Deep Inelastic Neutrino Interactions*

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Understanding neutrino interactions is an important task in searches for neutrino oscillations; e.g. the $\nu_\mu \to \nu_\tau$ oscillation hypothesis will be tested through ν_τ production of τ in long-baseline experiments as well as underground neutrino telescopes. An anomaly in the deep inelastic interaction of neutrinos has recently been observed by the NuTeV collaboration – resulting in a measured weak mixing angle $\sin^2\Theta_{\rm W}$ that differs by $\sim 3\sigma$ from the standard model expectation. In this contribution to the proceedings of NUINT02, we summarize results on the NLO neutrino structure functions and cross sections in which charm quark mass and target mass effects in the collinear approximation are included.

1. INTRODUCTION

The interpretation of the atmospheric muon neutrino deficit in the SuperKamiokande is that muon neutrinos oscillate to tau neutrinos with nearly maximal mixing [1]. Experiments such as ICANOE, OPERA and MONOLITH will be able to test the oscillation hypothesis by searching for tau neutrino conversion to taus [2] after the accelerator beam of muon neutrinos has passed over the long-baseline between CERN and the Gran Sasso Laboratory. The MINOS experiment run at lower energies [3] will look for the characteristic muon neutrino deficit. Ultimately, one would like to know the neutrino-nucleon charged current cross section to the level of a few percent for precision measurements of oscillation mixing angles. Mass corrections, from both the out-going charged lepton and from the target nucleon, are important at this level. Here, we report on lepton and target mass corrections in the collinear limit to neutrino-nucleon charged current interactions in deep inelastic scattering. Next-to-leading order QCD corrections and charm production are included. This is a portion of the total cross section which includes quasi-elastic scattering and few pion production at the energies of interest [4–6].

At the precision level, an anomaly in the interaction of neutrinos of higher $\mathcal{O}(100\text{ GeV})$ energies has recently been observed by the FERMILAB NuTeV experiment [7]. For this Paschos-Wolfenstein-like [8] measurement of the weak mixing angle $\sin^2\Theta_{\rm W}$, deep inelastic scattering is the dominant interaction channel. NLO QCD, charm mass (m_c) and target mass corrections are potentially important ingredients in the interpretation of the experimental result.

In the next section, we review the deep inelastic scattering (DIS) formalism, concentrating on charged current scattering (CC) where two additional structure functions F_4 and F_5 come into play for energies not too large compared to the outgoing lepton mass. We show our results for the structure functions with NLO QCD corrections, and m_c and nucleon mass M_N corrections in the collinear limit, in Section 3. Our cross section results for ν_τ CC scattering with isoscalar nucleons

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N and $\nu_\mu N$ CC scattering are shown in Section 4. Conclusions appear in Section 5. More complete references and further details of this work appear in Ref. [9], and extensions to include neutral current interactions as well as target mass corrections through the operator product expansion approach of Georgi and Politzer [10], through NLO QCD, will be presented in Ref. [11].

2. DIS FORMALISM

The QCD dynamical part of the evaluation of neutrino interactions is given through the hadronic tensor

$$\begin{aligned} W_{\mu\nu} &\equiv \frac{1}{2\pi}\int e^{iq\cdot z} d^4z \langle N|J_\mu(z)J_\nu(0)|N\rangle \qquad (1) \\ &= \frac{1}{\pi}\,\mathrm{Disc}\int e^{iq\cdot z} d^4z \langle N|iT\left(J_\mu(z)J_\nu(0)\right)|N\rangle \\ &= -g_{\mu\nu}W_1 + \frac{p_\mu p_\nu}{M^2}W_2 - i\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho q^\sigma}{M^2}W_3 \\ &+ \frac{q_\mu q_\nu}{M^2}W_4 + \frac{p_\mu q_\nu + p_\nu q_\mu}{M^2}W_5 \ . \end{aligned}$$

Sandwiched between the current operators in the first line of Eq. 1, an inclusive sum over a complete set of states has been performed:

$$\mathbf{1} = \sum_X |\, X\rangle\langle X\,| \qquad (2)$$

in which

$$\begin{aligned} |\,X\rangle &= |\,N\rangle \quad \text{for quasielastic} \\ |\,X\rangle &= |\,N\,n\pi\rangle \quad \text{resonances/few particles} \\ |\,X\rangle &= |\,\text{multiparticle}\rangle \quad \text{DIS continuum} \ . \end{aligned}$$

Our focus here is on the DIS contribution, however, there is an issue of double counting if one does the inclusive sum and adds to separate exclusive processes important in the few GeV energy region. One way to approximately avoid double counting [12] is to impose a cut on the hadronic energy

$$W^2 = (p_N + q)^2 > (1.4\ \mathrm{GeV})^2 \qquad (3)$$

for W-boson momentum q. In addition, in Ref. [9] we evaluate the contributions from a range of Q^2_{min} to assess the importance of non-perturbative or higher-twist contributions to the cross section.

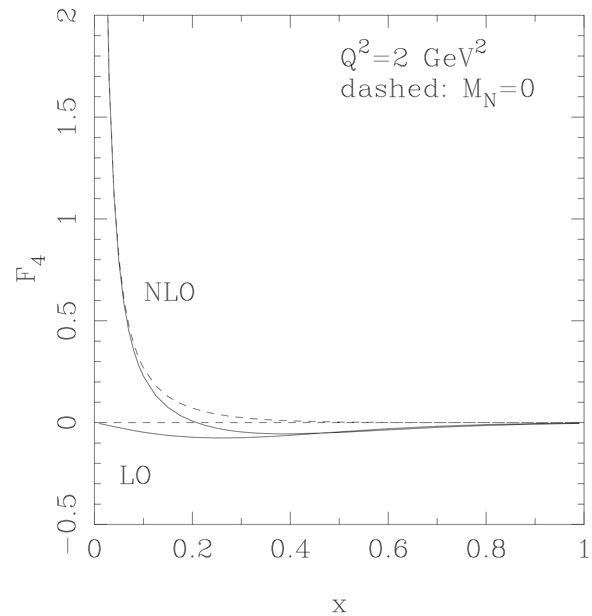

Figure 1. The LO and NLO structure function F_4 as a function of x for $Q^2 = 2$ GeV^2. The CTEQ6 parton distribution functions are used. Dashed lines show the case with target mass $M_N = 0$.

A general hadronic tensor has six structure functions W_i which can be rescaled into the more conventional F_i. The structure function W_6 which multiplies the antisymmetric tensor with p_N and q never appears because the tensor combination contracts to zero with the leptonic equivalent of $W_{\mu\nu}$ in the cross section. The full differential cross section for neutrino CC scattering, including the lepton mass (for definiteness set to m_τ) is

$$\begin{aligned} \frac{d^2\sigma^{\nu(\bar\nu)}}{dx\ dy} &= \frac{G_F^2 M_N E_\nu}{\pi(1+Q^2/M_W^2)^2} \\ &\times \left\{ (y^2x + \frac{m_\tau^2 y}{2E_\nu M_N})F_1^{W^\pm} \right. \\ &+ \left[(1 - \frac{m_\tau^2}{4E_\nu^2}) - (1 + \frac{M_N x}{2E_\nu})y \right] F_2^{W^\pm} \\ &\pm \left[xy(1-\frac{y}{2}) - \frac{m_\tau^2 y}{4E_\nu M_N} \right] F_3^{W^\pm} \end{aligned}$$

$$+ \quad \frac{m_\tau^2(m_\tau^2+Q^2)}{4E_\nu^2 M_N^2 x} F_4^{W^\pm} \qquad (4)$$
$$- \quad \frac{m_\tau^2}{E_\nu M_N} F_5^{W^\pm} \Bigg\} \ .$$

where $\{x, y, Q^2\}$ are the standard DIS kinematic variables related through $Q^2 = 2M_N E_\nu xy$ and where we have neglected factors of $m_\tau^2/2M_N E_\nu \cdot Q^2/M_W^2$ which come from the $q^\mu q^\nu/M_W^2$ part of the massive boson propagator. They are negligible both at low and at high neutrino energies so do not enter our numerics. They can be included by replacing:

$$F_i^{W^\pm} \to F_i^{W^\pm} \times (1+\epsilon_i) \qquad (5)$$

with

$$\begin{aligned}
\epsilon_1 &= \frac{m_\tau^2\ (Q^2+2M_W^2)}{2M_W^4} \\
\epsilon_2 &= -\frac{E_\nu^2\ m_\tau^2\ y\ [4M_W^2+y(Q^2+m_\tau^2)]}{M_W^4\ [4(y-1)E_\nu^2+m_\tau^2+Q^2]} \\
\epsilon_3 &= 0 \qquad (6) \\
\epsilon_4 &= \frac{Q^2\ (Q^2+2M_W^2)}{M_W^4} \\
\epsilon_5 &= \frac{Q^2}{M_W^2} + \frac{(M_W^2+Q^2)\ (m_\tau^2+Q^2)\ y}{2M_W^4}
\end{aligned}$$

In Eq. 4, the extra structure functions F_4 and F_5 appear with powers of the charged lepton mass, and so are usually neglected. At leading order, they are related to the usual structure functions by the Albright-Jarlskog relations [13], generalizations of the Callan-Gross relations:

$$F_4 = 0 \qquad (7)$$
$$2xF_5 = F_2 \ , \qquad (8)$$

These relations are violated by kinematic target mass corrections and at NLO in QCD when quark masses are retained. More specifically, Eq. 7 is violated by $M_N \neq 0$ and $\mathcal{O}(\alpha_s)$ corrections, but not by the charm mass m_c when the strange quark mass vanishes. Eq. 8 is violated by non-zero M_N and m_c, but for the massless case, this Albright-Jarlskog relation holds to any order in α_s.

The complete lepton mass correction comes from the above equation together with the limits of integration to get the full cross section. Details of the lepton mass dependence of the integration limits appear in Ref. [9].

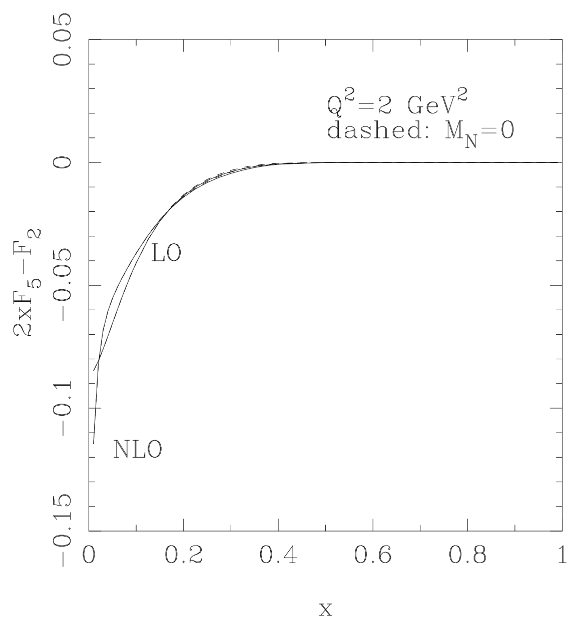

Figure 2. The LO and NLO structure function difference $2xF_5 - F_2$ as a function of x for $Q^2 = 2$ GeV^2. The CTEQ6 parton distribution functions are used. Dashed lines show the case with target mass $M_N = 0$.

Target mass corrections appear in Eq. 4, and they also appear implicitly in the F_i's. For neutrino scattering, it is conventional to rewrite F_i in terms of $\mathcal{F}_i$ which are written at leading order, neglecting the target and charm quark masses, as

$$\mathcal{F}_i = (1-\delta_{i4})q(x,Q^2) \qquad (9)$$

in terms of quark distribution functions $q(x, Q^2)$.

Target mass corrections appear in the collinear ($p_q^\perp = 0$) limit due to the fact that the *small* nucleon light cone momentum p_N^- is not a simple rescaling of the massless parton momentum p_q^- when we have $p_N^+ = \xi p_q^+$ for the *large* components. When one includes M_N in the collinear limit and m_c, where m_c is replaced by the parameter $\lambda \equiv Q^2/(Q^2+m_c^2)$, one gets for the case of CC charm production:

$$
\begin{aligned}
F_1^c &= \mathcal{F}_1^c && (10)\\
F_2^c &= 2\,\frac{x}{\lambda}\,\frac{\mathcal{F}_2^c}{\rho^2} && (11)\\
F_3^c &= 2\,\frac{\mathcal{F}_3^c}{\rho} && (12)\\
F_4^c &= \frac{1}{\lambda}\,\frac{(1-\rho)^2}{2\rho^2}\,\mathcal{F}_2^c + \mathcal{F}_4^c + \frac{1-\rho}{\rho}\,\mathcal{F}_5^c && (13)\\
F_5^c &= \frac{\mathcal{F}_5^c}{\rho} - \frac{(\rho-1)}{\lambda\rho^2}\,\mathcal{F}_2^c\ . && (14)
\end{aligned}
$$

Here,

$$\rho^2 \equiv 1 + \left(\frac{2M_N x}{Q}\right)^2\ . \qquad (15)$$

A further target mass correction come in through the Nachtmann variable [14] $\eta = 2x/(1+\rho)$ for massless quark production and $\tilde{\eta} = \eta/\lambda$ for charm production. For the result shown below

$$\mathcal{F}_i^c = (1-\delta_{i4})q(\tilde{\eta}, Q^2) + \mathcal{O}(\alpha_s)\ . \qquad (16)$$

Expressions for the $\mathcal{O}(\alpha_s)$ corrections appear in detail in Ref. [9]. The light quark contributions are obviously understood to be included in the $\lambda \to 1$ limit.

3. STRUCTURE FUNCTION RESULTS

In Figs. 1, 2 we quantify the violations of the Albright-Jarlskog relations discussed above. The violations are concentrated at low-x which is not the most relevant kinematic region for present neutrino energies. The naive Albright-Jarlskog relations therefore provide reasonable approximations in the evaluation of integrated cross sections.

The target mass corrections shown in Figs. 1,2 are in the collinear limit. These corrections differ by at most 10% from the target mass corrected structure functions evaluated in the operator product expansion approach [10] in the range of $Q^2 = 1 - 4$ GeV^2 [11]. The differences are smaller than 10% where the structure functions are largest.

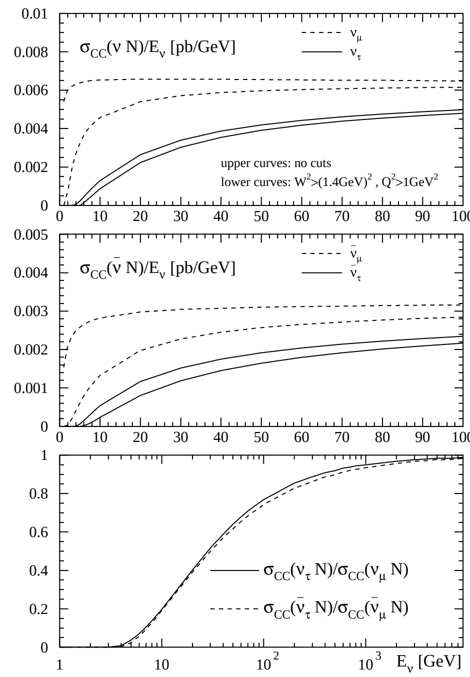

Figure 3. Comparative results for tau and muon neutrino cross section versus energy with and without various cuts applied.

4. CROSS SECTION RESULTS

The integrated cross sections for tau and muon (anti-)neutrino DIS are plotted in Fig. 3. The uncertainty in the cross sections from varying the parton distribution functions within their bounds is typically about a few percent: The results in the figures are based on CTEQ6 [15]. We have also compared to GRV98 [16].

4.1. $\sigma(\nu_\tau N \to \tau X)$

In terms of a purely DIS evaluation of the neutrino cross-section, τ production is a natural observable because the tau mass cuts off non-DIS interaction to a good extent. The general NLO

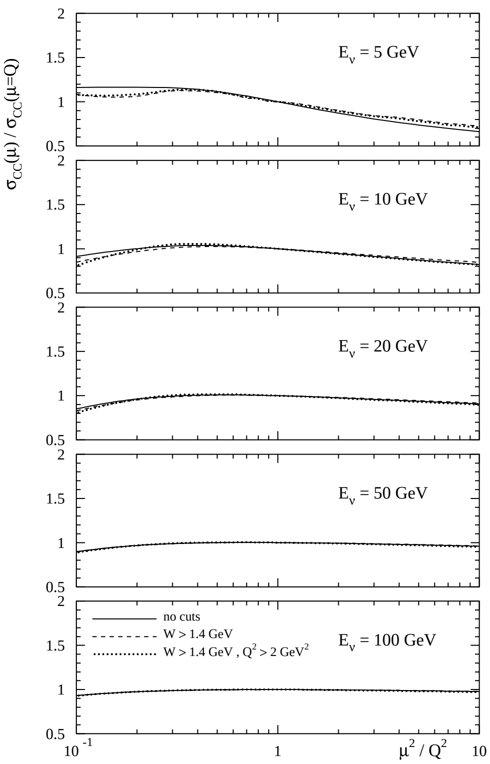

Figure 4. The ratio of $\sigma_{\rm CC}(\mu)/\sigma_{\rm CC}(\mu = Q)$ in NLO for ν_τN interactions, as a function of μ^2/Q^2 for several values of E_ν.

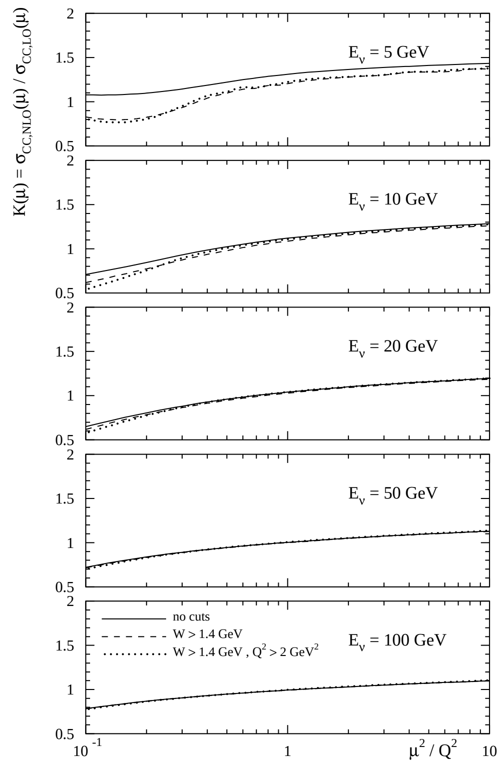

Figure 5. The K-factor K =NLO/LO versus factorization scale μ for tau neutrinos with no cuts (solid), $W_{\rm min}$ = 1.4 GeV (long dash) and $W_{\rm min} = 1.4$ GeV with $Q^2 > 2$ GeV^2 (short dash).

features of our results for ν_τ DIS are, therefore, equally valid for ν_μ DIS with kinematic DIS cuts. In Figs. 4 and 5 we show two typical features of the inclusion of NLO effects: Scale dependence and the size of the K-factor. The scale dependence is substantially reduced compared to a LO calculation and the K factor hints at the importance of NLO corrections towards low neutrino energies.

4.2. $\sigma(\nu_\mu N \to \mu X)$

Different from τ production, the deep inelastic component becomes small in ν_μ scattering at lower energies as shown in Fig. 6. Since the tau threshold in ν_τ DIS extends to large energies one may wonder about the impact of muon mass terms in low energy ν_μ scattering. Fig. 7 looks at such corrections to the DIS component. The results strongly depend on the kinematic cuts imposed. This observation, taken together with the smallness of the *bona fide* deep inelastic component at such energies do not allow to draw a conclusion from the perturbative framework which we consider here. Rather, one will have to look into the equivalent corrections to elastic and low-multiplicity form factors.

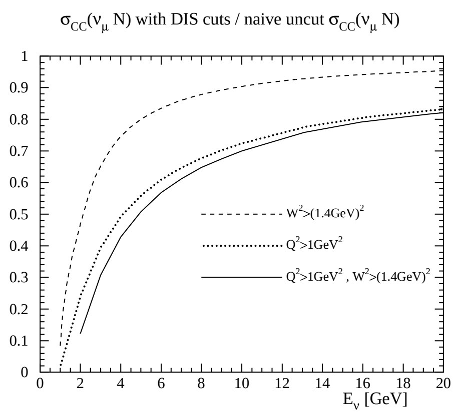

Figure 6. The muon neutrino cross section versus energy with DIS cuts.

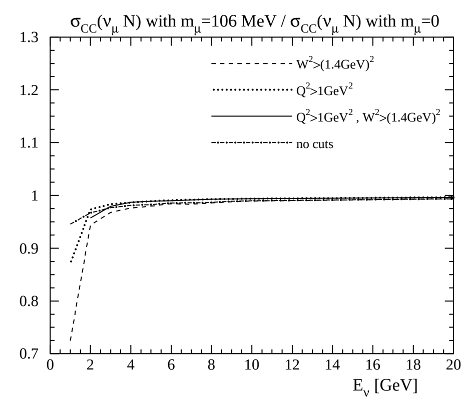

Figure 7. The muon mass effect on the DIS neutrino cross section with and without kinematic cuts (applied to both numerator and denominator).

5. CONCLUSIONS

The νN cross section is an important ingredient in current and future atmospheric and neutrino factory experiments. Our evaluation [9] of the NLO corrections for ν_τN CC interactions including charm mass corrections and an estimate of target mass effects is part of a larger theoretical program to understand the inelastic νN cross section over the full energy range relevant to experiments. Apart from oscillation searches, the recent observation of an anomaly in the ratios of neutral to charged current neutrino ineractions also necessitate a solid theory for neutrino interactions with nuclei. Research along these lines is progressing and further results are under completion [11].

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