Modeling Quark-Hadron Duality

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This talk gives a brief review of duality in electron scattering. Experimental data and practical applications of duality are discussed, as well as theoretical efforts to model duality.

1. What is Quark-Hadron Duality?

Quark hadron duality was first observed in inclusive, inelatic electron scattering at SLAC by Bloom and Gilman [1], but it appears in many reactions [3]. Recently, new high precision data from Jefferson Lab have confirmed duality and extended its range of validity down to low Q^2, and to several observables besides F_2 [2]. Much of the excitement about and interest in quark-hadron duality stems from the fact that it provides a connection between the resonance region and the deep inelastic region, and, if understood well enough, may allow us to extract information on one region by taking data in the other region. Thus, duality may be relevant for neutrino experiments in the resonance region.

In general, duality implies a situation in which two different pictures give an accurate description of Nature. While one may be more convenient than the other in certain situations, both are correct. If we are interested in hadronic reactions, the two relevant descriptions are the quark-gluon picture and the hadronic picture. In principle, we can describe any hadronic reaction in terms of quarks and gluons, by solving Quantum Chromodynamics (QCD). While this is correct, it is impractical, since in most cases we can neither perform nor interpret a full QCD calculation. In general, we also cannot perform a complete hadronic calculation. We will refer to the statement that, if one could perform and interpret the calculations, it would not matter at all which set of states - hadronic states or quark-gluon states - was used, as "degrees of freedom" duality.

However, there are cases where another, more practical form of duality applies: for some reactions, in a certain kinematic regime, properly averaged hadronic observables can be described by perturbative QCD (pQCD). This statement is much more practical than the "degrees of freedom" duality introduced above. In contrast to full QCD, pQCD calculations can be performed, and in this way, duality can be exploited and applied to many different reactions.

In the next part of this talk, I will illustrate the concept of duality in electron scattering with some recent experimental data and discuss possible applications. In the last part, I will discuss theoretical efforts to model duality. A pedagogic introduction to duality can be found in [4].

2. Applications - Why should we think about Duality?

The cross section for inclusive electron scattering is given by

$$\frac{d\sigma}{d\Omega dE_f} = \sigma_{Mott}(W_2 + 2W_1 \tan^2 \frac{\vartheta_e}{2}) \,, \qquad (1)$$

where $\sigma_{Mott} \propto Q^{-4}$. Therefore, the cross section for high Q^2 is dropping off rapidly. Traditionally, the region where W, the invariant mass of the final state, is smaller than 2 GeV, is called the resonance region, and $W > 2$ GeV is referred to as the deep inelastic region. This distinction is rather

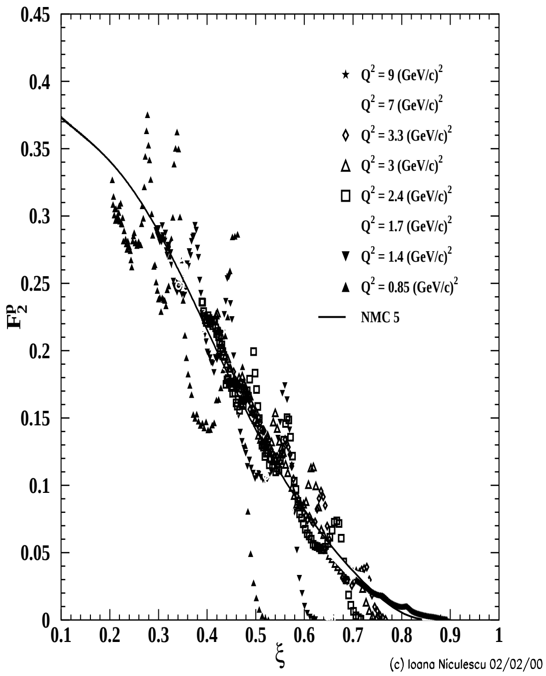

Figure 1. Experimental data for $F_2(\xi, Q^2) = \nu W_2(\xi, Q^2)$ from Jefferson Lab [2]. The data is plotted versus Nachtmann's variable ξ.

artificial, and one key point of quark-hadron duality is that these two regions are actually connected. It is clear that duality in inclusive electron scattering must hold in the scaling region, for $Q^2 \to \infty$, as perturbative QCD is valid there, and therefore will describe the hadronic reaction. In deep inelastic scattering, the kinematics are such that the struck quark receives so much energy over such a small space-time region that it behaves like a free particle during the essential part of its interaction. This leads to the compellingly simple picture that the electromagnetic cross section in this kinematic region is determined by free electron-quark scattering, i.e. duality is exact for this process in the scaling region. The really interesting question is if duality will be valid approximately at lower Q^2, in a region where the cross section is dominated by resonances, which are strongly interacting hadrons, after all. The experimental data, see Fig. 1, show that duality holds even at very low $Q^2 \approx 0.5$ GeV^2 [2].

One can see clearly that the resonance data follows the scaling curve, as given by the NMC parameterization evolved to $Q^2 = 5$ GeV ^{2}. In principle, one should compare the resonance results to the pQCD results evolved to the same Q^2 at which the resonance data were taken. As the resonance Q^2 values are too low for this, choosing 5 GeV^2 is a very reasonable approach. Finite energy sum rules formed for the scaling (pQCD) curve and the resonance regime further quantify the validity of duality, for details see [2]. Also, moments of the data have been considered [5]. The most striking feature of the moments is that they flatten out at rather low $Q^2 \approx 2$ GeV ^{2}.

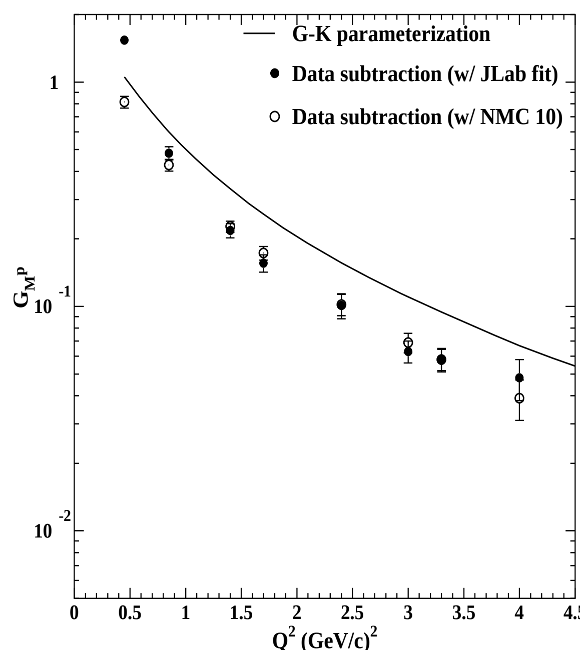

Figure 2. Extracted values for the magnetic form factor of the proton, from [7].

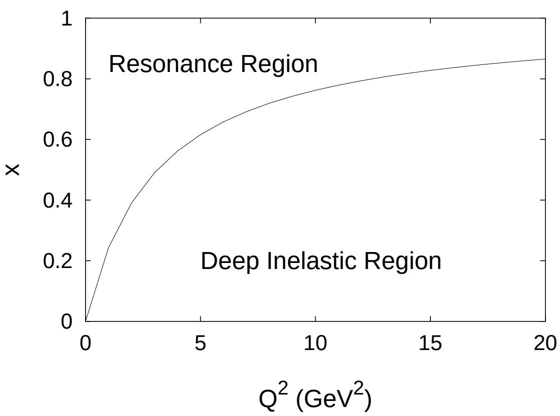

Figure 3. The kinematic plane. The line indicates that W = 2 GeV. The region below it corresponds to W > 2 GeV (deep inelastic region), the region above it corresponds to W < 2 GeV (resonance region).

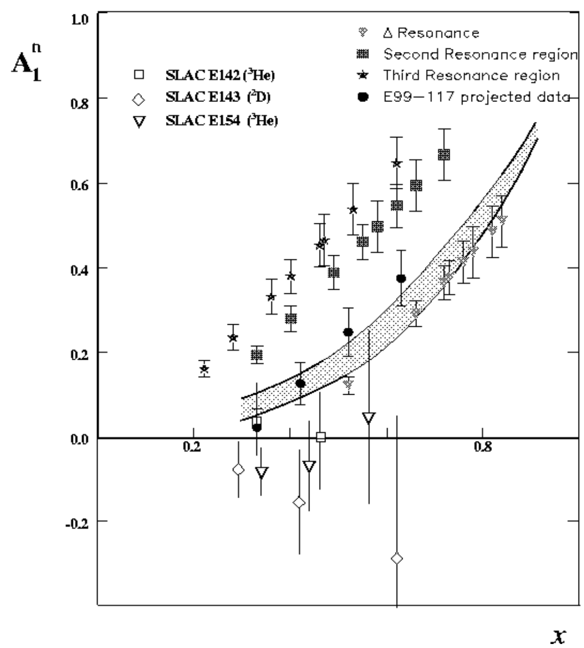

Figure 4. Data from SLAC for the polarization asymmetry of the neutron, A_1^n, and projected data for the Jefferson Lab experiment E01-012 [8].

If duality holds very locally, i.e. for just one resonance, instead of the whole resonance region, then one may use it to extract information on that one resonance from the deep inelastic region, and vice versa. A benchmark for applying duality in this, very local, way is the extraction of the magnetic form factor of the proton from the scaling curve [6,7]. The result is shown in Fig. 2. The qualitative agreement is very good, and quantitatively, one sees that the duality extraction undershoots the form factor parameterization somewhat. This result gives us a good idea where we are in our understanding of duality, and in our ability to extract information from the data.

While the extraction of G_M provides an interesting benchmark, it does not provide us with new information, as the nucleon form factor can be measured directly. For practical purposes, applying duality in order to extract information on the deep inelastic region from measurements in the resonance region is relevant. The physical quantities of interest here are observables at high x_{Bj}, i.e. in the valence quark region. Data in the deep inelastic region for x_{Bj} close to 1 are scarce, because high x_{Bj} values in this region can be attained only for high Q^2, see Fig. 3, and this, in turn, means very low count rates, as $\sigma_{Mott} \propto Q^{-4}$. However, if one wishes to measure high x_{Bj} values in the resonance region, this can be done at low Q^2 and correspondingly high count rates.

As an application for duality, I will discuss the experimental situation for the measurement of the polarization asymmetry of the neutron, A_1^n, in the valence region. This is a prime example for the potential usefulness of duality. The physics information we can glean from this observable is related to the valence quark spin distribution functions. Several, widely varying theoretical predictions exist for A_1^n in the limit of $x_{Bj} \to 1$ (for a review, see [9]). In Fig. 4, the old data from SLAC (open symbols) and projected data that will be taken at Jefferson Lab, both in the deep inelastic region (filled circles), and using duality (other filled symbols). In the latter case, data will be taken in the resonance region, e.g. the Δ region (filled triangles), and then averaged over, to yield the scaling region result. As can be seen, the current data set suffers from large error bars due to the low count rates. It also does not extend very far into the valence region. Measurement exploiting duality would be able to push the data up to x_{Bj}- values very close to 1, with very reasonable error. However, before we can go ahead and extract this information confidently, we need to better understand duality, its range of validity and the precision with which duality holds, and appropriate averaging procedures. Currently, we do not fully understand duality, and the main thrust towards understanding duality is in the direction of modeling the phenomenon [10–13]. While any solvable model will have to be much simpler than reality, one still may gain valuable insights into the workings of duality following this approach.

3. Modeling - What we do about duality

The obvious questions asked by theorists right now are: Why do we observe duality - how can there be precocious scaling in a region where the interactions between the quarks are strong? For which observables in which kinematic regimes can we apply quark-hadron duality, and how precise are our results going to be?

When modelling duality, the first goal is to gain a qualitative understanding of the phenomenon. Obviously, the situation as observed in nature is very complicated, necessitating various simplifications. Nevertheless, the goal is to incorporate the essential physical features into a model. The general approach is to choose a solvable model for hadrons, calculate the relevant observables, and compare these results to the - hypothetical - free quark results. At this point, all models assume that after the excitation from the ground state to an excited level N, the quark will remain in its excited state,i.e. the produced resonance will not decay. The results obtained for the transition of the quarks to a bound, excited state are summed over and compared to the case where in the final state, the binding potential is switched off, and the quark is "free". The latter case corresponds

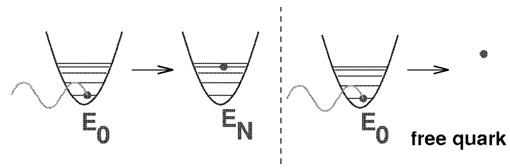

Figure 5. Schematic view of the model calculations. The left panel shows the bound-bound transition, the right panel shows the bound-free transition.

to the pQCD situation. A schematic view of the modelling is given in Fig. 5.

All models for duality must fulfill the following criteria:

1. The model must reproduce scaling. In addition, the scaling curve for the transition from the quark's ground state to the excited state must lead to the same scaling curve as the transition from the ground state to a free quark state.

2. The calculated moments must flatten out for large Q^2, as observed in the data.

3. The resonance region results should oscillate around the scaling curve.

Let us now turn to one particular model, which was introduced in [10,11]. The approach in this work was to construct a model with just a few underlying basic assumptions, which could be extended to the more realistic case. In [10,11], we assumed that it is sufficient to incorporate relativity and confinement in a valence quark model. We also treated the quarks as scalars - while spin is crucial in nature, we assumed that for duality to be observed, it would not be necessary. In principle, we are interested in nucleon targets, i.e. in a three-body system. As this poses some technical difficulties, we assumed that only one quark would carry charge and therefore interact with the photon, the other two quarks form a spectator system. One may think of them either as an anti-quark or as a diquark. In order to further

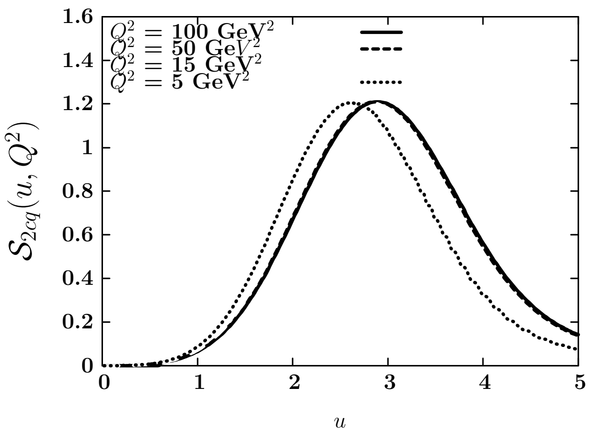

Figure 6. Scaling of the bound-bound transition for $Q^2 \to \infty$.

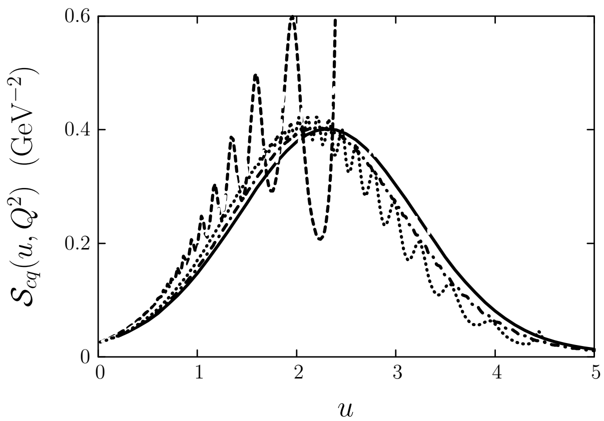

Figure 7. Duality at low Q^2 for the all scalar case. The solid lines show the result for large Q^2, the short dashed lines show $Q^2 = 0.5$ GeV^2, the long-dashed lines show $Q^2 = 1$ GeV^2, the dotted lines show $Q^2 = 2$ GeV^2, and the dash-dotted lines show $Q^2 = 5$ GeV^2.

simplify the task, we also assumed that the spectator system has infinite mass. This means that instead of solving the Bethe-Salpeter equation for the two-body problem, we have to solve only a one-body equation. As we are dealing with scalar quarks, the Klein-Gordon equation needs to be solved. We model the confinement using a scalar, linear potential, $V \propto r$. As the potential enters the Klein-Gordon equation as V^2, the resulting equation resembles the Schrödinger equation for the non-relativistic harmonic oscillator. This has the advantage that the wave functions obtained in the solution are exactly the wave functions obtained for the non-relativistic harmonic oscillator, whereas the energy spectrum is given by $E_N \propto \sqrt{N}$, which leads to a much higher density of excited states than in the non-relativistic case, where $E_N^{non-rel} \propto N$. A comparison between the relativistic and the non-relativistic solutions is easily feasible in this case. A nice feature of this model is that the solutions can be obtained analytically. The two parameters needed for this model are the constituent quark mass, $m = 0.33$ GeV and the string tension, which takes a value of 0.16 GeV^2. None of the results depend crucially on these precise values, and we have checked that variation of these values gives reasonable results, e.g. we obtain the free case in the limit of the sting tension going to zero. While all particles, including beam and exchange particles, were treated as scalars in [10], only the quarks were treated as scalars in [11]. In the latter case, with spin 1/2 electrons and spin 1 photons, one deals with a conserved current.

The first condition for duality that must be fulfilled by any model is scaling for large Q^2, both for the bound-bound transition, i.e. the calculation with final state interaction (FSI), and for the bound-free transition, i.e. the plane wave impulse approximation (PWIA). However, as duality is precocious scaling, we first have to answer the question which quantity should scale, and which scaling variable we must use. Bjorken's variable was derived explicitly assuming that all masses are small compared to Q^2, but this is not true in the low Q^2 region ($Q^2 \approx 0.5 GeV^2$) where duality was observed at Jefferson Lab. At low Q^2, the relevant degree of freedom is not the current quark mass, but the constituent quark mass which the quarks gain by spontaneous chiral symmetry breaking.

This situation demands a different scaling variable and scaling function. Bloom and Gilman used the ad hoc variable $x' = \frac{Q^2}{W^2+Q^2}$, and later on [14], a variable that treats target mass and constituent quark mass on the same footing was derived. It reads

$$x_{cq} = \frac{1}{2M}(\sqrt{\nu^2+Q^2}-\nu)(1+\sqrt{1+\frac{4m^2}{Q^2}}), \quad (2)$$

and was derived for the case of free quarks with a momentum distribution. When deriving a scaling variable, it turns out that it is intimately connected to a scaling function, which for our case (scalar quarks), reads $\mathcal{S}_{2,cq} = |\vec{q}|W_2$. Note that all scaling variables and scaling functions must reduce to Bjorken's variable x_{Bj} and F_2 in the limit of high Q^2. The y-scaling behavior is discussed in [15].

The results for the scaling in the bound-bound case are shown in Fig. 6. It is clear from the figure that scaling is present: once Q^2 is high enough, the curves for different Q^2 practically coincide. Analytically, it was shown [11] that

$$\mathcal{S}_{2,cq} = \frac{m^2 u_{Bj}^2}{\pi^{\frac{1}{2}}\beta E_0}\exp\left(-\frac{(E_0-mu_{Bj})^2}{\beta^2}\right) \quad (3)$$

, and that this is the same result which one obtains for the bound-free transition. It is interesting to note that the scaling function obtained in the all scalar case - where again, the bound-bound and bound-free transitions lead to the same scaling function - has a slightly different analytic form:

$$\mathcal{S}_{cq} = \frac{1}{4\pi^{\frac{1}{2}}\beta E_0}\exp\left(-\frac{(E_0-mu_{Bj})^2}{\beta^2}\right). \quad (4)$$

The approach to scaling at low Q^2 is shown for the all scalar case in Fig. 7, and for the electromagnetic current in Fig. 8. It is interesting to observe the changes caused by the introduction of a more realistic current operator.

4. Summary and Outlook

In this talk, we have shown that duality is well established experimentally, and that it has interesting and useful applications. Duality is not yet fully understood theoretically, but significant progress has been made in the past few years. With a few, very basic assumptions, one can reproduce all the qualitative features of duality as seen in the data. This may hint that duality is a rather fundamental property of hadronic reactions. Experimentally, duality is investigated further at Jefferson Lab and DESY, in various reactions, including meson production and reactions involving polarization. Theory will progress to more realistic models, including the spin of the quarks.

The use of electron or neutrino beams should not make a difference for the validity of duality - duality seems to be a very basic property, and in our model calculations, different beam and exchange particles (all scalars in one case, electrons and photons in the other case) reproduced the features of duality.

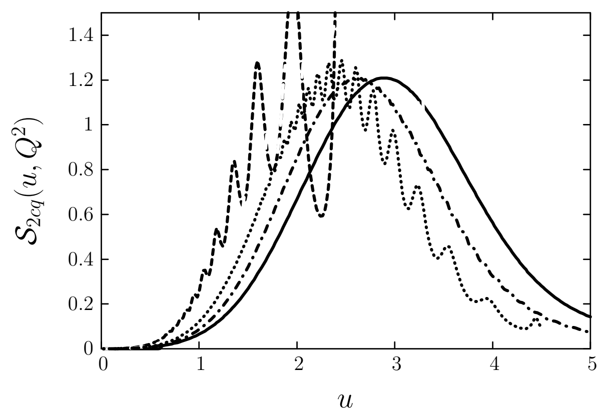

Figure 8. Duality at low Q^2 for the electromagnetic current. The solid lines show the result for large Q^2, the short dashed lines show $Q^2 = 0.5$ GeV^2, the long-dashed lines show $Q^2 = 1$ GeV^2, the dotted lines show $Q^2 = 2$ GeV^2, and the dash-dotted lines show $Q^2 = 5$ GeV^2.

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REFERENCES

1. E. D. Bloom and F. J. Gilman, *Phys. Rev. Lett.* **25**, 1140 (1970); E. D. Bloom and F. J. Gilman, *Phys. Rev. D* **4**, 2901 (1971).
2. I. Niculescu et al., Phys. Rev. Lett. **85**, 1182 (2000); **85**, 1186 (2000); R. Ent, C.E. Keppel and I. Niculescu, Phys. Rev. D **62**, 073008 (2000).
3. N. Isgur and M. B. Wise, *Phys. Rev. D* **43**, 819 (1991); R. Lebed and N. Uraltsev, *Phys. Rev. D* **62**, 094011 (2000); E. C. Poggio, H. R. Quinn, and S. Weinberg, *Phys. Rev. D* **13**, 1958 (1976); R. Rapp, Pramana **60**, 675 (2003) [arXiv:hep-ph/0201101]; M. A. Shifman, arXiv:hep-ph/0009131.
4. M. A. DeWitt and S. Jeschonnek, 17^{th} Annual Hampton University Graduate Studies at the Continuous Electron Beam Accelerator Facility (HUGS at CEBAF), June 2002, World Scientific, in press.
5. C. S. Armstrong, R. Ent, C. E. Keppel, S. Liuti, G. Niculescu and I. Niculescu, Phys. Rev. D **63**, 094008 (2001).
6. A. DeRujula, H. Georgi, and H. D. Politzer, *Ann. Phys. (N.Y.)* **103** 315 (1977).
7. R. Ent, C. E. Keppel and I. Niculescu, Phys. Rev. D **64**, 038302 (2001).
8. N. Liyanage, private communication.
9. N. Isgur, Phys. Rev. D **59**, 034013 (1999).
10. N. Isgur, S. Jeschonnek, W. Melnitchouk, and J. W. Van Orden, *Phys. Rev. D 64*, 054005 (2001).
11. S. Jeschonnek and J. W. Van Orden, Phys. Rev. D. **65**, 094038 (2002).
12. M. W. Paris and V. R. Pandharipande, *Phys. Lett. B* **514** 361 (2001); M. W. Paris and V. R. Pandharipande, *Phys. Rev. C* **65**, 035203 (2002).
13. F. E. Close and Q. Zhao, Phys. Rev. D **66** (2002) 054001; Q. Zhao and F. E. Close, arXiv:hep-ph/0305017; F. E. Close and W. Melnitchouk, arXiv:hep-ph/0302013.
14. R. Barbieri, J. Ellis, M. K. Gaillard, and G. G. Ross, *Phys. Lett.* **64B** 171 (1976); R. Barbieri, J. Ellis, M. K. Gaillard, and G. G. Ross, *Nucl. Phys.* **B117** 50 (1976).
15. J. W. Van Orden and S. Jeschonnek, Eur. Phys. J. **A17**, 391 (2003).