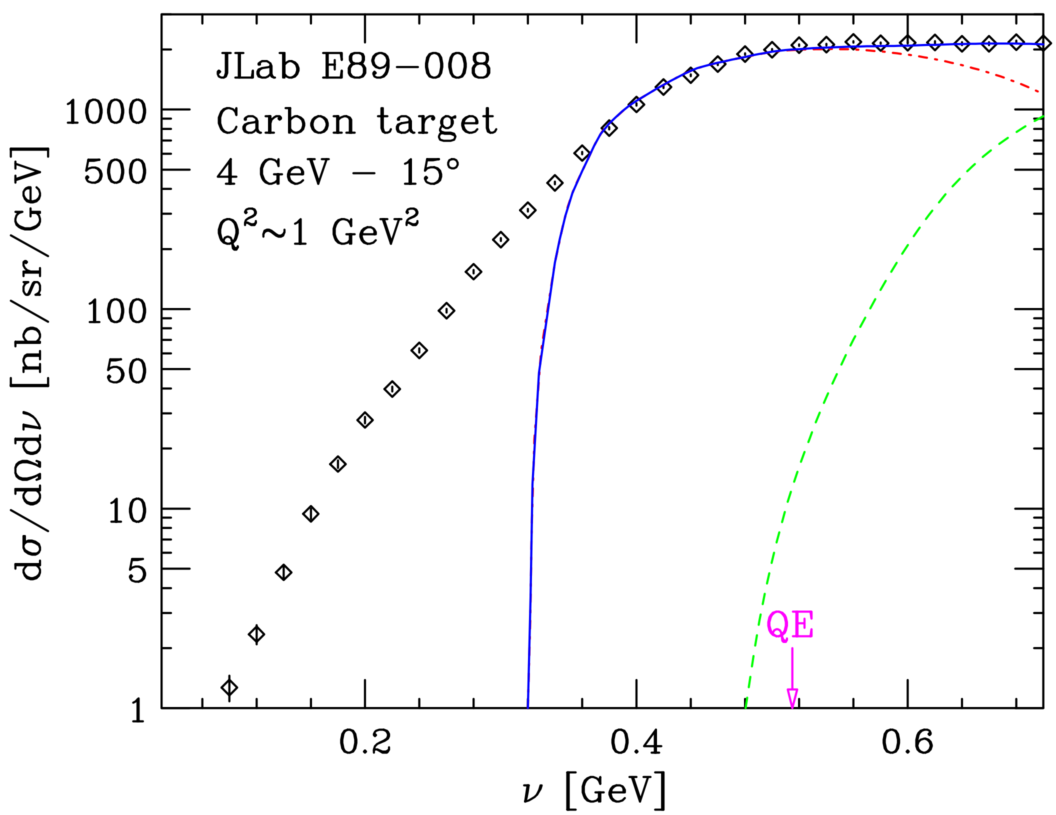
# Nuclear Response
# Beyond the Fermi Gas Model

## Outline

- Motivation: why go beyond the FG model ?
- How do we go beyond FG ?
  - ▷ What do we know about the energy and momentum distributions of nucleons in nuclei ?
  - ▷ What do we know about neutrino interactions with a bound nucleon ?
  - ▷ What do we know about nucleon propagation in the nuclear medium ?
- Inclusive cross section for quasielastic $\nu$ scattering off $^{16}O$ as a pedagogical example.
- Prospects and perspectives.

# Splendors and Miseries of the FG Model

- simple two-parameter model
- provides a good description of inclusive $e$A data in the region of quasi-free kinematics, $x = Q^2/2m_N \sim 1$
- dramatically fails to explain the data @ $x >> 1$, where dynamical NN correlation effects in both the initial and final states dominate




- does not include surface and shell effects. Cannot be applied to the study of exclusive channels

## The nuclear spectral function

- probability of removing a nucleon of momentum $\mathbf{p}$ from the nuclear ground state leaving the residual system with excitation energy $E$:

$$P(\mathbf{p}, E) = \sum_n \left| \langle \Psi_n^{(A-1)} | a_{\mathbf{p}} | \Psi_0^A \rangle \right|^2 \delta(E + E_0 - E_n)$$

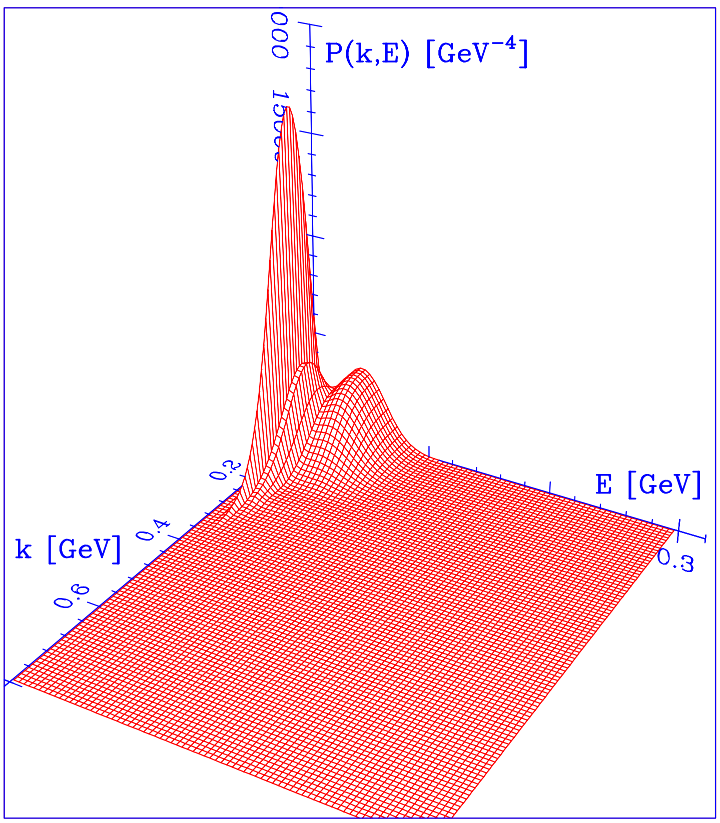
- $P(\mathbf{p}, E)$ at low $\mathbf{p}, E$ measured in nucleon knock-out $(e, e'p)$ reactions.

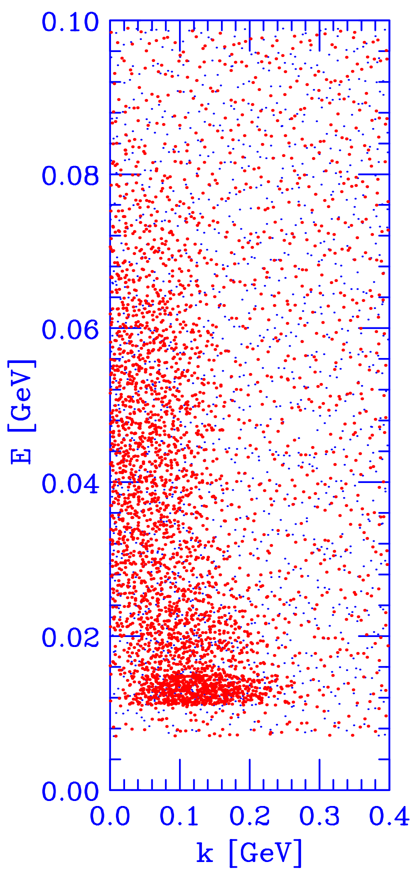
- $P(\mathbf{p}, E)$ calculable within non relativistic nuclear many-body theory for $A = \infty$.

- $P(\mathbf{p}, E)$ of finite nuclei can be obtained using the Local Density Approximation (LDA) to combine data and theoretical nuclear matter results.

# LDA spectral function of $^{16}O$

- Saclay $(e, e'p)$ data for nucleon knock-out from shell-model states
- continuum contribution calculated in infinite nuclear matter at different densities




- shell model states account for $\sim 80$ % of the strength.
- large energy and large momentum strongly correlated.
- $\sim 50\%$ of the strength @ $\mathbf{p} = 320$ MeV located @ $E > 80$ MeV

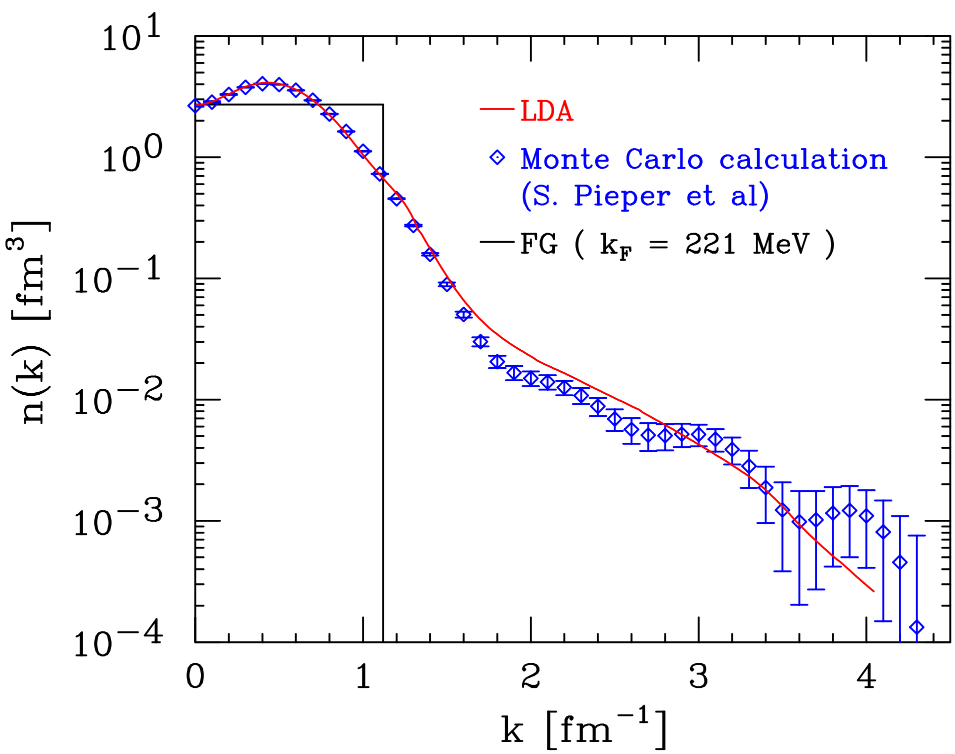
# Compare to FG model

- FG spectral function:

$$P(\mathbf{p}, E) \propto \Theta(k_F - |\mathbf{p}|)\delta\left(E + \frac{|\mathbf{p}|^2}{2m_N} - \overline{\epsilon}\right)$$

- momentum distribution:

$$n(\mathbf{p}) = \int dE \; P(\mathbf{p}, E)$$




- The LDA spectral function exhibits a sizable amount of strength @ $|\mathbf{p}| > k_F$, $E > \overline{\epsilon} = 25\text{MeV}$

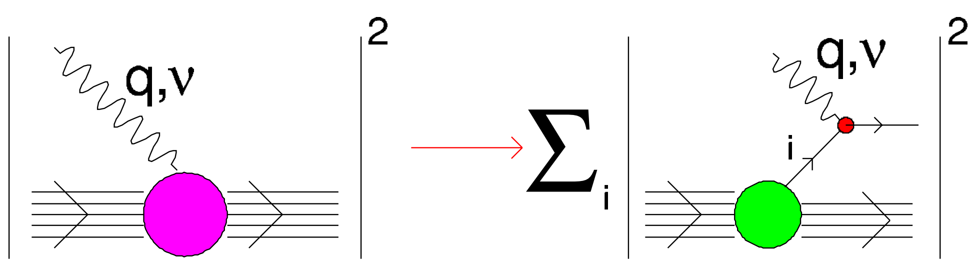
## Nuclear Cross Section

## Within the Impulse Approximation (IA)

Basic Assumptions:

- At large momentum transfer, scattering off nuclear targets reduces to the incoherent sum of scattering processes off individual nucleons

- There are no final state interactions (FSI) between the struck nucleon and the $(A-1)$ spectator particles




IA nuclear x-section:

$$\sigma_A(\mathbf{q},\nu) = \int d^3p \, dE \; \sigma_N(\mathbf{q},\nu,\mathbf{p},E) \; P(\mathbf{p},E)$$

- $\sigma_N$ is the x-section describing the elementary process involving a *bound* nucleon *carrying momentum* $\mathbf{p}$

Consider the weak charged current process:

$$\nu_\mu + A \to \mu^- + p + (A-1)$$

- IA cross section

$$\frac{d\sigma_A}{d\Omega_\mu dE_\mu} = \int d^3p \; dE \;\; P(\mathbf{p}, E) \; \frac{d\sigma_N}{d\Omega_\mu dE_\mu}$$

$$\times \delta(E_\nu - E_\mu - E - E_{\mathbf{p}+\mathbf{q}})$$

- cross section of the elementary process

$$\nu_\mu(k) + n(p) \longrightarrow \mu^-(k') + p(p')$$

$$\left( \frac{d\sigma}{d\Omega_\mu dE_\mu} \right)_N = \frac{1}{16\pi^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} \frac{1}{2E_{\mathbf{p}}} \frac{1}{2E_{\mathbf{p}'}} \frac{G_F^2}{2} L_{\mu\nu} W^{\mu\nu}$$

▷ $L_{\mu\nu}(k, k')$ lepton tensor

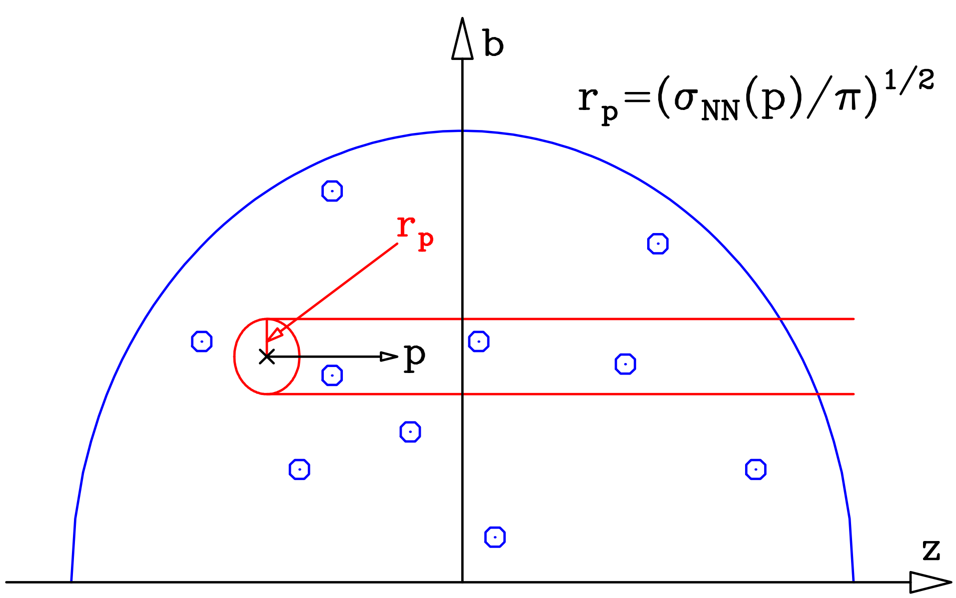
▷ $W^{\mu\nu}(p, \widetilde{q})$ describes the weak interactions of a *free* nucleon. Can be written in terms of the form factors $F_1$, $F_2$, $F_A$ & $F_p$

▷ within IA binding is taken care of replacing $q \to \widetilde{q} \equiv (\widetilde{\nu}, \mathbf{q})$, with $\widetilde{\nu} = \nu + M_A - E_{A-1} - E_{\mathbf{p}}$

## Inclusion of FSI

Why worry ?:

- Consider a nucleon, initially located at position $\mathbf{r} \equiv (\mathbf{b}, z)$, travelling along a *straight line* through the nucleus with momentum $\mathbf{p}$
- The probability of rescattering against the spectator nucleons, assumed to be *frozen* in their configuration at $t = 0$, can be estimated from the number of spectators found within the cylindrical volume described by a circle of area $\sigma_{\rm NN}$ (the NN scattering x-section) moving in the direction of $\mathbf{p}$



• $\langle N_{\mathbf{p}}(\mathbf{b}, z) \rangle$ is defined as $(R \equiv (\mathbf{r}_1, \ldots, \mathbf{r}_A))$

$$\langle N_{\mathbf{p}}(\mathbf{b}, z) \rangle = \frac{1}{\rho_A(\mathbf{r})} \int dR \, |\Psi_0(R)|^2 \, \frac{1}{A} \sum_{i=1}^{A} \delta(\mathbf{r} - \mathbf{r}_i)$$

$$\times \sum_{j \neq i=1}^{A} \Theta \left( \sqrt{\frac{\sigma_{\rm NN}(\mathbf{p})}{\pi}} - |\mathbf{b} - \mathbf{b}_j| \right) \Theta(z_j - z)$$

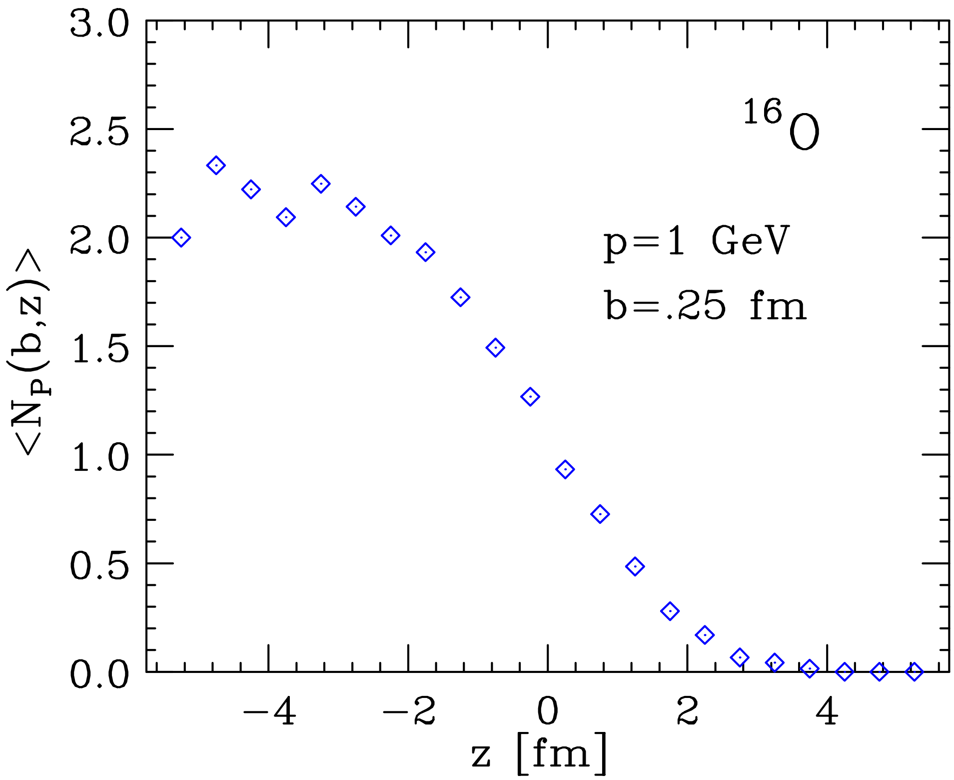
▷ $\rho_A(\mathbf{r})$ nuclear density

▷ $\Psi_0(R)$ nuclear ground state wave function

▷ $\sigma_{\rm NN}(\mathbf{p})$ total NN x-section at momentum $\mathbf{p}$

• $\langle N_{\mathbf{p}}(\mathbf{b}, z) \rangle$ calculated using the measured NN x-sections and Monte Carlo configurations sampled from the probability distribution associated with a realistic many-body wave function of $^{16}O$, obtained from a variational calculation of the ground state energy (S. Pieper *et al*)

- $\langle N_p(b,z) \rangle$ for oxygen, calculated @ $p = 1$ GeV and $b = .25$ fm




- nucleons hit in the backward emisphere are likely to undergo at least one rescattering

- there seem to be very good reasons to worry about FSI !

# Inclusion of FSI effects

- Use *eikonal* & *frozen spectators* approximations

- Write the ground state-averaged propagator of the struck nucleon in the factorized form

$$U_p(t) = U_p^0(t) U_p^{FSI}(t)$$

  ▷ $U_p^0(t)$ free-space propagator

$$U_p^{FSI}(t) = \langle \frac{1}{A} \sum_i \mathrm{e}^{i \sum_{j \neq i} \int_0^t \; dt' w_p(|\mathbf{r}_i + \widehat{v} t' - \mathbf{r}_j|)} \rangle$$

  ▷ $w_p(r)$ coordinate-space NN scattering t-matrix, parametrized in terms of total cross section, slope and real to imaginary part ratio

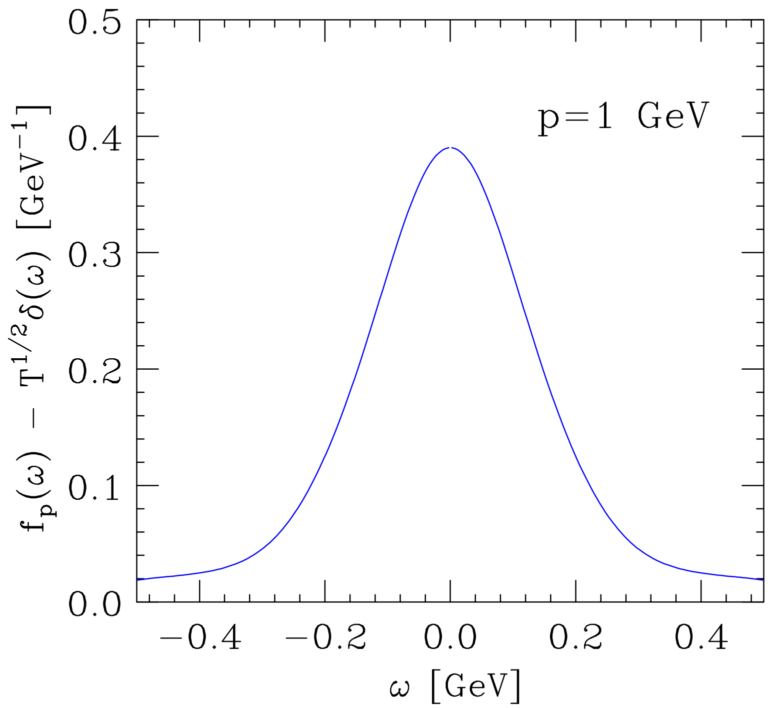
- Relation to total nuclear transparency measured in $(e, e'p)$

$$T = \lim_{t \to \infty} |U_p^{FSI}(t)|^2$$

- $U_p(t)$ calculable for $^{16}O$ using Monte Carlo

- FSI in *inclusive* processes driven by the *folding function*

$$f_p(\omega) = \int \frac{dt}{2\pi}\, e^{i\omega t} \left[U_p^{FSI}(t) - T^{1/2}\right] + T^{1/2}\delta(\omega)$$



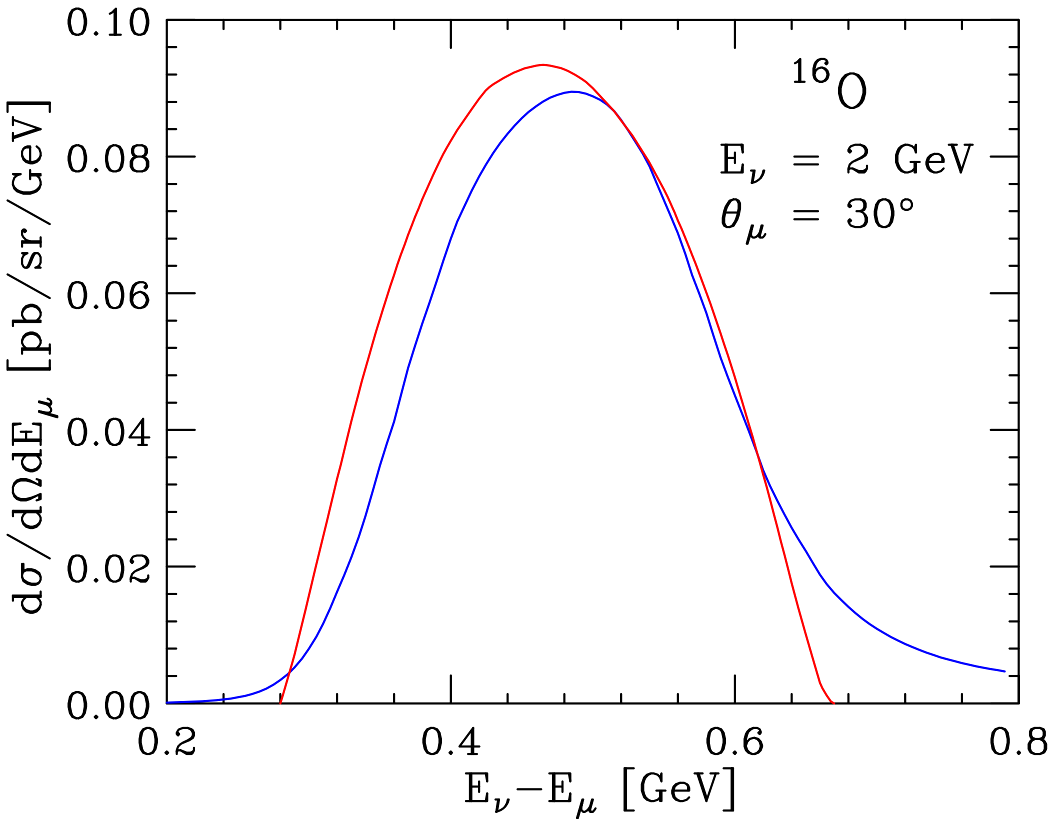

- strength of FSI measured by $T$ and the width of the folding function. In absence of FSI $T \equiv 1$ and $f_p(\omega) \rightarrow \delta(\omega)$

- inclusive $\nu$-nucleus x-section

$$\frac{d\sigma^A}{d\Omega_\mu dE_\mu} = \int dE'_\mu\, f_p(E_\mu - E'_\mu) \left(\frac{d\sigma^A}{d\Omega_\mu dE'_\mu}\right)_{IA}$$

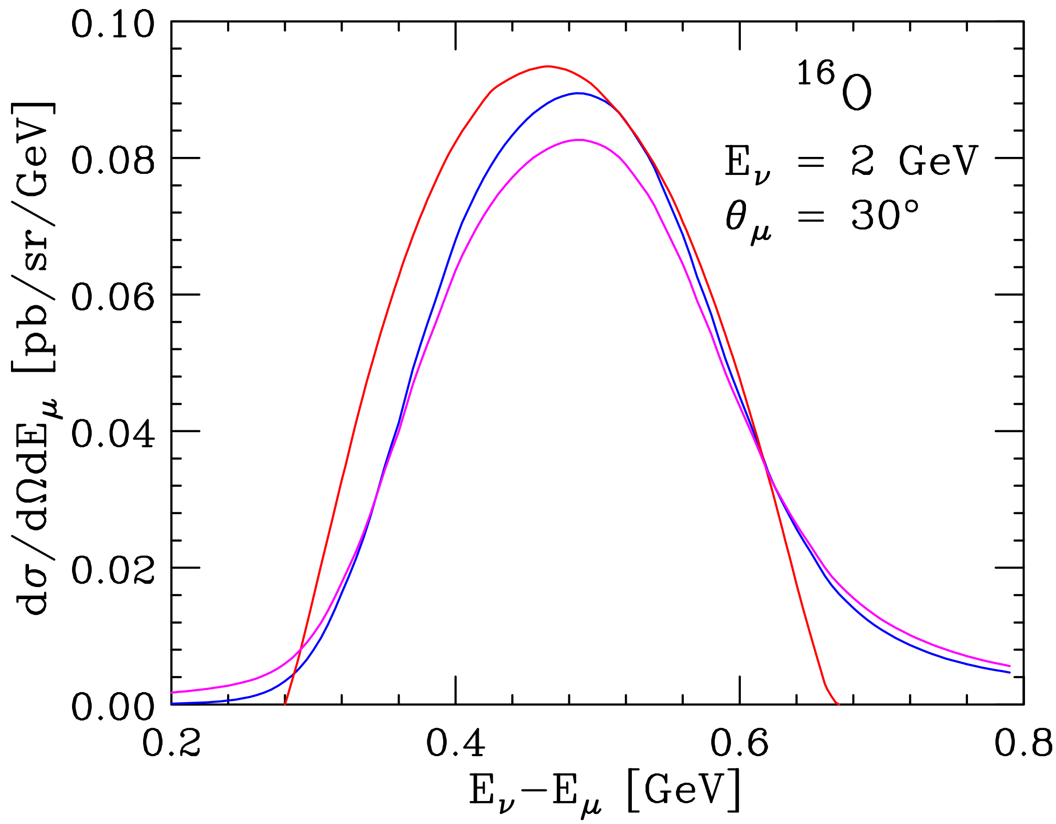
- differential x-section of the process $^{16}O(\nu_\mu, \mu^-)X$ as a function of the energy of the outgoing muon (quasielastic only)

- Impulse approximation: FG (red line) *vs* LDA (blue line) spectral function




- QE peak quenched and shifted. $\langle E \rangle_{LDA} = 42$ MeV, to be compared to $\overline{\epsilon} = 25$ MeV of the FG model

- Effect of FSI (LDA spectral function, purple line)




- inclusion of NN correlation effects in both the initial and final states produces a significant redistribution of the inclusive strentgh, leading to a quenching and a shift of the QE bump and to a sharp enhancement of the tails

## Prospects & perspectives

- Improving upon the FG model of the nuclear response at large momentum transfer possible
- Nonrelativistic nuclear many-body theory provides a consistent framework to include NN correlation effects in both the initial and final states
- Nonrelativistic many-body theory of the nuclear response extensively and succesfully tested against electron-nucleus scattering data
- The extension of the approach to study exclusive and inelastic channels, along the line of the work done for electron-nucleus scattering, appears to be feasible
- Warning: the applicability of IA at neutrino energies $E_\nu \lesssim 1$ GeV needs to be quantitatively investigated