

Modelling nuclear effects in neutrino interactions in 1 GeV region

Jan T. Sobczyk^a *

^aInstitute of Theoretical Physics, Wrocław University,
pl. M. Borna 9, 50-204 Wrocław, Poland

We evaluate nuclear effects in neutrino reactions in a framework based on a model proposed by Marteau with quasi-elastic and Δ production processes treated together. Nuclear effects include RPA corrections and Δ width modification in nuclear matter.

1. INTRODUCTION

Description of Δ excitation region in neutrino-nucleus interactions is the most problematic ingredient in Monte Carlo codes [1]. Experiments provide cross sections with a precision $\sim 25\%$ [2,3]. MC implementations are based on a combination of Rein-Sehgal pion production model [4] with Fermi gas model with different level of sophistication in kinematical assumptions. The basic dynamical rule is a factorization of interaction in two steps: (i) neutrino-nucleon interaction and (ii) re-interactions of outgoing particles inside nucleus.

In this contribution we wish to present computations done in a framework of more ambitious theoretical scheme. Our model is based on Marteau model [5] but it includes several modifications and also simplifications made in order to be able to compare better its predictions with experimental data. Marteau model describes on equal footing Δ excitation and quasi-elastic processes. It is based on the non-relativistic Fermi gas with RPA corrections due to contact interaction terms and exchange of pions and ρ mesons [6]. Elementary $2p - 2h$ excitations are also included. A modification of Δ width in a nuclear matter is done using Oset's results [7]. The model provides inclusive cross section for quasi-elastic and Δ excitation reactions and also contributions from several exclusive channels. The original model is rather complicated as local density effects are taken into

account from the very beginning [8].

Our modifications are:

- i) We avoid complications due to local density profile of nucleus keeping a constant value of Fermi momentum $k_F = 225 \text{ MeV}$. The model becomes easier to handle and local density effects can be included at the very end in the MC approach.
- ii) We adopt relativistic nucleons kinematics (we use relativistic generalization of the Lindhard function).
- iii) We adopt original Oset results for the Δ width in nuclear matter to specific kinematics of neutrino-nucleus reaction.
- iv) We do not include $2p - 2h$ part as it requires further study [9].

2. FERMION GAS

The basic cross section formula is:

$$\frac{d^2\sigma}{dq d\nu} = \frac{G_F^2 \cos^2\theta_c^2 q}{32\pi E^2} L_{\mu\nu} H^{\mu\nu}. \quad (1)$$

where

$$L_{\mu\nu} = 8(k'_\mu k'_\nu + k_\mu k'_\nu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\alpha\beta} k'^\alpha k^\beta) \quad (2)$$

is the leptonic tensor, E is the neutrino energy, k, k' denote lepton initial and final four-momenta, $q^\mu = k^\mu - k'^\mu = (\nu, \vec{q})$ is four-momentum transfer. The hadronic tensor is of the form

$$H^{\mu\nu} = H_{NN}^{\mu\nu} + H_{N\Delta}^{\mu\nu} + H_{\Delta N}^{\mu\nu} + H_{\Delta\Delta}^{\mu\nu} \quad (3)$$

We use the hadronic weak current

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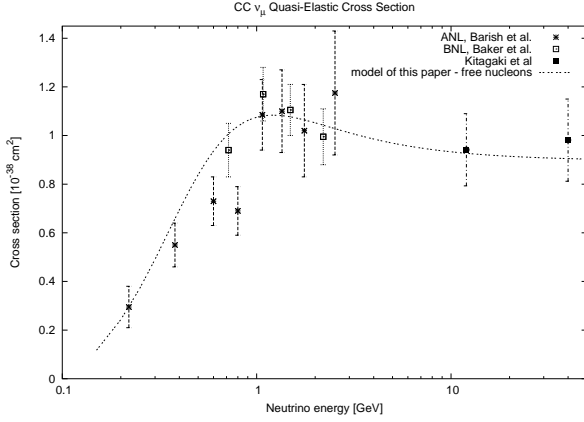


Figure 1. CC Quasi-elastic ν_μ cross section on free nucleons. Experimental points are taken from [10–12].

$$J^\mu = F_1(Q^2)\gamma^\mu + iF_2(Q^2)\sigma^{\mu\nu}\frac{q_\nu}{2M} + G_A(Q^2)\gamma^\mu\gamma_5 + G_P(Q^2)\gamma_5\frac{q^\mu}{2M} \quad (4)$$

where F_1 , F_2 , G_A and G_P are the standard form-factors. In the frame $\vec{q} = (0, 0, q)$ we calculate ($x, y \in \{N, \Delta\}$)[8]:

$$H_{xy}^{00}(\nu, q) = \sqrt{\frac{M_x + M + \nu}{2M_x}} \sqrt{\frac{M_y + M + \nu}{2M_y}} \times \left(\alpha_{0x}^0(\nu, q)\alpha_{0y}^0(\nu, q)R_{xy}^c(\nu, q) + \beta_{0x}^0(\nu, q)\beta_{0y}^0(\nu, q)R_{xy}^l(\nu, q) \right) \quad (5)$$

$$\alpha_{0x}^0(\nu, q) = F_1(Q^2) - F_2(Q^2)\frac{q^2}{2M(M_x + M + \nu)} \quad (6)$$

$$\beta_{0x}^0(\nu, q) = q \left(\frac{G_A(Q^2)}{M_x + M + \nu} - \frac{\nu}{2M} \cdot \frac{G_P(Q^2)}{M_x + M + \nu} \right), \quad (7)$$

etc. The free Fermi gas is characterized by

$$R_{N\Delta} = R_{\Delta N} = 0, \quad (8)$$

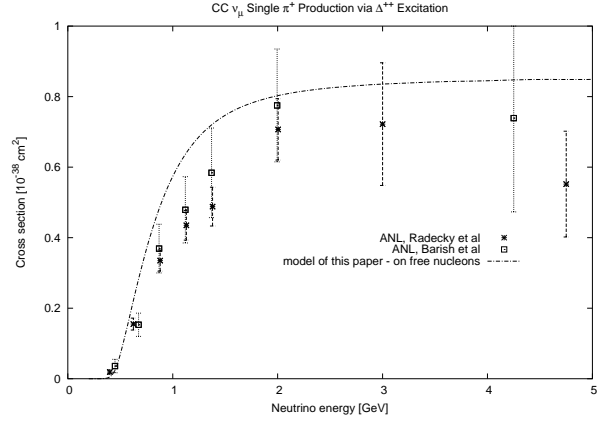


Figure 2. CC Δ^{++} excitation ν_μ cross section on free nucleons. Experimental points are taken from [2,3] and refer to $\nu_\mu p \rightarrow \mu\pi^+p$.

$$R_{NN}^{c,l,t}(\nu, q) = -\frac{Vol}{\pi} \text{Im}\Pi_{N-h}^0(\nu, q), \quad (9)$$

$$R_{\Delta\Delta}^{l,t}(\nu, q) = -\left(\frac{f_{\pi N\Delta}}{f_{\pi NN}}\right)^2 \frac{Vol}{\pi} \text{Im}\Pi_{\Delta-h}^0(\nu, q), \quad (10)$$

$$R_{\Delta\Delta}^c = 0. \quad (11)$$

$$Vol = \frac{3\pi^2 A}{2k_F^3}, \quad (12)$$

$$\text{Im}\Pi_{N-h}^0(\nu, \vec{q}) = -\frac{2M^2}{(2\pi)^2} \int d^3p \theta(k_F - |\vec{p}|) \times \frac{\delta(\nu + E_{\vec{p}} - E_{\vec{q}+\vec{p}})}{E_{\vec{p}}E_{\vec{q}+\vec{p}}} \theta(|\vec{q} + \vec{p}| - k_F) \quad (13)$$

$$\text{Im}\Pi_{\Delta-h}^0(\nu, \vec{q}) = -\frac{16}{9} \frac{M_\Delta^2}{(2\pi)^3} \int d^3p \frac{\Gamma_\Delta \cdot \theta(k_F - |\vec{p}|)}{(s - M_\Delta^2)^2 + M_\Delta^2 \Gamma_\Delta^2} \quad (14)$$

$$\Gamma_\Delta = PBL \cdot \Gamma_{\pi N} - 2\text{Im}(\Sigma_\Delta) \quad (15)$$

where $PBL \in [0, 1]$ is the Pauli blocking factor defined as follows [7]:

$$PB = \frac{|\vec{p}_\Delta| |\vec{q}_{cm}| - \sqrt{s} E_F + E_\Delta E_{cm}}{2|\vec{p}_\Delta| |\vec{q}_{cm}|} \quad (16)$$

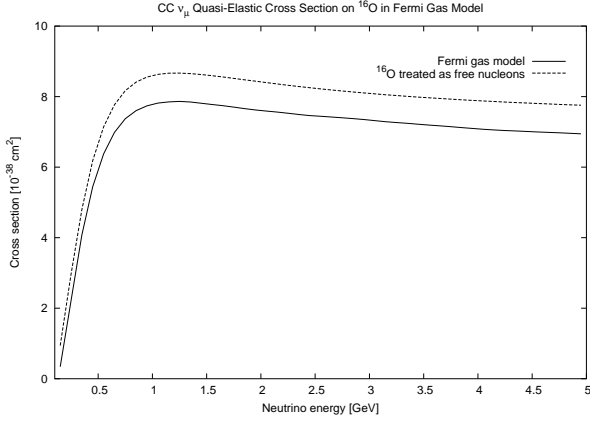


Figure 3. CC Quasi-elastic ν_μ cross section on ^{16}O in Fermi gas model.

$PBL = 1$ if $PB > 1$, $PBL = 0$ if $PB < 0$, otherwise $PBL = PB$. $(E_\Delta, \vec{p}_\Delta)$ is Δ 4-momentum, (E_{cm}, \vec{q}_{cm}) is pion (from Δ decay) 4-momentum in the center of mass frame,

$$\Gamma_{\pi N} = \Gamma_0 \frac{q_{cm}(\sqrt{s})^3}{q_{cm}(M_\Delta)^3} \frac{M_\Delta}{\sqrt{s}}, \quad (17)$$

$\Gamma_0 = 115\text{MeV}$, $M_\Delta = 1232\text{MeV}$. $\mathcal{I}m(\Sigma_\Delta)$ describes nuclear effects in the form of extra contributions to the Δ width from channels $\Delta \rightarrow \pi N$, $N\Delta \rightarrow NN$, $NN\Delta \rightarrow NNN$.

$$\mathcal{I}m(\Sigma_\Delta) = \mathcal{I}m(\Sigma_\Delta^\pi) + \mathcal{I}m(\Sigma_\Delta^{NN, NNN}). \quad (18)$$

$\mathcal{I}m(\Sigma_\Delta)$ was calculated by Oset-Salcedo [7] in two kinematical situations: pion-nucleon and photon-nucleon scatterings. Because the kinematical region for neutrino induced reaction is different ($\nu^2 - \vec{q}^2 < 0$) we adopt an approximation

$$\mathcal{I}m(\Sigma_\Delta)_\nu = \mathcal{I}m(\Sigma_\Delta)_\gamma + (\mathcal{I}m(\Sigma_\Delta)_\gamma - \mathcal{I}m(\Sigma_\Delta)_\pi) \quad (19)$$

with $\mathcal{I}m(\Sigma_\Delta)_\gamma$, $\mathcal{I}m(\Sigma_\Delta)_\pi$ taken from Oset-Salcedo. Our prescription introduces an increase with respect to the value of $\mathcal{I}m(\Sigma_\Delta)_\gamma$ by amount of 5-10%.

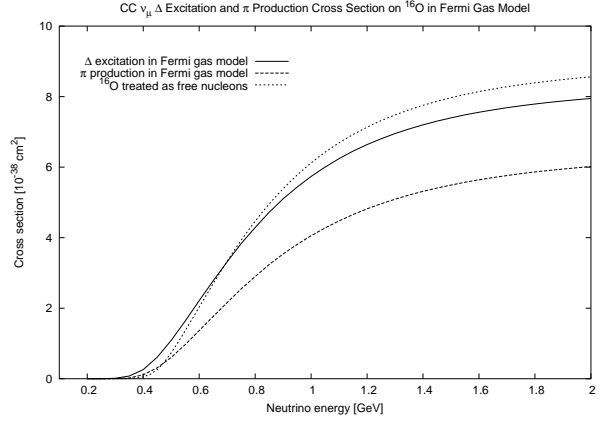


Figure 4. CC ν_μ π production cross section on ^{16}O in Fermi gas model is smaller than Δ excitation cross section since Δ can decay without π 's in final state

Normalization factors are checked by performing the limit $k_F \rightarrow 0$ in which we recover quasi-elastic (Fig. 1) and Δ excitation (Fig. 2) cross sections on free nucleons. In the case of Δ excitation Marteau model provides a sum of cross sections over isospin states i.e. a sum of Δ^+ and Δ^{++} productions. We recover Δ^{++} production cross section assuming isospin branching ratio rule $\sigma(\Delta^{++}) = 3\sigma(\Delta^+)$. We perform a comparison for this particular channel of pion production since it is known that in the reaction $\nu_\mu p \rightarrow \mu^- p \pi^+$ resonance contribution is dominant. Experimental points are taken from papers [10–12] in the case of quasi-elastic reaction and [2,3] in the case of Δ^{++} production.

In calculating $H^{\mu\nu}$ we adopted an approximation in which calculation of sums over hadronic spins is done in the limit in which target nucleon is at rest. In the case of quasi-elastic one can verify by explicit computations that this approximation is very good (on the level of 2-3%). In the case of Δ excitation computations of response functions done by Marteau [8] show that in the relevant kinematical region the approximation is

also very good.

Results in the Fermi gas model are presented in Fig. 3 (quasi-elastic reaction) and Fig. 4 (Δ excitation and π production). In these and next computations we have assumed that nucleus in question is ^{16}O i.e. $A = 16$. As mentioned in the introduction we do not take into account ^{16}O density profile keeping a constant value of Fermi momentum.

3. RPA CORRECTIONS

RPA equations read (separately for c,l,t; we omit the arguments ν and q of all the functions below):

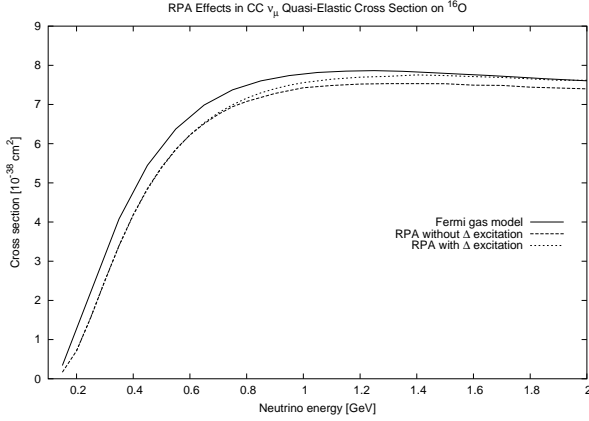


Figure 5. CC ν_μ quasi-elastic cross section on ^{16}O with RPA corrections.

$$\begin{aligned}
 \Pi_{NN} &= \Pi_{N-h}^0 + \Pi_{N-h}^0 V_{NN} \Pi_{NN} + \Pi_{N-h}^0 V_{N\Delta} \Pi_{\Delta N}, \\
 \Pi_{\Delta\Delta} &= \Pi_{\Delta-h}^0 + \Pi_{\Delta-h}^0 V_{\Delta N} \Pi_{N\Delta} + \Pi_{\Delta-h}^0 V_{\Delta\Delta} \Pi_{\Delta\Delta}, \\
 \Pi_{N\Delta} &= \Pi_{N-h}^0 V_{NN} \Pi_{N\Delta} + \Pi_{N-h}^0 V_{N\Delta} \Pi_{\Delta\Delta}, \\
 \Pi_{\Delta N} &= \Pi_{\Delta-h}^0 V_{\Delta N} \Pi_{NN} + \Pi_{\Delta-h}^0 V_{\Delta\Delta} \Pi_{\Delta N}.
 \end{aligned} \tag{20}$$

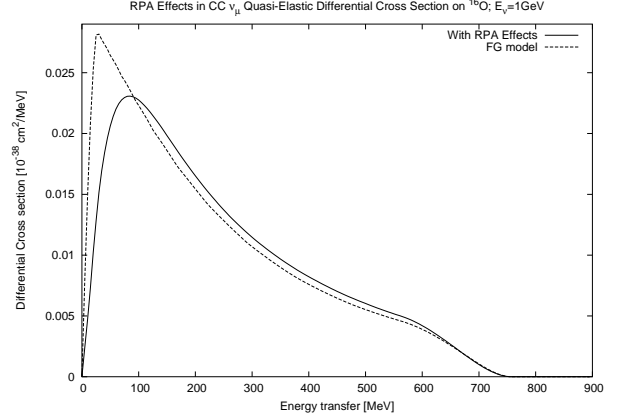


Figure 6. CC ν_μ differential quasi-elastic cross section on ^{16}O with RPA corrections; neutrino energy is 1GeV .

The solutions are found to be

$$\begin{aligned}
 \Pi_{NN} &= \Pi_{N-h}^0 (1 - V_{\Delta\Delta} \Pi_{\Delta-h}^0) D^{-1} \\
 \Pi_{\Delta\Delta} &= \Pi_{\Delta-h}^0 (1 - V_{NN} \Pi_{N-h}^0) D^{-1} \\
 \Pi_{N\Delta} &= \Pi_{\Delta N} = V_{N\Delta} \Pi_{\Delta-h}^0 \Pi_{N-h}^0 D^{-1}
 \end{aligned} \tag{21}$$

where

$$\begin{aligned}
 D &= (1 - V_{NN} \Pi_{N-h}^0)(1 - V_{\Delta\Delta} \Pi_{N-h}^0) \\
 &\quad - V_{N\Delta}^2 \Pi_{N-h}^0 \Pi_{\Delta-h}^0
 \end{aligned} \tag{22}$$

After substitution

$$R_{\Delta\Delta}^{l,t} = -\left(\frac{f_{\pi N\Delta}}{f_{\pi NN}}\right)^2 \frac{Vol}{\pi} \mathcal{I}m \Pi_{\Delta\Delta}^{l,t}, \tag{23}$$

$$R_{NN}^{c,l,t} = -\frac{Vol}{\pi} \mathcal{I}m \Pi_{NN}^{c,l,t}, \tag{24}$$

$$R_{N\Delta}^{l,t} = R_{\Delta N}^{l,t} = -\frac{f_{\pi N\Delta}}{f_{\pi NN}} \frac{Vol}{\pi} \mathcal{I}m \Pi_{N\Delta}^{l,t} \tag{25}$$

we obtain the final expression for inclusive the cross section.

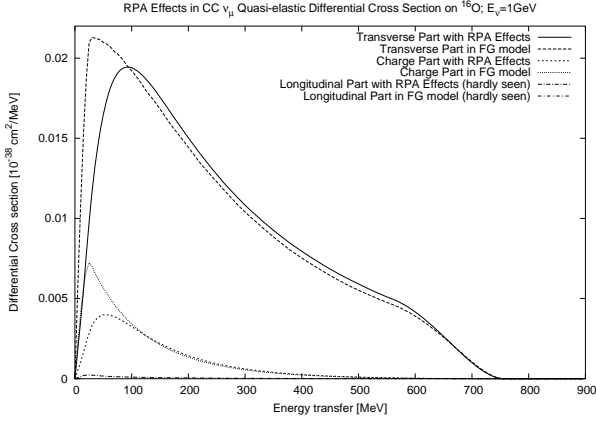


Figure 7. CC ν_μ differential quasi-elastic cross section on ^{16}O with RPA corrections in decomposition to charge, longitudinal and transverse parts; neutrino energy is 1GeV .

In numerical computations we use the interaction terms [6]:

$$\begin{aligned}
 V_c^{NN} &= \frac{f'}{m_\pi^2}, \\
 V_l^{NN} &= \frac{f_{\pi NN}^2}{m_\pi^2} \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \nu^2 + q^2} \right)^2 \\
 &\quad \times \left(g' + \frac{q^2}{\nu^2 - q^2 - m_\pi^2} \right), \\
 V_t^{NN} &= \frac{f_{\pi NN}^2}{m_\pi^2} \left(g' \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \nu^2 + q^2} \right)^2 \right. \\
 &\quad \left. + C_\rho^2 \frac{q^2}{\nu^2 - q^2 - m_\rho^2} \left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - \nu^2 + q^2} \right)^2 \right),
 \end{aligned} \tag{26}$$

$$\begin{aligned}
 V_l^{N\Delta} &= V_l^{\Delta N} = \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2} \\
 &\quad \times \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \nu^2 + q^2} \right)^2 \left(g'' + \frac{q^2}{\nu^2 - q^2 - m_\pi^2} \right) \\
 V_t^{N\Delta} &= V_t^{\Delta N} = \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2} \\
 &\quad \times \left(g'' \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \nu^2 + q^2} \right)^2 \right. \\
 &\quad \left. + C_\rho^2 \frac{q^2}{\nu^2 - q^2 - m_\rho^2} \left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - \nu^2 + q^2} \right)^2 \right),
 \end{aligned} \tag{27}$$

$$\begin{aligned}
 V_l^{\Delta\Delta} &= \frac{f_{\pi N\Delta}^2}{m_\pi^2} \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \nu^2 + q^2} \right)^2 \\
 &\quad \times \left(g''' + \frac{q^2}{\nu^2 - q^2 - m_\pi^2} \right), \\
 V_t^{\Delta\Delta} &= \frac{f_{\pi N\Delta}^2}{m_\pi^2} \left(g''' \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - \nu^2 + q^2} \right)^2 \right. \\
 &\quad \left. + C_\rho^2 \frac{q^2}{\nu^2 - q^2 - m_\rho^2} \cdot \left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - \nu^2 + q^2} \right)^2 \right),
 \end{aligned} \tag{28}$$

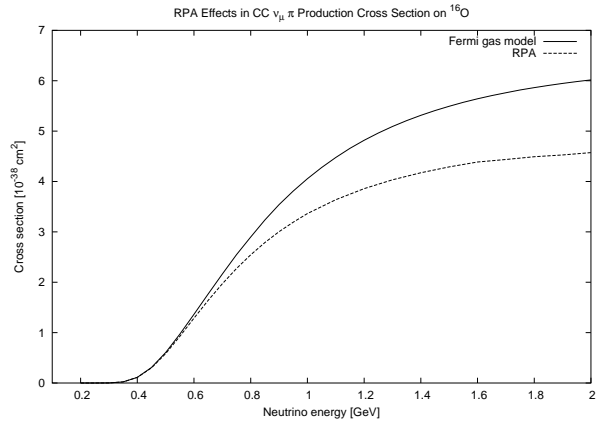


Figure 8. ν_μ CC π production cross section on ^{16}O with RPA corrections.

with the following values of free parameters:
 $f_{\pi NN}^2 = 4\pi \cdot 0.08$, $(\frac{f_{\pi N\Delta}}{f_{\pi NN}})^2 = 4.78$, $C_\rho^2 = (\frac{m_\pi}{m_\rho}) \cdot$

$$\left(\frac{f_{\rho NN}}{f_{\pi NN}}\right)^2 = 2, f' = 0.6, g' = 0.7, g'' = 0.5, g''' = 0.5, \Lambda_\pi = 1000 \text{ MeV}, \Lambda_\rho = 1500 \text{ MeV}.$$

4. EXCLUSIVE CHANNELS

In identifying contributions to exclusive channels we distinguish quasi-elastic processes and reactions with and without pions in the final state. For this purpose we split Δ width and response functions in two parts according to

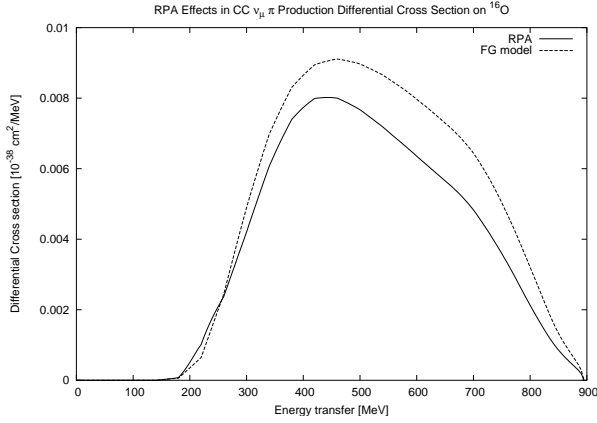


Figure 9. ν_μ CC differential π production cross section on ^{16}O with RPA corrections; neutrino energy is 1GeV .

$$\Gamma_\Delta^{(\pi)} = PBL \cdot \Gamma_{\pi N} - 2\text{Im}(\Sigma_\Delta^\pi) \quad (29)$$

$$\text{Im}\Pi_{\Delta-h}^0 = \text{Im}(\Pi_{\Delta-h}^0)_\pi + \text{Im}(\Pi_{\Delta-h}^0)_N \quad (30)$$

$$\begin{aligned} \text{Im}(\Pi_{\Delta-h}^0)_\pi(\nu, \vec{q}) = \\ -\frac{16}{9} \frac{M_\Delta^2}{(2\pi)^3} \int d^3p \frac{\Gamma_\Delta^{(\pi)} \cdot \theta(k_F - |\vec{p}|)}{(s - M_\Delta^2)^2 + M_\Delta^2 \Gamma_\Delta^2} \end{aligned} \quad (31)$$

Contributions to pion production come from all four $\text{Im}\Pi_{xy}$ sectors of the theory. They are cal-

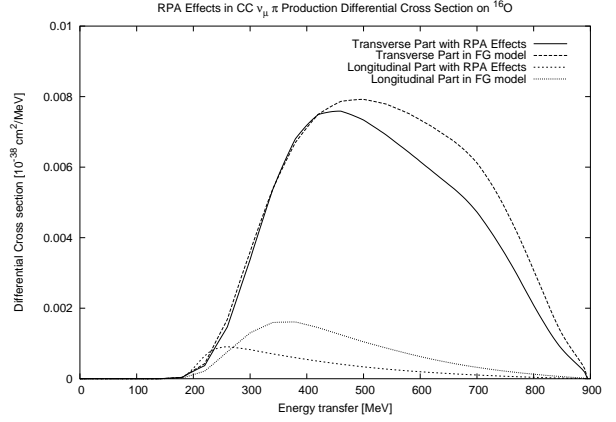


Figure 10. ν_μ CC differential π production cross section on ^{16}O with RPA corrections in decomposition to longitudinal and transverse parts; neutrino energy is 1GeV .

culated by making substitutions

$$\begin{aligned} R_{N\Delta}^{l,t} &\rightarrow (R_{N\Delta}^{l,t})_\pi = -\frac{f_{\pi N\Delta}}{f_{\pi NN}} \frac{Vol}{\pi} (\text{Im}\Pi_{N\Delta}^{l,t})_\pi \\ R_{NN}^{l,t} &\rightarrow (R_{NN}^{l,t})_\pi = -\frac{Vol}{\pi} (\text{Im}\Pi_{NN}^{c,l,t})_\pi, \end{aligned} \quad (32)$$

$$R_{\Delta\Delta}^{l,t} \rightarrow (R_{\Delta\Delta}^{l,t})_\pi = -\left(\frac{f_{\pi N\Delta}}{f_{\pi NN}}\right)^2 \frac{Vol}{\pi} (\text{Im}\Pi_{\Delta\Delta}^{l,t})_\pi,$$

$$\text{Im}(\Pi_{NN})_\pi = \frac{|\Pi_{N-h}^0|^2 (V_{N\Delta})^2 \text{Im}(\Pi_{\Delta-h}^0)_\pi}{|D|^2}, \quad (33)$$

$$\frac{1}{2} (\text{Im}(\Pi_{N\Delta})_\pi + \text{Im}(\Pi_{\Delta N})_\pi) = \quad (34)$$

$$\frac{(V_{N\Delta} (\text{Re}(\Pi_{N-h}^0) - V_{NN}) |\Pi_{N-h}^0|^2) \text{Im}(\Pi_{\Delta-h}^0)_\pi}{|D|^2},$$

$$\text{Im}(\Pi_{\Delta\Delta})_\pi = \frac{|(1 - V_{NN} \Pi_{N-h}^0)|^2 \text{Im}(\Pi_{\Delta-h}^0)_\pi}{|D|^2} \quad (35)$$

5. RESULTS AND DISCUSSION

Our results concerning relevance of RPA are summarized in a series of plots.

In Fig. 5 total cross sections for CC quasi-elastic reaction are presented. We compare three situations: Fermi gas without RPA correlations, RPA correlations without taking into account $\Delta - h$ excitation and full RPA computations. It is seen that RPA does not introduce much change in the total cross section. Inclusion of $\Delta - h$ excitations increases cross section for neutrino energies above 0.8GeV and one arrives at results very close to those for Fermi gas.

In Fig. 6 CC quasi-elastic differential cross section $\frac{d\sigma}{d\nu}$ is shown for neutrino energy $E = 1\text{GeV}$. A well known result is reproduced: the quasi-elastic peak is cut and some increase of differential cross section is observed at bigger values of energy transfer [6].

In Fig. 7 transverse, longitudinal and charge contributions (according to spin operators present in the transition amplitude) are distinguished in the CC quasi-elastic differential cross section and patterns of RPA modifications for them is seen. The transverse contribution dominates and the longitudinal one is negligible.

In Fig. 8 total cross sections for CC π production are shown. We compare two situations: with and without RPA correlations. Inclusion of RPA correction reduces the cross section significantly by about 25%.

In Fig. 9 CC π production differential cross section $\frac{d\sigma}{d\nu}$ is shown for neutrino energy $E = 1\text{GeV}$.

In Fig. 10 in the CC π production differential cross section longitudinal and transverse contributions (according to spin operators present in the transition amplitude) are distinguished and patterns of RPA modifications can be followed. There is a substantial reduction of the dominant transverse part at bigger values of energy transfer. The longitudinal part is in general reduced (order of 50 %) but some increase is also seen for smaller values of energy transfer.

To summarize: we have presented a scheme to calculate nuclear effects in neutrino-nucleus interactions. As a candidate to implement in MC codes the model presented in this paper has to be supplemented with a non-resonant contribution to π production, perhaps after [13]. It is

unclear how much of nuclear effects described by RPA and Δ width modification are covered in FSI (Final State Interactions) models of existing MC codes. This has to be understood in order to avoid double counting. Finally, as mentioned in the introduction, local density effects can be investigated by repeating present computations for several values of Fermi momentum and by taking an appropriate average.

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