Different G_F and $\sin^2\theta_W$ for different processes

Ernest Ma

Physics Department, University of California, Riverside, California 92521, USA

A natural theoretical framework is presented which allows for small departures from quark-lepton universality such that different G_F and $\sin^2\theta_W$ values are applicable for different processes. In particular, the inequality $(G_F)^{NC}_{lq} < (G_F)^{CC}_{lq} < (G_F)^{CC}_{ll} < (G_F)^{NC}_{ll}$ holds. New physics is predicted at the TeV scale.

1. Introduction

In the Standard Model, the low-energy effective weak interactions are of the form

$$\frac{4G_F}{\sqrt{2}}\left[j^{(+)}j^{(-)} + \left(j^{(3)} - \sin^2\theta_W j^{(em)}\right)^2\right], \quad (1)$$

where

$$\frac{4G_F}{\sqrt{2}} = \frac{g^2}{2M_W^2} = \frac{g^2+g'^2}{2M_Z^2} = \frac{1}{v^2}. \quad (2)$$

Note that G_F is independent of g and g'.

As a result of Eq. (1), there are 3 predictions:

$$(A) \quad G_F^q = G_F^l, \quad \sin^2\theta_W^q = \sin^2\theta_W^l; \quad (3)$$

$$(B) \quad G_F^e = G_F^\mu = G_F^\tau; \quad (4)$$

$$(C) \quad G_F^{CC} = G_F^{NC}. \quad (5)$$

Possible experimental deviations of (A) and (C) have now been observed at the 3σ level. Whereas it is too early to tell for sure that these are real effects, it is clearly desirable to have a theoretical framework where departures from quark-lepton universality are naturally expected and which reduces to the Standard Model in the appropriate limit.

2. Three Experimental Discrepancies

(1) A recent measurement [1] of the neutron $\beta-$decay asymmetry has determined that

$$|V_{ud}| = 0.9713(13), \quad (6)$$

which, together with [2] $|V_{us}| = 0.2196(23)$ and $|V_{ub}| = 0.0036(9)$, implies the apparent nonunitarity of the quark mixing matrix, i.e.

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9917(28). \quad (7)$$

However, if $(G_F)^{CC}_{lq} < (G_F)^{CC}_{ll}$, as we will show, then the above is actually expected.

(2) The NuTeV experiment [3] which measures ν_μ and $\bar{\nu}_\mu$ scattering on nucleons reported a value of

$$\sin^2\theta_W = 0.2277 \pm 0.0013 \pm 0.0009, \quad (8)$$

as compared to the Standard-Model expectation of 0.2227 ± 0.00037, assuming that $(G_F)^{NC}_{lq}/(G_F)^{CC}_{lq} = 1$. In our model, this ratio will be smaller than one, which would explain the data if it is $0.9942 \pm 0.0013 \pm 0.0016$ and $\sin^2\theta_W$ does not change. However, we do expect the latter to change, but since its precise determination comes from Z decay, we need to consider also data at the Z resonance.

(3) In precision measurements of $e^-e^+ \to Z \to q\bar{q}$ and $l\bar{l}$, there seem to be two different values of $\sin^2\theta_{eff}$, i.e. [4]

$$(\sin^2\theta_{eff})_{hadrons} = 0.23217(29), \quad (9)$$

$$(\sin^2\theta_{eff})_{leptons} = 0.23113(21). \quad (10)$$

This may be an indication of a small deviation from quark-lepton universality.

In this talk I will show that (1) is naturally explained by a gauge model of quark-lepton nonuniversality [5], the prototype of which was proposed over 20 years ago [6] for generation nonuniversality. As a result, effects indicated by (2) and (3) are also expected, but the observed deviations are too large.

3. Gauge Model of Quark-Lepton Nonuniversality

Consider the gauge group $SU(3)_C \times SU(2)_q \times SU(2)_l \times U(1)_q \times U(1)_l$ with couplings g_s and

$g_{1,2,3,4}$ respectively. The quarks and leptons transform as

$$(u,d)_L \sim (3,2,1,1/6,0), \quad (11)$$
$$u_R \sim (3,1,1,2/3,0), \quad (12)$$
$$d_R \sim (3,1,1,-1/3,0), \quad (13)$$
$$(\nu,e)_L \sim (1,1,2,0,-1/2), \quad (14)$$
$$e_R \sim (1,1,1,0,-1). \quad (15)$$

The scalar sector consists of

$$(\phi_1^+,\phi_1^0) \sim (1,2,1,1/2,0), \quad (16)$$
$$(\phi_2^+,\phi_2^0) \sim (1,1,2,0,1/2), \quad (17)$$
$$\chi^0 \sim (1,1,1,1/2,-1/2), \quad (18)$$

and a bidoublet

$$\eta = \frac{1}{\sqrt{2}}\begin{pmatrix} \eta^0 & -\eta^+ \\ \eta^- & \bar{\eta}^0 \end{pmatrix} \sim (1,2,2,0,0), \quad (19)$$

which is assumed to be self-dual, i.e. $\eta = \tau_2\eta^*\tau_2$. Note that g_1 may be different from g_2, and g_3 may be different from g_4, so there is no quark-lepton symmetry at this level. The remarkable fact is that the effective low-energy weak interactions of the quarks and leptons will turn out to be independent of $g_{1,2,3,4}$ and become all equal in a certain limit, as shown below.

Consider

$$\langle\phi_{1,2}^0\rangle = v_{1,2}, \quad \langle\chi^0\rangle = w, \quad \langle\eta^0\rangle = u, \quad (20)$$

then the 2×2 charged-gauge-boson mass-squared matrix is given by

$$\mathcal{M}_W^2 = \frac{1}{2}\left[\begin{array}{cc} g_1^2(v_1^2+u^2) & -g_1g_2u^2 \\ -g_1g_2u^2 & g_2^2(v_2^2+u^2) \end{array}\right]. \quad (21)$$

Thus the effective lepton-lepton charged-current weak-interaction strength, i.e. that of μ decay, is

$$\frac{4(G_F)_{ll}^{CC}}{\sqrt{2}} = \frac{g_2^2}{2}\left(\mathcal{M}_W^{-2}\right)_{22} = \frac{u^2+v_1^2}{(v_1^2+v_2^2)u^2+v_1^2v_2^2}, \quad (22)$$

whereas the analogous expression for nuclear β decay is

$$\frac{4(G_F)_{lq}^{CC}}{\sqrt{2}} = \frac{g_1g_2}{2}\left(\mathcal{M}_W^{-2}\right)_{12} = \frac{u^2}{(v_1^2+v_2^2)u^2+v_1^2v_2^2}. \quad (23)$$

Note that both are independent of g_1 and g_2, and their ratio is not one, but rather

$$\frac{(G_F)_{lq}^{CC}}{(G_F)_{ll}^{CC}} = \frac{u^2}{u^2+v_1^2} \simeq 1 - \frac{v_1^2}{u^2}. \quad (24)$$

The apparent nonunitarity of the quark mixing matrix, i.e. Eq. (7), is then naturally explained with

$$\frac{v_1^2}{u^2} = 0.0042(14). \quad (25)$$

As for the effective neutral-current interactions, we have

$$\frac{4(G_F)_{lq}^{NC}}{\sqrt{2}} = \frac{u^2w^2}{(v_1^2+v_2^2)u^2w^2+v_1^2v_2^2(u^2+w^2)} \simeq \frac{4(G_F)_\mu}{\sqrt{2}}\left[1-\frac{v_1^2}{u^2}-\left(\frac{v_2^2}{v_1^2+v_2^2}\right)\frac{v_1^2}{w^2}\right], \quad (26)$$

$$\frac{4(G_F)_{ll}^{NC}}{\sqrt{2}} = \frac{u^2w^2+v_1^2(u^2+w^2)}{(v_1^2+v_2^2)u^2w^2+v_1^2v_2^2(u^2+w^2)} \simeq \frac{4(G_F)_\mu}{\sqrt{2}}\left[1+\left(\frac{v_1^2}{v_1^2+v_2^2}\right)\frac{v_1^2}{w^2}\right]. \quad (27)$$

This implies that the ratio

$$\frac{(G_F)_{lq}^{NC}}{(G_F)_{lq}^{CC}} \simeq 1 - \left(\frac{v_2^2}{v_1^2+v_2^2}\right)\frac{v_1^2}{w^2} \quad (28)$$

is what NuTeV actually measures [3]. The corresponding $\sin^2\theta_W$ expressions depend on the identification of the observed Z boson as a linear combination of the 3 massive neutral gauge bosons of this model, which will be discussed in the next section.

4. Observables at the Z Pole

There are 4 electroweak gauge couplings in this model. The electromagnetic coupling e is given by

$$\frac{1}{e^2} = \frac{1}{g_1^2}+\frac{1}{g_2^2}+\frac{1}{g_3^2}+\frac{1}{g_4^2}. \quad (29)$$

Defining $g_{ij}^{-2} \equiv g_i^{-2}+g_j^{-2}$, the photon A and 3 orthonormal Z bosons are given in the basis (W_q^0, W_l^0, B_q, B_l) by

$$A = e\left(\frac{1}{g_1},\frac{1}{g_2},\frac{1}{g_3},\frac{1}{g_4}\right), \quad (30)$$

$$Z_1 = e\left(\frac{g_{12}}{g_{34}g_1}, \frac{g_{12}}{g_{34}g_2}, \frac{-g_{34}}{g_{12}g_3}, \frac{-g_{34}}{g_{12}g_4}\right), \tag{31}$$

$$Z_2 = g_{12}\left(\frac{1}{g_2}, \frac{-1}{g_1}, 0, 0\right), \tag{32}$$

$$Z_3 = g_{34}\left(0, 0, \frac{1}{g_4}, \frac{-1}{g_3}\right). \tag{33}$$

The observed Z boson is approximately $Z_1 - \epsilon_2 Z_2 - \epsilon_3 Z_3$, where

$$\epsilon_2 \simeq \frac{g_{34}g_{12}^4}{eg_1^3 g_2^3}\left(\frac{g_1^2 v_1^2 - g_2^2 v_2^2}{u^2}\right), \tag{34}$$

$$\epsilon_3 \simeq \frac{g_{12}g_{34}^4}{eg_3^3 g_4^3}\left(\frac{-g_3^3 v_1^2 + g_4^2 v_2^2}{w^2}\right). \tag{35}$$

Deviations from the Standard Model must occur and quark-lepton universality in Z decay is violated if $\epsilon_2 \neq 0$ or $\epsilon_3 \neq 0$.

We have obtained [5] all the appropriate expressions for the expected deviations from the Standard Model in terms of 5 parameters:

$$\frac{v_1^2}{u^2},\ \frac{v_1^2}{w^2},\ r \equiv \frac{v_2^2}{v_1^2},\ y \equiv \frac{g_2^2}{g_1^2+g_2^2},\ x \equiv \frac{g_4^2}{g_3^2+g_4^2}, \tag{36}$$

and performed a global fit to 22 observables. The best-fit values are

$$\frac{v_1^2}{u^2} = 0.00489, \quad \frac{v_1^2}{w^2} = 0.00238, \tag{37}$$

$$r = 10.2, \quad y = 0.0955, \quad x = 0.135. \tag{38}$$

Our results are summarized in Table I.

We see that we are able to explain the apparent nonunitarity [1] of the quark mixing matrix and reduce the NuTeV discrepancy [3] while maintaining excellent agreement with precision data at the Z resonance, except for the $b\bar{b}$ forward-backward asymmetry measured at LEP, which is also not explained by the standard model. In fact, the shift of $A_{fb}^{0,b}$ is given in our model by

$$\Delta A_{fb}^{0,b} = \frac{3}{4}(A_e \Delta A_b + A_b \Delta A_e) = -0.07\Delta\sin^2\theta_q - 5.57\Delta\sin^2\theta_l. \tag{39}$$

Because of the dominant coefficient of the second term, it measures essentially the same quantity as A_l and there is no realistic means of reconciling the discrepancy of $\sin^2\theta_{eff}$ at the Z resonance using $b\bar{b}$ versus using leptons in the final state.

5. Other Effects

The new polarized $e^-e^- \to e^-e^-$ experiment (E158) at SLAC (Stanford Linear Accelerator Center) is designed to measure the left-right asymmetry which is proportional to $G_F(1-4\sin^2\theta_W)$ to an accuracy of about 10%. Using the standard-model prediction of $\sin^2\theta_W = 0.238$, our expectation is that the above measurement will shift by only −2.2% from its standard-model prediction. The new polarized ep elastic scattering experiment (Qweak) at TJNAF (Thomas Jefferson National Accelerator Facility) is designed to measure Q_W of the proton to an accuracy of about 4%. We expect a shift of only +3.0%. Using Eq. (37), we see also that the scale of new physics, i.e. u and w, is at the TeV scale. Specifically, using the best-fit values of r, y, and x, we find $M_{W_2} \simeq M_{Z_2} \simeq 1.2$ TeV, and $M_{Z_3} \simeq 0.8$ TeV.

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Table 1
Fit Values of 22 Observables

Observable	Measurement	Standard Model	Pull	This Model	Pull
Γ_l [MeV]	83.985 ± 0.086	84.015	-0.3	83.950	$+0.4$
Γ_{inv} [MeV]	499.0 ± 1.5	501.6	-1.7	501.2	-1.5
Γ_{had} [GeV]	1.7444 ± 0.0020	1.7425	$+1.0$	1.7444	-0.0
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01649	$+0.7$	0.01648	$+0.7$
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1483	-0.6	0.1482	-0.5
R_b	0.21644 ± 0.00065	0.21578	$+1.0$	0.21582	$+1.0$
R_c	0.1718 ± 0.0031	0.1723	-0.2	0.1722	-0.1
$A_{fb}^{0,b}$	0.0995 ± 0.0017	0.1040	-2.6	0.1039	-2.6
$A_{fb}^{0,c}$	0.0713 ± 0.0036	0.0743	-0.8	0.0740	-0.8
A_b	0.922 ± 0.020	0.935	-0.7	0.934	-0.6
A_c	0.670 ± 0.026	0.668	$+0.1$	0.665	$+0.2$
A_l(SLD)	0.1513 ± 0.0021	0.1483	$+1.4$	0.1482	$+1.5$
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314	$+0.8$	0.2322	$+0.2$
m_W [GeV]	80.449 ± 0.034	80.394	$+1.6$	80.390	$+1.7$
Γ_W [GeV]	2.139 ± 0.069	2.093	$+0.7$	2.093	$+0.7$
$g_V^{\nu e}$	-0.040 ± 0.015	-0.040	-0.0	-0.039	-0.1
$g_A^{\nu e}$	-0.507 ± 0.014	-0.507	-0.0	-0.507	-0.0
$(g_L^{eff})^2$	0.3001 ± 0.0014	0.3042	-2.9	0.3032	-2.2
$(g_R^{eff})^2$	0.0308 ± 0.0011	0.0301	$+0.6$	0.0299	$+0.8$
Q_W(Cs)	-72.18 ± 0.46	-72.88	$+1.5$	-72.26	$+0.2$
Q_W(Tl)	-114.8 ± 3.6	-116.7	$+0.5$	-115.7	$+0.3$
$\sum_{i=d,s,b} \|V_{ui}\|^2$	0.9917 ± 0.0028	1.0000	-3.0	0.9902	$+0.5$