Treatment of Small Mass Hadronic Systems in the Lund Model

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The Lund String Fragmentation Model has been very successful in describing experimental data in high-energy multi-particle processes. For small invariant masses of the string, the effects of diminishing phase space have to be considered more carefully than in the standard implementation of the model in the Monte Carlo Pythia .

1. Introduction

The Lund String Fragmentation Model [1,2] is a phenomenological model to describe the process of hadronisation in high energy particle collisions. It is based upon a few physically well motivated assumptions, and leads to intuitively appealing results. The well known Monte Carlo implementation of the model, Pythia [3], has been used in the analysis of multiparticle production events with great success.

However, at relatively low energies when only a few particles are produced, as is usual in nutrino nucleus interactions, the usefulness of the model in the description of hadronisation is relatively scarcely explored. In this case, the assumption of an asymptotically large available phase space, which is used in deriving the fragmentation function in the standard treatment of the Lund model, does not apply. It is therefore necessary to find approximations for the available phase space for few particle final states. An effort to provide a faithful implementation of the Lund model at low energies in a new Monte Carlo called Alfs [9] is currently underway.

1.1. Lund Model Basics

The string in the Lund model is the colour force field between a coloured and an anti-coloured particle. It is assumed that the QCD force field exists as a flux tube which can be approximated by a string stretched between the partons in a colour singlet system. For example, when a $q\overline{q}$ pair is produced in e^+e^- collision, a string is thought to be stretched between these particles. The q and $\overline{q}$ move away from each other and deposit their energy in the string between them. When sufficient amount of energy is stored in the string, new $q\overline{q}$ pairs can be formed along the string field, breaking the field between the original q and $\overline{q}$. After a series of such "breakups" of the string, the system consists of a set of small $q\overline{q}$ pairs connected by a little piece of string between them, c.f. Fig.1. These small $q\overline{q}$ systems (roughly) are used as models for mesons in the model.

Since the new $q\overline{q}$ pairs are produced from the string, the production points are well localised within the transverse size of the string. One would therefore expect, from the uncertainty principle, that the pair would be generated with some transverse momentum relative to the string. Pair production and the generation of transverse momentum have been modelled as a tunnelling process across a potential barrier in [6]. The result is that the probability for the production of a $q\overline{q}$ pair with quark mass μ is a Gaussian in the transverse mass $\mu_\perp$, $\mu_\perp^2 = \mu^2 + k_\perp^2$ of the quark, so that generation of large tranverse momenta and the creation of heavy quark flavours are supressed during hadronisation.

From Lorentz invariance, causality, and a symmetry between the coloured and anti-coloured ends of the string, it can be shown that in the Lund model, the probability for the production of a set of hadrons with energy–momenta $\{p_\alpha\}$ and masses $\{m_\alpha\}$, cf. Fig.1, is given by the following result, known as the area law:

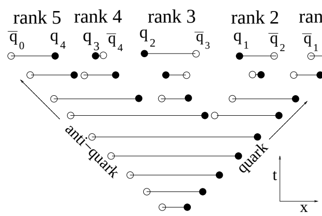

Figure 1. A sequence of "snapshots" of a high energy string stretched between a $q_0\overline{q}_0$ pair. The string breaks through the formation of $q\overline{q}$ pairs at many "vertices", and the resulting small mass systems which are projected on to hadron states.

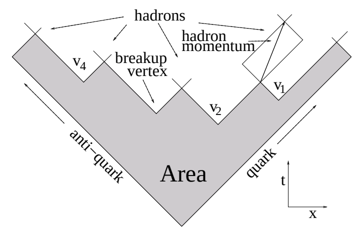

Figure 2. For the situation shown in Fig.1 this figure shows the area traced by the string before its decay, which is used in the Lund model probability for the process.

$$dP_n(\{p_\alpha\};P_{tot}) \;\propto\; d\overline{P}_n(\{p_\alpha\};P_{tot}) = \quad (1)$$
$$\left(\prod_{\alpha=0}^{n-1} N_\alpha d^4p_\alpha \delta(p_\alpha^2 - m_\alpha^2)\right) \delta(\sum_{\alpha'=0}^{n-1} p_{\alpha'} - P_{\rm tot})$$
$$\exp(-bA)\exp(-\frac{1}{2\sigma_q^2}\sum_{\beta=0}^{n} k_{\perp\beta}^2)$$

In this expression, A is the area swept by the string before it decays due to pair production as shown in Fig.2, and the parameter b has a phenomenological value of about 0.6 GeV^{-2}. The exponential suppression with respect to area favours production of vertices close to the light-cones, which leads to fewer particles and hence less available phase space. The competition between the exponential suppression with area and the size of the phase space leads to a hyperbolic average production region, and also to a rough correlation between the rapidity of a particle and the mean hyperbolic angles of its production vertices. A Gaussian suppression factor for the hadronisation transverse momenta produced at each of the vertices also appears in the expression.

In qualitative terms, Eqn.1 contains the following factors. It contains an energy momentum conserving δ-function. There is also one δ-function for each particle, which ensures that the particles are produced on mass shell. Apart from these terms which, along with the d^4p_α terms, constitute the final state phase space, there is a Gaussian "cost" factor for the transverse momenta produced at the vertices, and an exponential suppression factor for the total area traced by the string before it has decayed into hadrons.

2. Iterative Formulation

The area law is well suited for an iterative algorithm to simulate hadronisation. In such a procedure, the hadrons are generated in what is known as a "rank" order. From the properties of the massless relativistic string, it follows that the production vertices where new $q\overline{q}$ pairs are produced along the field, must be separated by space–like intervals. This means that time ordering of the production vertices, and hence the production of individual hadrons is Lorentz frame dependent. Rank ordering is a Lorentz invariant ordering, which may be identified as the ordering of the vertices along one light-cone direction.

By partitioning the area A into contributions associated with the production of individual particles, the area law could be formulated in terms of an iterative scheme. To do this, we define the phase space integrals as the integrals of Eqn.1 over all particle properties. Since all details about the particles are summed over, Lorentz invariance implies that the function depends only on the invariant mass of the system and the flavours of the endpoint $q\overline{q}$ pair:

$$g_n(s) = \int_{\text{all particle properties}} d\overline{P}_n \tag{2}$$

$g_n(s)$ represents the relative probability or density for the production of all possible n particle states from a system with invariant mass s. The (normalised) probability for the decay into exactly n particles is then:

$$\overline{g}_n(s) \equiv \frac{g_n(s)}{\sum_l g_l(s)} = \frac{g_n(s)}{g(s)} \tag{3}$$

For any finite s the $g_l(s)$ in the sum above is zero for l above a certain maximum value. The probability for one particular n particle state should then be evaluated as:

$$dP_n(s) = \frac{g_n(s)}{g(s)} \cdot \frac{d\overline{P}_n}{g_n(s)} \tag{4}$$

If a hadron takes a fraction z of the remaining light–cone momentum along (say) the positive light–cone, the invariant mass of the system before and after the production of the hadron can be related as $s' = (1-z)(s - \frac{m^2}{z})$. Eqn.1 can then be implemented by choosing the light–cone fractions z according to

$$\frac{dP}{dz}dz = \frac{dz}{z}\exp(-b\frac{m_{\perp\alpha}^2}{z}) \cdot \frac{g(s')}{g(s)} \tag{5}$$

At each step of the iteration, by choosing the negative light–cone fraction appropriately, it is ensured that the hadron is on mass shell.

One could rewrite Eqn.5 as an integral equation for the function $g(s)$:

$$g(s) = N\int_0^1 \frac{dz}{z}\exp(-b\frac{m_{\perp\alpha}^2}{z}) \cdot g((1-z)\cdot(s - \frac{m_{\perp\alpha}^2}{z})) \tag{6}$$

For large s, this equation has solutions of the form $g(s) \propto s^a$ which, when combined with Eqn.(5) gives

$$\frac{dP}{dz} = f(z) = N\frac{(1-z)^a}{z}\exp(-b\frac{m_{\perp\alpha}^2}{z}) \tag{7}$$

This is known as the Lund symmetric fragmentation function. To satisfy Eqn.(6), the value of the power a must depend on N and b in such a way that the distribution $\frac{dP}{dz}$ is properly normalised.

In arriving at the above result, effects of different possible quark flavours generated at the vertices, different hadron masses, transverse momentum etc have all been neglected. None of these factors seems to affect the resulting distributions if the invariant mass of the decaying string is large. In a generalisation to account for different quark flavours, the power a in the function $g(s)$ could in principle depend on the flavour of the produced quark. Phenomenologically this dependence is weak for large s, and it is possible to describe experimental data with a single value for a, with the exception of diquark production in connection with baryon production. Therefore, an implementation of the Lund model based on the generation of light cone momentum fraction z from the distribution in Eqn.(7), would be consistent with the area law and the available phase space of Eqn.(1), in all but the situations with very small invariant masses, of the order of a few GeV, for “flat” string systems[1].

The Monte Carlo Pythia uses this asymptotic form of the fragmentation function, so that the area law is implemented quite accurately for strings without gluons with invariant masses large compared to the hadronic mass scales. In every event however, when the invariant mass of the remaining system (after the production of a set of hadrons in the iterative procedure) becomes small, it is necessary to allow the remaining system to decay into one or two hadrons in order to conserve energy–momentum. In Pythia , it is arranged that this last step of the production process occurs at a random location on the string. If a large number of particles are produced, the

[1]In this article, only strings without gluonic excitations are considered

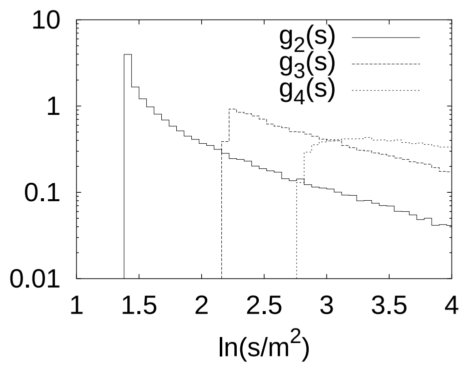

Figure 3. The phase space functions $g_i(s)$ for $i = 2, 3, 4$. $g_3(s)$ and $g_4(s)$ are obtained by approximate numerical integration. Transverse momentum is neglected in these plots. A single hadron mass $m = 0.7$ GeV was used.

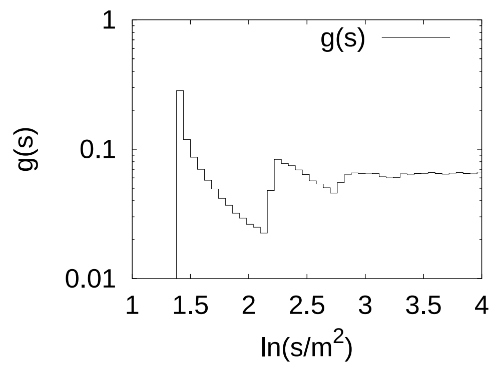

Figure 4. Approximate illustration of the function $g(s)$. Threshold behaviour is important when the available energy is of the order of a few hadron masses.

effects of this step are almost always negligible. This reasoning can not be applied if on total the string typically decays into three or four particles.

3. Systems with small invariant mass

The asymptotic solution of Eqn.(6) can not be expected to apply for small values of s. In fact, if s is of the order of a hadron mass, $g(s)$ defined in Eqn.(3) would have only one or two non-zero terms, as the states with more particles can not contribute because of the δ functions in Eqn.(1). The functions $g_l(s)$ can be related to each other using Eqn.(1) and Eqn.(2) with an equation very similar to Eqn.(6) :

$$g_n(s) = N \int_0^1 \frac{dz}{z} \exp(-b \frac{m_{\perp\alpha}^2}{z}) \cdot \qquad (8)$$
$$g_{n-1}((1-z) \cdot (s - \frac{m_{\perp\alpha}^2}{z}))$$

From Eqn.(1) and Eqn.(2) one can write, $g_1(s) = N \exp(-bm_{\perp 1}^2)\delta(s - m_1^2)$, for one hadron flavour. From this, using Eqn.(8), one can derive an equation for $g_2(s)$ (without including transverse momenta):

$$g_2(s) = N^2 \Theta(\sqrt{s} - m_1 - m_2) \quad (9)$$
$$\frac{2 \exp(-\frac{b(s+m_2^2-m_1^2)}{2}) \cosh(\frac{b}{2}\sqrt{\lambda(s, m_1^2, m_2^2)})}{\sqrt{\lambda(s, m_1^2, m_2^2)}}$$

where,

$$\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$$

The above expression should be integrated with respect to transverse momenta and summed over flavours to yield the full $g_2(s)$, as well as $g_3(s)$, $g_4(s)$... Unfortunately the integrals can not be expressed in closed form. Useful approximations could however be found. Fig.3 shows the first few $g_i(s)$ functions, while Fig.4 shows the function $g(s)$. These plots are obtained by approximate numerical integration. They neglect transverse momentum and use the same mass for the hadrons, and are therefore not representative of what is required in a realistic situation. But they illustrate the increasing importance of the threshold behaviour as the invariant mass of the system is reduced.

4. Concluding Remarks

A detailed calculation of the phase space functions is interesting for several reasons. It is well known that the string model gives a very good description of multi-particle production for large invariant masses. But it is not clear if the physical idea of a string, and the semi-classical picture of hadron formation in the Lund model could make sense in the few GeV region. It should be kept in mind that the phenomenological value for the string constant, or energy per unit length along the string is about 1 GeV/fm , while the transverse size of the string is estimated to be about 1 fm. Use of the asymptotic fragmentation function as done in PYTHIA and an energy momentum conserving step creating two particles is not equivalent to the result in Eqn.1 in this limit.

A framework for a careful treatment of the phase space at low energies in string fragmentation has been developed and implemented by this author, in a new Monte Carlo routine, ALFS [9]. It is designed to take into account the diminishing phase space for low energies, even though only very basic approximations have been derived so far, for the $g_i(s)$ functions. The original aim behind the development of ALFS was to implement the area law more accurately for multi-gluon strings [5], for which the procedure in PYTHIA makes small errors. ALFS has developed into a reformulation of the string fragmentation picture based on the idea that the string surface is a minimal area surface in space–time, and hence totally described by its boundary. ALFS implements string fragmentation as a process along the boundary curve. For a multi-gluon string, this provides a very tractable intuitive picture for fragmentation. It was understood that the procedure in ALFS allowed an extension of a model for Bose Einstein correlations in the Lund model to multi-gluon systems [8]. At the time of writing this, there are indications that the small errors made on the area law by the energy momentum conserving step in the iterative fragmentation process are quite important for the correlations obtained in our procedure. Since the phase space calculations also affect the last steps of the iterative fragmentation procedure, even for strings with large invariant masses, they are of interest from the point of view of correlation studies.

REFERENCES

1. B. Andersson, G. Gustafson and B. Söderberg, *Z. Phys.* **C20** (1983) 317.
2. B. Andersson, The Lund Model, (Cambridge University Press 1998).
3. T. Sjöstrand, P. Edén, C. Friberg, L. Lönnblad, G. Miu, S. Mrenna and E. Norrbin, *Computer Phys. Commun.* **135** (2001) 238.
4. B. Andersson, G. Gustafson, G. Ingelman, T. Sjostrand, *Phys. Rept.* **97** (1983) 31
5. B. Andersson, S. Mohanty, F. Söderberg, *Eur. Phys. J.* **C21** (2001) 631.
6. B. Andersson, G. Gustafson and T. Sjöstrand, *Z. Phys.* **C6** (1980) 235.
7. B. Andersson, G. Gustafson, T. Sjöstrand, *Phys. Scripta.* **32** (1985) 574.
8. S.Mohanty, F. Söderberg, LU TP 03-08
9. S. Mohanty LU TP 03-09